Tutorial Sheet 2 (EE640) Multivariable Control: 6^{th} Aug 2024.

1. Show that if b is an eigenvector of A, $A \in \mathbb{R}^{n\times n}$, $n \geq 2$, then A, b is uncontrollable.

2. Why $n \geq 2$ is needed?

3. What is a possible converse of Q-1? True or false? (There can be multiple converses.).

4. For any $A \in \mathbb{R}^{3\times 3}$, at least one $b \neq 0$ would exist such that (A, b) is uncontrollable. Why?
(why should size be 3×3?

5. Suggest a 2×2 A such that for any nonzero $b \in \mathbb{R}^2$, A, b is controllable.

6. To show for the 1×2 case that
$Pv = w$, with $P \in \mathbb{R}^{1\times 2}$, P full row rank,
that $v := P^T(PP^T)^{-1}w$ is least length soln,
Consider $\begin{bmatrix} 1 & a \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 1$ (solve for $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$.).
Check that $P^T(PP^T)^{-1}w$ is least length (for arbitrary $a \in \mathbb{R}$).

7. In what way is
$$P_T := \int_0^T e^{A\tau} BB^T e^{A^T\tau} d\tau$$
an increasing function of T? (check for $A \in \mathbb{R}$ first.).
(A matrix P_T is called increasing fn of T if $P_{T_1} - P_{T_2} \geq 0$ for $T_1 \geq T_2$, (for symmetric matrices only).
and $Q \geq 0 :\iff v^T Q v \geq 0 \ \forall v \in \mathbb{R}^n$.