Tutorial Sheet 2, EE640 Multivariable Control, 4^{th} Sept 2023

<u>Q-1</u>: Suppose $P \in \mathbb{R}^{n \times q}$ is full row rank. (which means $\text{Im}\, P = \mathbb{R}^n$).

Suppose we are trying to find x such that $Px = y$ (y is given).

<u>the</u> least length x is $\quad x := P^T (PP^T)^{-1} y$

verify this for $P = [3 \;\; 4]$ & $y = 2$ & a brute force $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

<u>Q-2</u>: Explain parallel of Q-1 with solving for $u(t)$ for $p, q \in \mathbb{R}^n$ & $T > 0$ fixed (assumed arbitrarily)

& ~~u(t) =~~ $\cancel{B e^{(T-t)A^T}}$ $\quad u(t) := B^T e^{A^T (T-t)} P_T^{-1} v$

with $\displaystyle P_T := \int_0^T e^{A(T-t)} BB^T e^{A^T(T-t)} dt$

<u>Q-3</u>: Theorem 3.1, Hautus / Stoorvogel / Trentelman book: understand proof completely.

<u>Q-4</u>: Consider $\dot{x} = -3x + u$, $x(0) = 0$.

Suppose we desire $\quad x(T) = 2 \quad$ at $T_1 = 0.1$

and at $T_2 = 10$

- Explain why $P_{T_2} > P_{T_1}$
- Find $\displaystyle \int_0^T (u(\tau))^2 d\tau$ for each of the cases T_1, T_2.

<u>Q-5</u>: Consider $\dot{x} = Ax + bu$, $A = \begin{bmatrix} 2 & 3 \\ 0 & 6 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $x(0) = 0$.

- Suggest 2 different & independent directions q_1, q_2 which are <u>not</u> reachable for any time using any input u.
- Suggest 2 different & independent vectors q_3 & q_4 such that each of q_3 & q_4 <u>are</u> reachable for some input at some time.

<u>Q-6</u>: Let $A = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$. Obtain e^{At} by diagonalizing A.

- obtain e^{At} by $\mathcal{L}^{-1}(sI - A)^{-1}$

Q-7: Consider $A = \begin{bmatrix} 0 & 1 & & \\ & & 1 & \\ & & \ddots & 1 \\ a_0 & a_1 & \cdots & a_{n-1} \end{bmatrix}$. find $\det(sI-A)$.

Q-8: for $A = \begin{bmatrix} 0 & 1 & \\ & & 1 \\ 2 & 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, find F (of suitable size)

such that $\chi_{A+BF}(s) = \underbrace{2s^3 + 4s^2 + 7s + 11}_{d(s)}$ or has same roots as $d(s)$.

Q-9

This problem is for concluding the robustness of feedback controller w.r.t. changes in system pole and/or initial condition $x(0)$.

Consider $\dot{x} = 3x + u$ & $x(0) = 4$.
model

- Check input $u(t)$ that gives $x(t) = x(0)e^{-2t}$ (after control action)

  ($u = -5x$ would have sufficed: but obtain $u(t)$ explicitly :)

- Now apply $u(t)$ from above explicitly to $\dot{x} = 3x + u$ & $\mathcal{L}(\quad) = \mathcal{L}(\quad)$ to get

$$sX(s) - 3X(s) - x(0) = U(s).$$

thus $X(s) = \frac{1}{(s-3)}\left[\qquad\right]$ & check that the $u(t)$ indeed gives $x(0)e^{-2t}$.

Now use same $u(t)$ for actual system $\dot{x} = 3.1x + u$

and/or actual initial conditions $x(0) = 3.9$

Check if $x(t)$ is still $x(0)e^{-2t}$ or how different $x(t)$ could be due to the mismatch in system equations/initial condition.

Q-10: Consider single input system $\dot{x} = Ax + bu$, $b \in \mathbb{R}^n$ and let $\mathcal{C} := [b \; Ab \cdots A^{n-1}b]$ be invertible. Check $A[b \; Ab \cdots] = \mathcal{C}[\;?\;]$

Get $[\;?\;]$ for simple cases 3×3 & then $n \times n$.