Three phase system

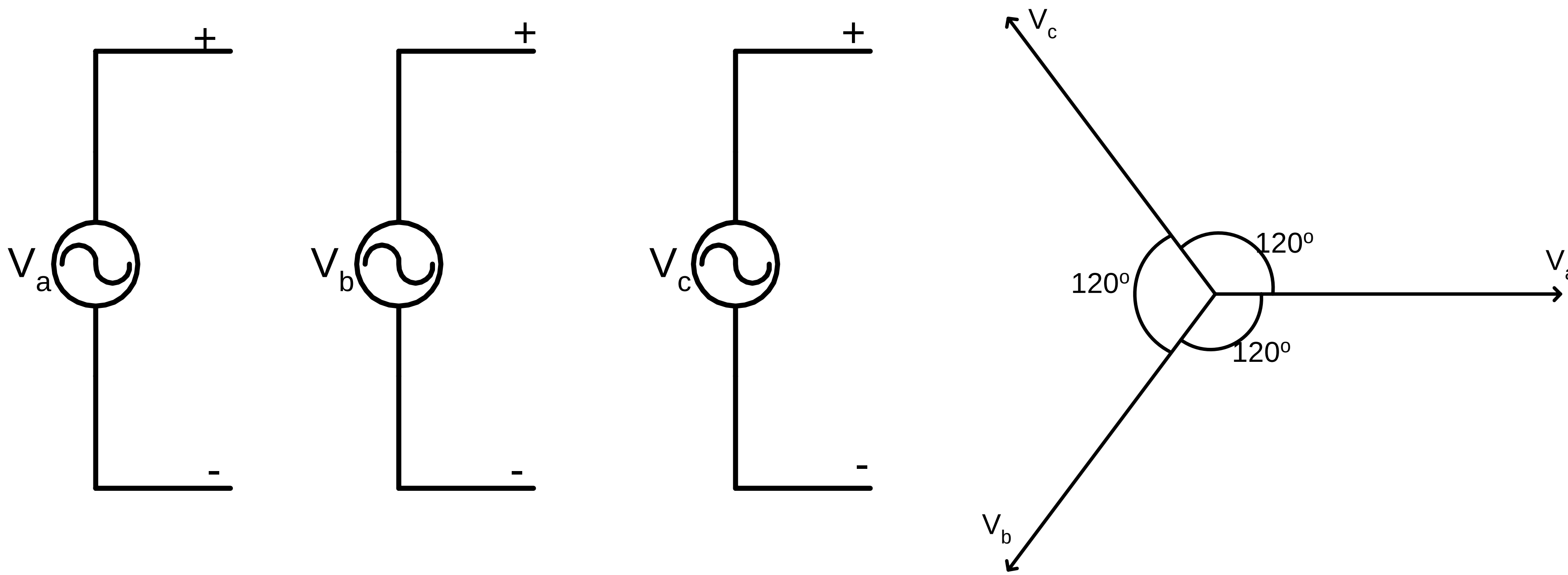
Motivation:

- Optimum utilization of the ac generator if three phase voltages are generated

- Improvement in transmission efficiency

- Torque development in ac motors

Creating a three phase system of voltages from three single phase sources

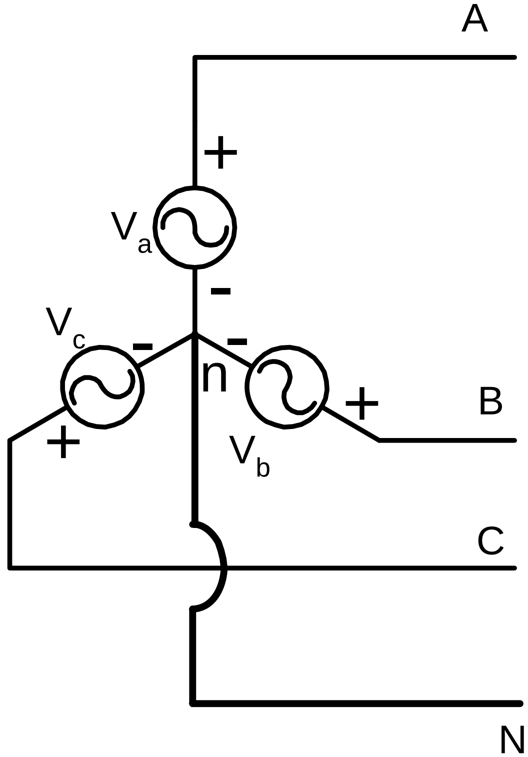


$$v_a = \sqrt{2}V_{ph}\sin\omega t \qquad v_b = \sqrt{2}V_{ph}\sin\left(\omega t - 120^0\right)$$

