Tutorial Sheet EE640 Multivariable Control, 7th Oct 2024
Attempt/Read all questions before tutorial.

Q-1: Use $(I-P)^{-1} = I + P + P^2 + P^3 + \cdots$ (as formal power series (i.e. without bothering about convergence)) to get

$$\frac{CB}{s} + \frac{CAB}{s^2} + \frac{CA^2B}{s^3} + \cdots = C(sI-A)^{-1}B \qquad \left(sI - A = s\left(I - \frac{A}{s}\right)\right)$$

Q-2: Just like for a biproper transfer fn, d can be got as just ratio of leading terms, check that Markov parameters h_i of $G(s) = \frac{6s^2+3s+2}{s^2-9s-1} \to h_0 = 6$, $h_1 = \lim_{s\to\infty} s\,(G(s)-6)$

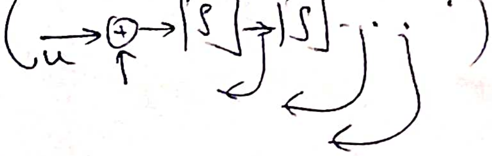
$$h_2 = \lim_{s\to\infty} s\,\big(s(G(s)-6) - h_1\big)$$

(This is same as long division). ⋮

Q-3 check that the Markov parameters do not change with $A \to T^{-1}AT$, $B \to T^{-1}B$, $C \to CT$, $D \to D$.

Q-4: Show that $(sI-A)^{-1}$ is strictly proper and where are the rel. degree 1 entries & rel. degree ≥ 2 entries (in $(sI-A)^{-1}$)?

Q-5: Consider controller canonical form for A & $b = e_n$. Obtain (by adjugate/co-factor method) just $(sI-A)^{-1}b$ as $\frac{\begin{bmatrix} ? \end{bmatrix}}{\det(sI-A)}$.
Compare relative degrees of $u \to x_1$, $\to x_2$, $\to x_3$ w.r.t. the block diagram of controller canonical form x_i ? 2



Q-6: Suppose relative degree of a strictly proper $G(s) = 7$, then deduce $h_i = 0$ for $i = 0, 1, \ldots$? What about converse? (Markov parameters). Also obtain how many initial derivatives $= 0$ for step response.

Q-7 Prove $\Lambda_{un}^{(A,B)} = \bigcap_F \sigma(A+BF)$ (where F is varied over $F \in \mathbb{R}^{m\times n}$)
$B \in \mathbb{R}^{n \times m}$. $\sigma \equiv$ spectrum $\equiv$ set of eigenvalues.
What if an eigenvalue λ is repeated & "both" controllable & uncontrollable?
(Q7: For simplicity, assume distinct eigenvalues of A & real eigenvalues).

Q-8: Let $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $C = [2 \quad 3]$. $x(0) =$ unknown.
Check observability O matrix & its inverse: O^{-1}.
Use O^{-1} to get $x(0)$ from $\begin{bmatrix} y(0) \\ \dot{y}(0) \end{bmatrix}$

Q-9: Assume (only in this exercise) that B is <u>not</u> full column rank.
Show that $B \in \mathbb{R}^{n\times m}$, & suppose B has rank only r & $r < m$.
then $k_m = 0$, $k_{m-1} = 0$, .. $k_{m-r+1} = 0$ ($k_i \equiv$ controllability indices).
(kappa).

~~Q-10~~: - Thus prove that $k_i \geq 1 \Leftrightarrow B$ is full column rank.
(for all i)

Q-10: Let B be f.c.r. and (A, B) controllable.
For each condition below, construct (if possible/exists)
A, B satisfy that condition

(a) all $k_i = 2$, $m = 3$, $n = ?$
(b) $k_1 = 2$, $k_2 = 1$, A is cyclic.
(c) $(k_1, k_2, k_3) = (3, 2, 1)$, A is <u>not cyclic</u>.
(d) —"— , A is of size 5
(e) —"— , A is of size 7.
(f) $(k_1, k_2, k_3) = (1, 1, 1)$, A is of size ?

Q-11: Consider the Lyapunov operator $L_A : P \longrightarrow A^T P + PA$
$L_A : \mathbb{R}^{n\times n} \rightarrow \mathbb{R}^{n\times n}$ with $P, A \in \mathbb{R}^{n\times n}$.
Assume A has all real & distinct eigenvalues and for each eigenvalue λ_i, let v_i^L, v_i^R denote left & right eigenvectors of A.

(a) Verify that
$P_{ij} := v_i^L (v_j^L)^T$ ~~$v_i^L (v_j^R)^T =: P_{ij}$~~ is an eigenvector
is an eigenvector.
(b) what is corresponding eigenvalue?
(c) Is # of eigenvalues = dimension of $\mathbb{R}^{n\times n}$?
(d) Find an A → singular / nonsingular / Hurwitz such that L_A is a singular operator, & find matrix P in its nullspace.
(e) Assume $P = v_i^L (v_j^R)^T + v_j^R (v_i^L)^T$ to get P symmetric.
& check $A^T P + PA$ also is symmetric.
(f) What is the dimension of symmetric $n\times n$ matrices? Do eigenvalue count add up?

(If you prefer, for simplicity, assume A is symmetric for all Q-11 a—f (thus <u>not</u> worrying about left/right eigenvectors.)