Construction of periodic timetables on a suburban rail network-case study from Mumbai

Soumya Dutta [a,1], Narayan Rangaraj [b,2], Madhu Belur [a,3],
Shashank Dangayach [c,4], Karuna Singh [d,5]

[a] Department of Electrical Engineering, Indian Institute of Technology Bombay
[b] Department of Industrial Engineering and Operations Research, Indian Institute of Technology Bombay
[c] ITS Planners and Engineers Pvt. Ltd. (ITSPE) [d] Central Railway, Indian Railways
[1] Email: soumya.besuee@gmail.com [2] Email: narayan.rangaraj@iitb.ac.in
[3] Email: belur@ee.iitb.ac.in [4] Email: shashank.dangayach@gmail.com [5] Email: kn.singh@rediffmail.com

Abstract
In this paper we design a feasible schedule of suburban trains in an urban transport rail network. The services that will be run on the network are decided earlier, from line planning activities. The inputs to our decision are the hourly demands of different services in the network. With these inputs we aim to create a cyclic timetable that can be physically implemented on the network. For this we use an extension of the Periodic Event Scheduling Problem (PESP) framework. A type of constraint, the rake-linkage constraints at terminals, has been introduced in the model. Also, in our system, the platform availability is a serious constraint in some terminals. These have been modeled in an implicit manner.
The entire problem has been modeled as a MILP and solved using Gurobi. The number of rakes required has been computed as well.
The method seems to be applicable generally to suburban rail networks of the kind which are operated in India. It would now permit more experimentation in timetabling options and it is hoped that it leads to more integration of line planning options with timetabling, in future.

Keywords
Cyclic Timetables, Periodic Event Scheduling Problem, Suburban Rail network

1 Introduction

Suburban railway networks form an integral part of the transport system in some of the major cities of India. Although these networks in Mumbai, Kolkata and Chennai together account for a mere 7% of the total track length of Indian Railways, they contribute more than 50 % of the total number of passengers.

Planning forms a large number of activities such as:

- **Line Planning**: This is concerned with working out the different railway routes that should be provided. With the different infrastructural constraints and passenger demands as input this consists of deciding the number and types of services that should be provided to the passengers
- **Timetabling**: Once the number and type of different services have been decided, the planner must schedule them with the knowledge of the infrastructural constraints at the terminals. Considerations of safety and quality of services provided are of utmost importance to the planner in this step
- **Rake-Linking**: This step is extremely important from the economic point of view. In this step, the timetable is realized with minimum rake-requirement. This problem is often considered while scheduling the trains in the previous step
- **Crew scheduling**: Assigning manpower for properly running these services is part of crew scheduling.

All the stages of planning are interlinked. In this paper we try to provide a framework for Timetabling and Rake-linking.

2 Literature Survey

The creation of periodic timetables appeared as a natural extension and application of the Periodic Event Scheduling Problem (PESP) introduced by Serafini and Ukovich (Serafini and Ukovich (1989)). An extensive coverage of the different requirements of a railway timetable and the way of handling them efficiently have been presented in Peeters in his thesis Cyclic Railway Timetable Optimization (Peeters (2003)). In (Peeters (2003)) most of the different types of constraints such as headway, traversal time, frequency, dwell time constraints have been described in detail. Symmetry constraints are described in detail in (Liebchen (2004)). The linking of the train services at the terminals is an issue which has been discussed at length in the paper (Kroon et al. (2013)). This paper discusses the importance of keeping the service links open at the terminals for optimality.

A timetable for the Berlin subway has been designed by Liebchen using two techniques, namely, the Max T-PESP and a heuristic method called Cut-Heuristic (Liebchen (2008)). A case study with timetable construction for the same network is also included in (Liebchen and Möhring (2002)). A hierarchical decomposition method for solving the PESP problem has been discussed in (Herrigel et al. (2013)). Such a method has then been used to prepare timetables for central Switzerland (Herrigel et al. (2013)). Case studies of preparing timetables by this PESP framework have also been reported in (Kroon et al. (2009)). The national timetable has been designed by two separate tools-CADANS and STATIONS.

The PESP problem is known to be hard to solve. It has been shown to be NP-complete in (Serafini and Ukovich (1989)). Several techniques such as Integral Cycle basis have been introduced for solving the PESP in (Liebchen and Peeeters (2009)). Another heuristic method of solving the problem has been shown in (Caprara et al. (2002)).