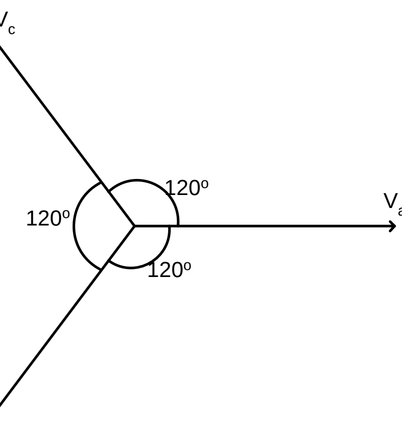
$$v_c = \sqrt{2}V_{ph}\sin\left(\omega t - 240^0\right) = \sqrt{2}V_{ph}\sin\left(\omega t + 120^0\right)$$

Two ways of connection possible:

- Star connection
- Delta connection



- Caution: Polarity of connections



$$\bar{V}_{AB} = \bar{V}_{An} - \bar{V}_{Bn}$$

$$= \bar{V}_{an} - \bar{V}_{bn}$$

$$= V_{ph}\angle 0^0 - V_{ph}\angle -120^0$$

$$= V_{ph} + \frac{V_{ph}}{2} + j\frac{\sqrt{3}}{2}V_{ph}$$

v_{AB}, v_{BC}, v_{CA} : Line to line voltages

v_{An}, v_{Bn}, v_{Cn} : Phase voltages

$$\bar{V}_{AB} = V_{ph} + \frac{V_{ph}}{2} + j\frac{\sqrt{3}}{2}V_{ph}$$

$$= \left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right)V_{ph}$$

$$= \sqrt{\frac{9}{4} + \frac{3}{4}}V_{ph}\angle\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \sqrt{3}V_{ph}\angle 30^0$$

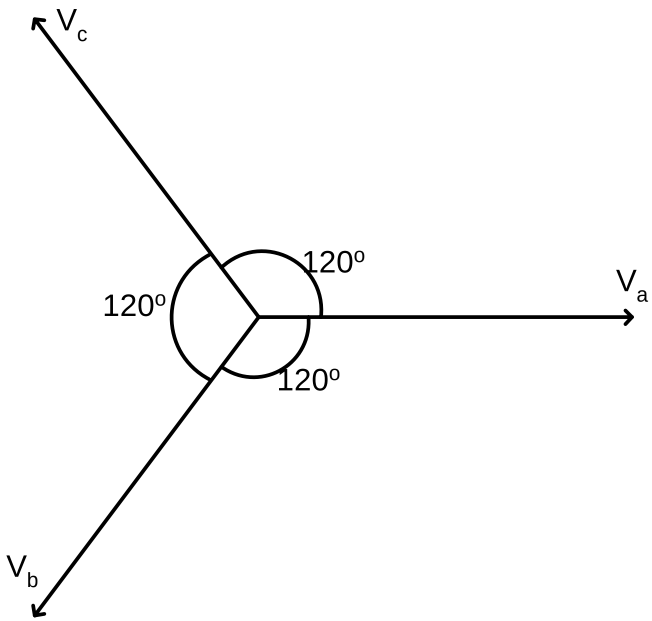
$$\bar{V}_{BC} = \bar{V}_{Bn} - \bar{V}_{Cn}$$

