EE640 Multivariable Control, Tutorial Sheet 1

7th Aug 2023

Q-1 Find rank of following matrices:

(a) $\begin{bmatrix} 3 & 2 \\ 3 & 2 \\ -6 & -4 \end{bmatrix}$ $\begin{bmatrix} 4 & 9 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 7 & 2 \\ 5 & 8 & 3 \\ 6 & 9 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \\ 2 & 3 & 4 \\ 7 & 8 & 9 \end{bmatrix}$ (d)

Q-2: Find eigenvalues and corresponding eigenvectors of

$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 2 \\ 0 & 0 & 9 \end{bmatrix}$ $\begin{bmatrix} 6 & -5 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & 2 \end{bmatrix}$

Q-3: Look up definition of nilpotent matrix, idempotent matrix kernel, image

Q-4: Is $b = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ in the image of $A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 3 & 2 \end{bmatrix}$: Yes/No. Why?

Q-5: Is $\begin{bmatrix} -2 & 6 \end{bmatrix}^T$ in the kernel of A in Q 4.

Q-6: Find a matrix A such that image A = kernel A.

(a) Why should A be square?
(b) Why should A be of row/col dimension even?
(c) Why should A be nilpotent?
(d) Why should A not be diagonalizable?, Why should all eigenvalues be 0?

Q-7 Check/verify rank nullity theorem for matrices above. (All.).

Q-8 Check whether the 3 vectors: 1st col. of A, 2nd col of A & vector b : of Question 4 are linearly independent or not. If not independent, then which one can be expressed as linear combination of other 2?

Tut Sheet 1 (contd) EE640, 7th Aug 2023

Q-9 : (a) Find ~~eigen~~ eigenvalue of matrix A below after checking if $\begin{bmatrix}1\\1\\1\end{bmatrix}$ is an eigenvector.

$$A = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 6 & -11 & 6\end{bmatrix}$$

. Same for $\begin{bmatrix}1\\2\\1\end{bmatrix}$ & $\begin{bmatrix}2\\-3\\1\end{bmatrix}$

Also check for $\begin{bmatrix}1\\3\\9\end{bmatrix}$ & $\begin{bmatrix}0\\0\\0\end{bmatrix}$

(First check if vector is an eigenvector & if yes, then find eigenvalue.)

Q-9(b) Now for $A = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 6 & -11 & 6\end{bmatrix}$, find

characteristic polynomial, eigenvalues & eigenvectors.
Find if 2 or 3 eigenvectors are - independent &
- mutually "orthogonal" &
(i.e. dot-product = 0)

Q-9c Same for A^T.

You can try using Scilab
(Scilab can be executed using browsers (for free, without login, even from mobile browsers) using http://cloud.scilab.in
(this is maintained by IIT Bombay!
& Scilab is just like Matlab (& better)