However the PESP approach has certain drawbacks in what it is able to model. Its shortcomings were discussed in (Liebchen and Möhring (2007)). Symmetry, balanced reduction of services is shown to be beyond the scope of pure PESP constraints.

3 Objective

Timetabling in India is presently done manually with some computer based visualization and decision support.. The planner schedules trains based on track availability and historical demand patterns. This is an iterative procedure which starts by modifying the already existing timetable. The existing approach completely ignores any sort of optimization that one might use while designing such timetables. The Periodic Event Scheduling Problem (PESP) proposed by (Serafini and Ukovich (1989)) led to generating timetables by solving Mixed Integer Linear Programs (MILP). We have used a similar approach as proposed by (Peeters (2003)) for modeling the timetabling problem.

The timetable which we wish to generate is a cyclic one, in that it repeats after a specific period of time which in our case we have taken to be 1 hour. Cyclic Timetables are also more acceptable to the railway authorities than aperiodic ones as activities such as rake assignments, crew-rostering become easier. Cyclic Timetables are also easier to comprehend for the passenger because of their concise presentation. At every station the arrival and departure (in minutes past the hour) of the trains are all that is specified in such a timetable. Periodicity ensures that the same timetable repeats after every one hour. Estimating the waiting time at stations or the amount of time required to reach the destination becomes much easier for the commuter.

However it must be mentioned that Cyclic Timetables may be difficult to implement practically. Planning for a single time period suffices for the entire day if the timetable is cyclic. This requires identical situations at the end of every time period. This requirement may not be fulfilled, which will then lead to planning activities for the entire day. Cyclic timetables are most beneficial when they are implemented during the peak hours when the demand is much higher. However we can propose a cyclic timetable, with lower frequencies during off-peak hours as well and integrate the two timetables as a single one as another exercise.

We also consider the problem of realizing the cyclic timetable with minimum number of rakes. The entire problem is modeled as a Cyclic Railway Timetabling Problem (CRTP) which has been implemented extensively in Europe. Apart from these considerations, station capacity constraints have also been taken into account.

4 Problem Formulation

We now describe the way we have modeled the Timetabling Problem. The entire timetabling exercise consists of assigning a time to the arrival or departure of trains at the different stations. Since we are considering a cyclic timetable, all these events are periodic events with a time period of *T*. As shown by (Peeters (2003)) the constraints in the MILP linking these periodic events must also be periodic in nature. Periodicity of constraints is ensured by *modulo T* operations. The formulation of the CRTP is briefly described below.

4.1 Decision Variables

- **d[i]**: Departure event of a train
- **a[i]**: Arrival event of a train
- **p[i][j]**: Integer variables to denote crossing of the hour mark between i^{th} and j^{th} event

- **X[i][j]**: Binary variables to denote linkage of an arrival and departure event

4.2 Objective Function

The MILP we have designed has no objective function.

4.3 Constraints

Headway Constraints
Trains leaving from a particular station using the same set of tracks must maintain a certain minimum distance among themselves for safety (Peeters (2003)). However we note that headway in both the situations below is 3 minutes:-

- Trains leaving at 8:01 and 7:58
- Trains leaving at 8:01 and 8:04

Let the minimum headway distance between two trains sharing a particular track be *m* minutes. Generalizing the constraint we define headway constraints as:

$$\begin{aligned} d[j] - d[i] + Tp[j,i] &\geqslant m \\ d[j] - d[i] + Tp[j,i] &\leqslant T - m \end{aligned} \tag{1}$$

Traversal Constraints
In our model we have considered the traversal time to be constant between two pairs of stations. Considering the traversal time between stations *i* and *j* to be t_{ij} the periodic traversal constraints can be written as (Peeters (2003)):

$$a[i] - d[j] + Tp[i,j] = t_{ij} \tag{2}$$

Dwell time Constraints
Trains must stop at stations for some time for passengers to get in and come out. These are modeled by the dwell time constraints. However trains cannot wait in the stations for too long as this would introduce delays within the system. Let *l* and *u* denote the minimum and the maximum time for which trains stop at any station. The periodic dwell time constraints can be written as (Peeters (2003)):

$$\begin{aligned} d[j] - d[i] + Tp[j,i] &\geqslant l \\ d[j] - d[i] + Tp[j,i] &\leqslant u \end{aligned} \tag{3}$$

