Tutorial 1 for EE640, 7^{th} Aug 2015.

For A, B of $\dot{x} = Ax + Bu$ ($A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n \times m}$)

F.A.E. (Following are equivalent).

1. System is controllable (i.e. for any $a, b \in \mathbb{R}^n$ there exist $u: \mathbb{R} \to \mathbb{R}^m$ and $T > 0$ such that $x(0) = a$ & $x(T) = b$).

2. $[B \;\; AB \;\; A^2B \; \ldots \; A^{n-1}B]$ is full row rank (i.e. has rank n).

3. $[A - \lambda I \;\; B]$ has rank n for each $\lambda \in \mathbb{C}$.

Proof of $2 \Leftrightarrow 3$ is not priority today.

Note: $A - \lambda I$ is already full row rank whenever λ is not an eigenvalue.

(Prove this).

So $[A - \lambda I \;\; B]$ is full row rank whenever λ is not an eigenvalue of A.

In other words,

If $[A - \lambda I \;\; B]$ loses rank for some $\lambda \in \mathbb{C}$ $\Rightarrow$ λ is an eigenvalue of A.

Thus ~~if~~ $\cancel{[B \; AA}$ $[B \;\; AB \;\; A^2B \; \ldots \; A^{n-1}B]$ is not full row rank

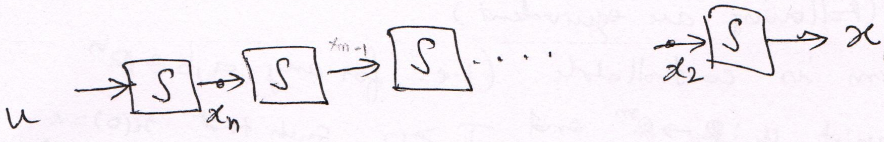
$\Updownarrow$

then are some eigenvalues λ such that $[A - \lambda I \;\; B]$ is not full row rank.

Call those λ as uncontrollable eigenvalues (of A)

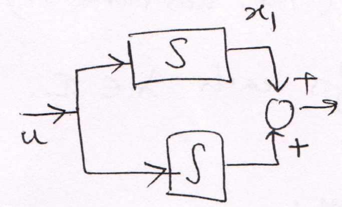
~~Draw~~ write A & B for

(1)



Prove controllability for arbitrary n.

(2)



write A, B such that

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \\ \end{bmatrix} u$$

Show that this system is uncontrollable.

$\int \equiv$ integrator.

3. Show that the characteristic polynomial of

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ & 0 & 1 & & 0 \\ & & \ddots & \ddots & \\ & & & & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{bmatrix}$$ is $s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$.

Prove that for $b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$, above A & b is controllable for any $a_0, a_1, \ldots a_{n-1} \in \mathbb{R}$.

4. For each pair below, decide controllable or uncontrollable and if uncontrollable, then ~~for~~ classify eigenvalues of A into controllable & uncontrollable eigenvalue.

(a) $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $b_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $b_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (Check (A, b_1) & (A, b_2)).

(b) $\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$, $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, $\begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$, $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ (for arbitrary $\lambda, b_1, b_2 \in \mathbb{R}$)

(c) $\begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$, $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, $\beta \neq 0$ (and arbitrary α, β, b_1, b_2).