$$= V_{ph}\angle -120^0 - V_{ph}\angle 120^0$$

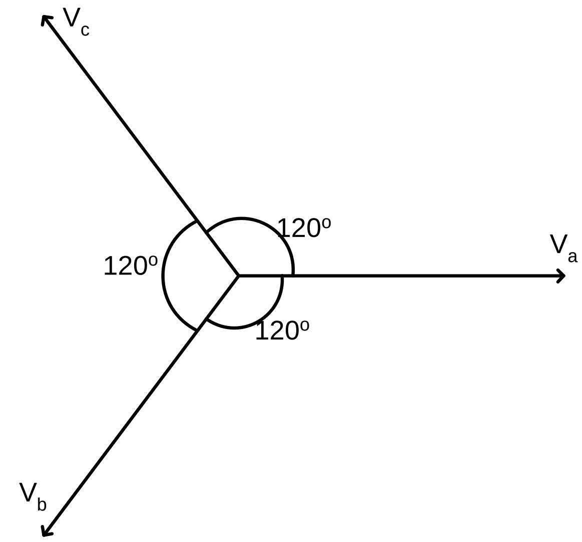
$$= -\frac{1}{2}V_{ph} - j\frac{\sqrt{3}}{2}V_{ph} + \frac{1}{2}V_{ph} - j\frac{\sqrt{3}}{2}V_{ph}$$

$$= 0 - j\sqrt{3}V_{ph}$$

$$= \sqrt{3}V_{ph}\angle -90^0$$



$$\bar{V}_{CA} = \bar{V}_{Cn} - \bar{V}_{An}$$



$$= V_{ph}\angle 120^0 - V_{ph}\angle 0^0$$

$$= -\frac{1}{2}V_{ph} + j\frac{\sqrt{3}}{2}V_{ph} - V_{ph} - j0$$

$$= -\frac{3}{2}V_{ph} + j\frac{\sqrt{3}}{2}V_{ph}$$

$$= \sqrt{\frac{9}{4}+\frac{3}{4}}V_{ph}\angle \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$= \sqrt{3}V_{ph}\angle 150^0$$

$$\bar{V}_{AB} = \sqrt{3}V_{ph}\angle 30^0$$

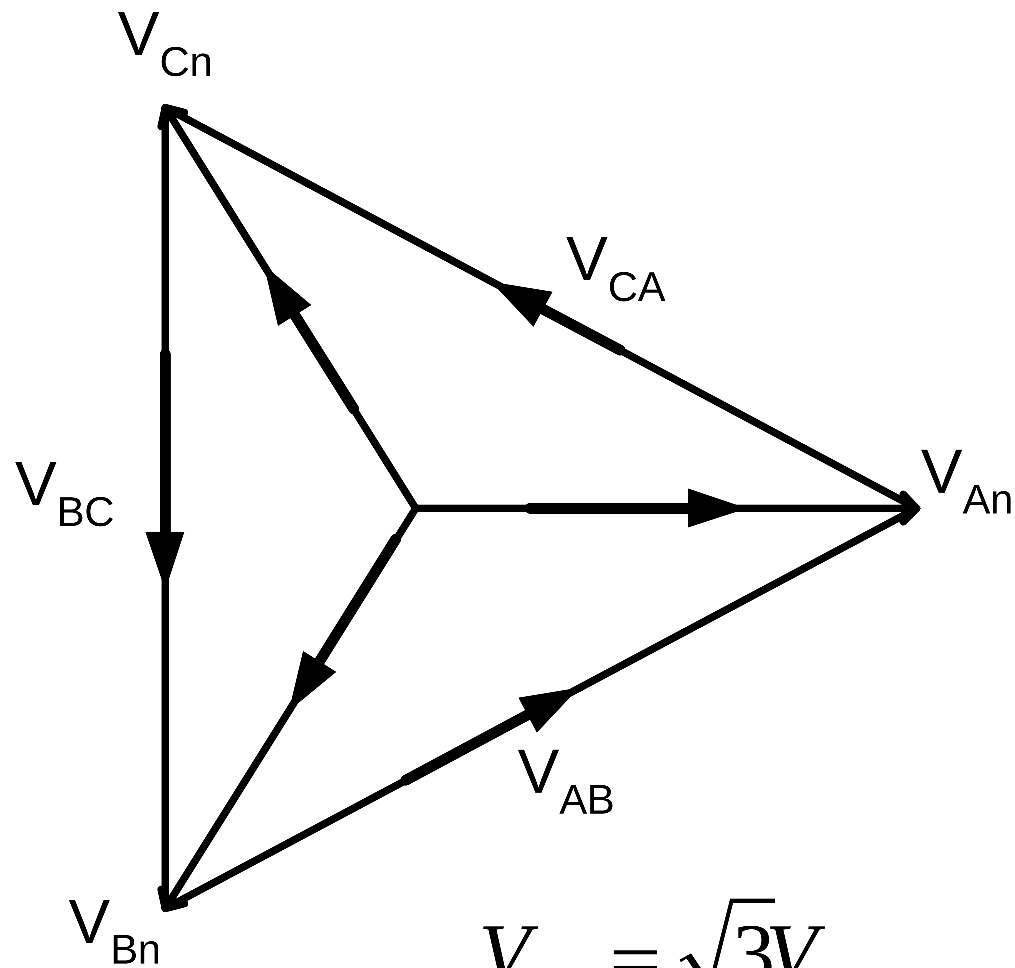
$$\bar{V}_{BC} = \sqrt{3}V_{ph}\angle -90^0$$