Frequency Constraints
The idea behind introducing these constraints is to evenly distribute the departure events of the trains from the terminal stations in an hour. Suppose the number of services between two stations *i* and *j* are N_{ij}. Then the commuter must get a train every $\frac{T}{N_{ij}}$ minutes, where *T* is the time period. However such equality constraints might lead to infeasibility of the MILP. Thus we relax the constraints slightly to get periodic frequency constraints (Peeters

(2003)):

$$\begin{aligned} d[j] - d[i] + Tp[j,i] &\geqslant \frac{T}{N_{ij}} - \delta \\ d[j] - d[i] + Tp[j,i] &\leqslant T - (\frac{T}{N_{ij}} + \delta) \end{aligned} \quad (4)$$

In the above equation δ is the relaxation of the hardness of the equality frequency constraints.

Symmetry constraints
As has been mentioned in (Liebchen and Möhring (2007)), symmetry constraints are very important for any periodic timetabling problem formulation. However these constraints cannot be modeled by pure PESP type constraints.

We now describe the way we have included symmetry constraints in our model. Suppose we have n types of services in a particular network. For each of these service types we have the same number of services in both directions. Let us consider the i^{th} service in the up direction and the j^{th} service in the down direction. Let the arrival time of the two services at a particular station on the route be given by a_i and a_j respectively. Then we include symmetry constraints in the following way:

$$a_i + a_j = T \quad (5)$$

We know that departure of services of the same type are already constrained by frequency constraints. Making the sum of arrival times of any one pair of up-down service at any station T, will make the sum of all the other pairs to be very close to T.

Turnaround constraints
These constraints are important in all the terminal stations. All incoming and outgoing services have to be linked at all terminal stations. As suggested in (Kroon et al. (2013)), we do not fix these linkages manually. Rather we make use of the binary variables *X[i,j]*, to denote the connections between incoming and outgoing services. If two services *i* and *j* are linked at any given terminal then *X[i,j]* = 1 else *X[i,j]* = 0. We first ensure that all incoming services are linked with at least one outgoing service. If *i* denotes an incoming service,

$$\sum_j X[i,j] = 1 \quad \text{for all terminal stations} \quad (6)$$

where *j* denotes any service that may be leaving a particular station.
Similarly we must ensure that every outgoing train is linked with only one incoming train. Therefore,

$$\sum_i X[i,j] = 1 \quad \text{for all terminal stations} \quad (7)$$

With the linkages decided by (6) and (7), we constrain the amount of time an incoming train waits at a terminal before leaving the station. However while including these constraints we do not know the values of *X[i,j]* for arbitrary *i* and *j*. We include the periodic turnaround constraints as:

$$\begin{aligned} d[j] - d[i] + Tp[j,i] &\geqslant l * X[j,i] \\ d[j] - d[i] + Tp[j,i] &\leqslant T + u_s - T * X[j,i] \end{aligned} \quad (8)$$

u_s denotes the maximum time that a train can wait at a particular terminal station s, while l denotes the minimum time a train takes to turnaround at any station. For un-linked services (8) does not play any role.
These constraints can also be used to model capacity constraints in an implicit manner. For stations with smaller capacity, u_s needs to be made smaller. Therefore these constraints can also be called *Platform Constraints*. It has been shown in (Kroon et al. (2013)) that keeping the rake-linkages flexible enables one to implement the same timetable with fewer rakes. Thus, these constraints can also be called *Rake-linkage constraints*. To the best of our knowledge this is the first attempt at modeling turnaround and platform capacity constraints together as one group.

Summary of the model
Thus we have seen that headway, traversal, dwell-time, frequency constraints can be modeled by pure PESP type constraints. For adding symmetry constraints we included simple additive constraints between a pair of up-down services. Frequency constraints ensure symmetry between all the services of the same type. However we require some *Assignment variables* which in conjunction with some PESP type constraints give us the complete set of *Turnaround constraints.*

4.4 Scheme for rake-linkages and counting number of rakes

We propose a scheme for computing rake-linkages and counting the number of rakes required for realizing the timetable. We have taken the number of rakes required to increase by one if

- The arrival time of a train is less than its departure time.
- The train takes more than *T* to complete its journey.

