Tutorial Sheet, EE640, Multivariable Control 23 Oct 2023

Q-1: Suppose $G(s) = \dfrac{5s+3}{s^6+100}$

give values of CB, CAB, CA^2B, CA^3B, CA^4B without calculating state space realization.

Q-2 Suppose $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $A = \begin{bmatrix} 6 & 7 & 8 \\ 9 & 0 & -1 \\ -2 & 3 & -1 \end{bmatrix}$ & $C = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 0 \end{bmatrix}$

By how much can observer order be reduced ?.

[i.e. Reduced from 3 to ?].

Q-3: Suppose $A = \begin{bmatrix} 0 & 1 \\ 9 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

Design an observer (full-order) such that error (i.e. $x - \hat{x}$) goes to zero as e^{-2t} (with repeated ~~roots~~ poles).

Q-4: Suppose $A = \begin{bmatrix} 0 & 1 \\ 9 & a \end{bmatrix}$, Find what values of a makes (A, B) uncontrollable & (A, C) unobservable, $B = C^T = e_2$ (check all 4 combinations).

Q-5: ~~State~~ & prove separation principle for pole-placement of control & observer poles ($B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$, $D = 0$).

Q-6: Suppose $G(s) = \dfrac{1}{s^5+6s^4+6}$, $\dfrac{s-1}{(s-1)^5}$, $\dfrac{s+1}{(s-1)^4}$ and $\dfrac{(s+1)^4}{(s-3)^2}$, & state (without calculating Markov params) the rank of the Hankel matrix

($G(s) = \underbrace{p(s)}_{\in \mathbb{R}[s]} + \dfrac{h_1}{s} + \dfrac{h_2}{s^2} + \dfrac{h_3}{s^3} + \cdots$ then only h_i are used to construct the Hankel matrix $H_{N,N} \in \mathbb{R}^{N \times N}$

Q-7: Suggest an $A \in \mathbb{R}^{n \times n}$ such that for any $b \in \mathbb{R}^n$, $b \neq 0$, (A, b) is controllable.