$$\bar{V}_{CA} = \sqrt{3}V_{ph}\angle 150^0$$

$$\bar{V}_{An} = V_{ph}\angle 0^0$$

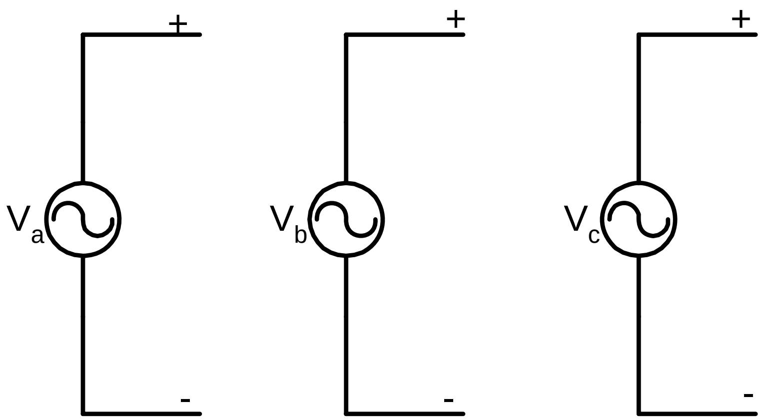
$$\bar{V}_{Bn} = V_{ph}\angle -120^0$$

$$\bar{V}_{Cn} = V_{ph}\angle -240^0$$



$$V_{LL} = \sqrt{3}V_{ph}$$

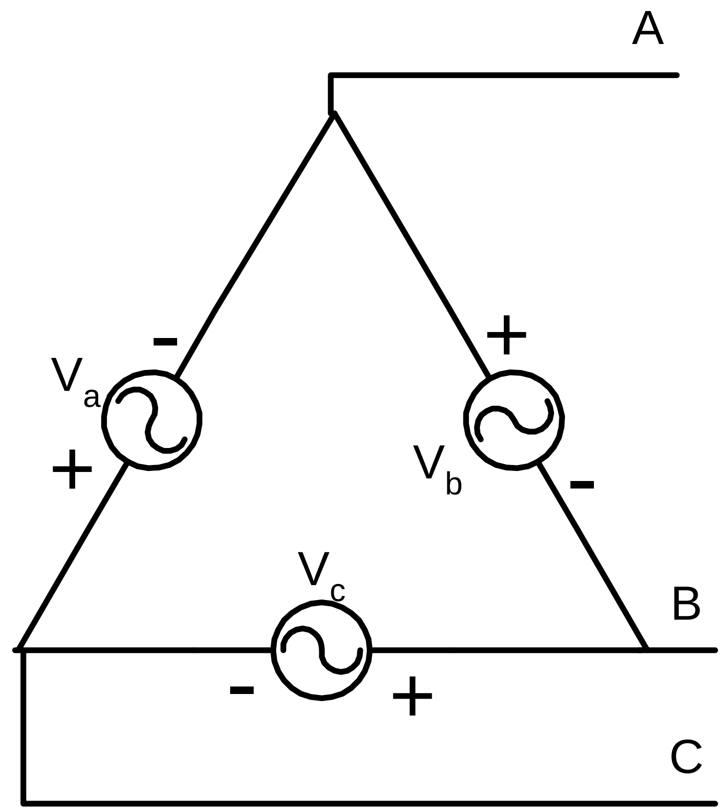
- Delta connection



$$\overline{V}_{AB} = \overline{V}_b = V\angle -120^0$$