Let us consider *T* to be 60 minutes. Both of the points above correspond to the hour mark being crossed while the train is still to reach its journey. The justification of such a algorithm is explained by the following simple example

Suppose a particular train goes from station A to B. Let us consider it leaves A at 0815 hrs. Let us consider the following cases

- It reaches B at 0840 hrs, in which case it might be back in A by 0910 hrs
- It reaches B at 0913 hrs, in which case the same rake cannot service the train that will leave around 0915 hrs from A
- It reaches B after 0915 hrs, in which case the same rake cannot service the train that will leave around 0915 hrs from A

In the last two cases we need to have extra rakes introduced into the system at station A for maintaining periodicity of the timetable.

As we have included the turnaround time for any terminating train, we must have a originating train 3 to 10 minutes later. In this way rake-linkages are done.

5 Description of the Case Study

In this section we briefly discuss the network and the model used for our analysis. The suburban network in Mumbai consists of mainly three parts-the Central line, the Western line and the Harbour-Transharbour line. We have selected the Harbour-Transharbour line for our analysis. A network diagram of the network with the major stations is shown in Fig 1 for reference.
All the stations except Turbhe have trains terminating in them. Of all these stations siding lines are available near Vashi, Belapur, Turbhe and Panvel. It is worthwhile to note that all siding lines are towards one side of the network which poses problems when the transition from the periodic to aperiodic timetable is made.

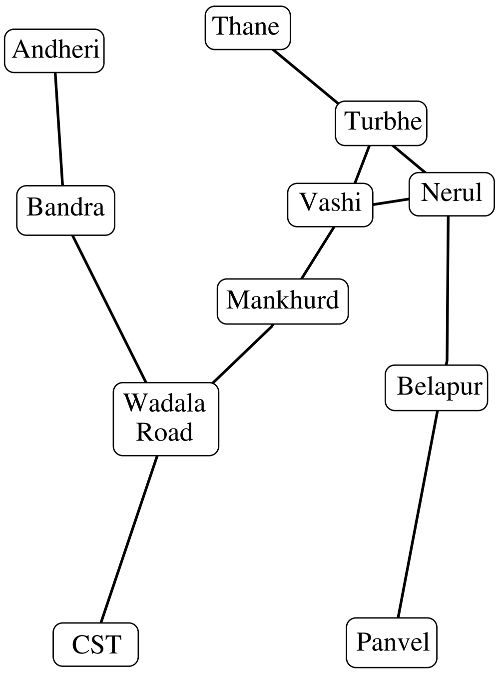

Figure 1: A schematic of the suburban network

5.1 Headways

Headway has to be specified for trains sharing a particular length of track. For our network we have taken a headway of 3 minutes between any such train pair.

5.2 Dwell Times

A dwell time of a minimum of 30 seconds and maximum of one minute has been kept at all the stations.

5.3 Traversal Times

The minimum traversal times between the different stations have been taken according to Table 1:

Table 1: Traversal Times

Origin Station	Destination Station	Traversal Time in minutes
CST	Wadala Road	17
Wadala Road	Mankhurd	20
Mankhurd	Vashi	7
Vashi	Nerul	8
Nerul	Belapur	5
Belapur	Panvel	16
Wadala Road	Bandra	9
Bandra	Andheri	14
Vashi	Turbhe	8
Nerul	Turbhe	11
Turbhe	Thane	20

5.4 Frequency constraints

The demanded frequency in the section is shown in Table 2 along with the existing frequency. The number of services shown in the table are for a period of 3 hours (during morning peak and evening peak). We can see from the present scenario that timetables are roughly periodic, while we try to make the timetable completely periodic.

Table 2: Comparison of demanded frequency

Origin Station	Destination Station	Present Frequency	Demanded Frequency
CST	Panvel	13	15
CST	Belapur	7	9
CST	Vashi	6	6
CST	Mankhurd	2	0
CST	Bandra	7	6
CST	Andheri	7	9
Wadala Road	Panvel	3	6
Wadala Road	Belapur	1	3
Wadala Road	Vashi	0	3
Panvel	Andheri	2	3
Thane	Panvel	4	6
Thane	Nerul	7	9
Thane	Vashi	11	15
CST	Chembur	1	0

The frequency of all the reverse services are the same as the ones in the table. Inclusion of frequency constraints in the Integer Program is straightforward.

5.5 Symmetry constraints

Adding symmetry constraints to the model is straightforward. We provide a simple example below for the CST-Panvel service.

As can be seen from Table 2, we have 5 services of this type every hour. We consider the arrival event of any one service in up and down direction each at the station Vashi. Let these events be a^{up}_{vashi} and a^{down}_{vashi}.

$$a^{up}_{vashi} + a^{down}_{vashi} = 60 \tag{9}$$

(9) adds symmetry constraints between all the 5 services of type CST-Panvel.

5.6 Turnaround constraints

The number of platforms in CST are 2. Due to the shortage of the platforms, the turnaround time is kept between 3 and 5 minutes at CST. At all the other stations it is kept between 3 and 10 minutes.

5.7 Integer Variables

We have two types of integer variables in our MILP formulation.

- **p[i][j]**: Present in all the PESP type constraints in the model
- **X[i][j]**: Binary variables used for linkages at the terminals

In the network that we have considered, the two events linked by PESP constraints cannot be separated by a time interval in which the hour marks is crossed twice. For this reason in our case we modify the integer variables **p[i][j]** and make them binary as well. This reduces the search space of the MILP considerably.

6 Results

The network is described by nodes and edges. Nodes refer to stations in the network, while edges refer to direct connections between two stations. Since our objective is to generalize the entire timetabling exercise, we need to work out the paths between two stations just by getting the distances between each consecutive pair of stations in the network. We use the shortest-path algorithm to actually work out the routes of all the trains.

The problem consists of approximately 200000 variables which is then being solved in an Intel Xeon server with 128 GB RAM and 20 cores using Gurobi. The computation time for the timetable is shown in Table (3). The output of the MILP is a feasible timetable that respects headway, frequency, dwell time and traversal constraints. The turnaround constraints that we have included result in a timetable which satisfies station capacity constraints. In the table turn-time refers to the turnaround time of the trains at stations other than CST.

We note from Table (3) that including symmetry reduces computation time. It is worth noting that the turnaround time is something that needs to be decided carefully. As turnaround

Table 3: Comparison of demanded frequency

	turn-time=10mins No Symmetry	**turn-time=10mins with Symmetry**	**turn-time=8mins**
Simplex iterations	2104048	1846794	229464598
Branch-and-cut nodes	35635	27409	10740969
Solved in	49.62s	42.87s	Not solved

time has been included by using some non-PESP constraints changing them might lead to the problem not being solved in reasonable time.

Now that we get a feasible timetable, we consider the problem of rake-linking. Using the scheme in 4.4, the number of rakes required for operating the timetable is 53. The rake linkages help us create the station occupancy charts. These charts also allow us to check the practical aspect of the timetable with respect to station capacity. As CST is the most constrained terminal in our case study we include the station capacity chart and platform allocated at CST for reference.

Table 4: CST platform occupancy

Arriving Service	**Arrival Time**	**Departure time**	**Departing Service**	**Platform**
Vashi-CST	0.5	5.5	CST-Panvel	1
Andheri-CST	4.5	8.5	CST-Belapur	2
Panvel-CST	7.5	11.5	CST-Bandra	1
Bandra-CST	10.5	15.5	CST-Panvel	2
Belapur-CST	15.5	20.5	CST-Andheri	1
Panvel-CST	21.5	24.5	CST-Panvel	2
Andheri-CST	25	30	CST-Vashi	1
Panvel-CST	30	33	CST-Belapur	2
Vashi-CST	33	36	CST-Panvel	1
Belapur-CST	36	39	CST-Andheri	2
Bandra-CST	39	42	CST-Bandra	1
Panvel-CST	45	50	CST-Belapur	1
Andheri-CST	48	53	CST-Panvel	2
Belapur-CST	54	57	CST-Vashi	1
Panvel-CST	57	0	CST-Andheri	2

As seen from Table (4), our timetable satisfies all the platform constraints at CST. This can be concluded from the fact that the minimum interval between arrival and departure of trains at this terminal is 2 minutes, which is acceptable.

7 Conclusion

In this paper we have considered a part of the Mumbai Suburban Network and applied the PESP framework based Cyclic Timetable formulation. We have calculated the number of rakes and also prepared rake linkage charts for the timetable.

However, feasibility of such a timetable in practice requires it to be integrated with the existing off-peak hour timetable. Wadala Road is the station that has an issue in our case study in this respect. Since we do not have any siding lines in Wadala Road and the nearest car shed is at Vashi, the number of services at Wadala Road has to be increased in the existing off-peak hour timetable as well. Such a change needs to be introduced at least one hour before the periodic timetable comes into force.

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