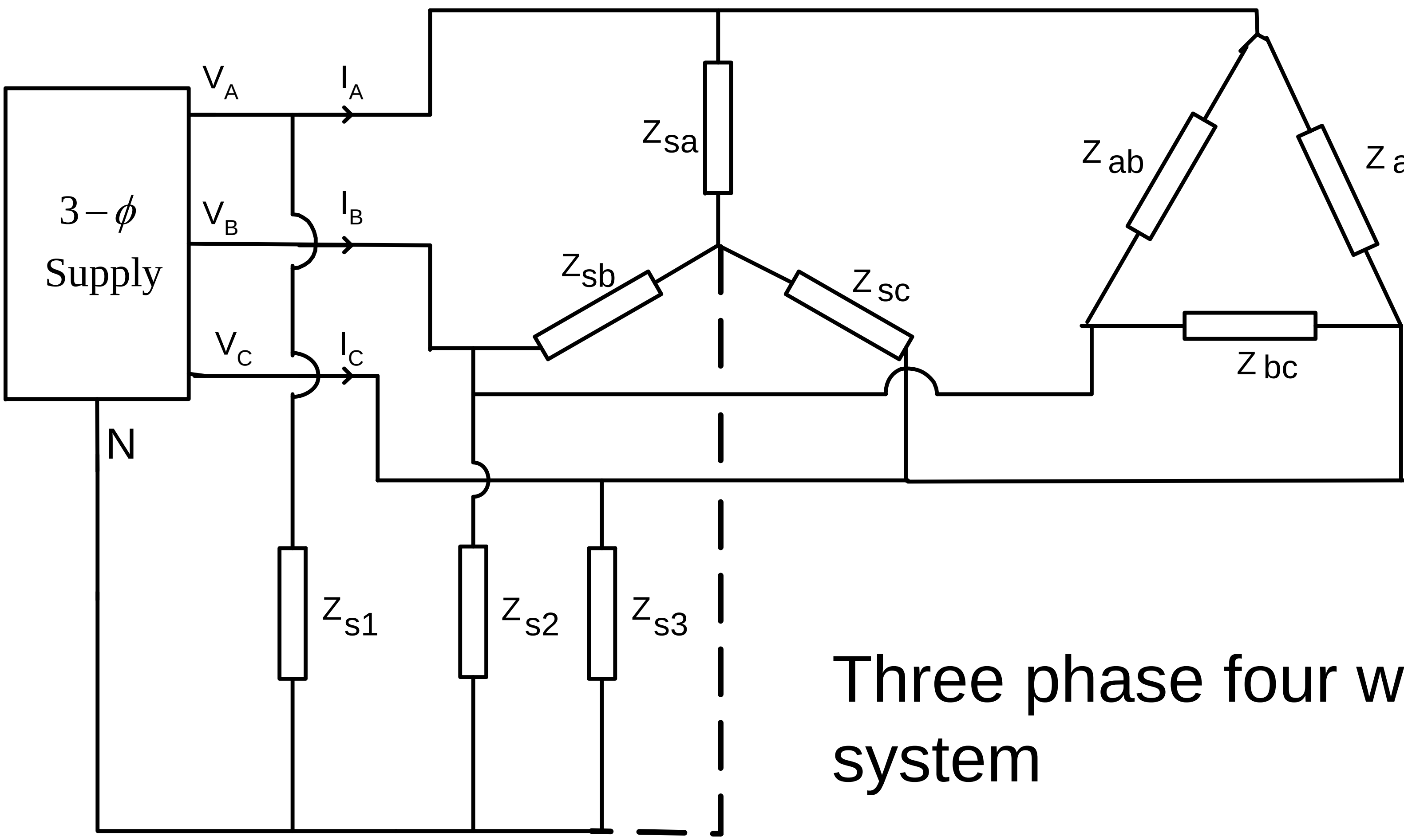
$$\overline{V}_{BC} = \overline{V}_c = V\angle +120^0$$

$$\overline{V}_{CA} = \overline{V}_a = V\angle 0^0$$



Neutral not available

Configuration of a three phase system



Three phase four wire system

Balanced three phase system

$$\bar{V}_{AB} = \sqrt{3}V_{ph}\angle 0^0$$

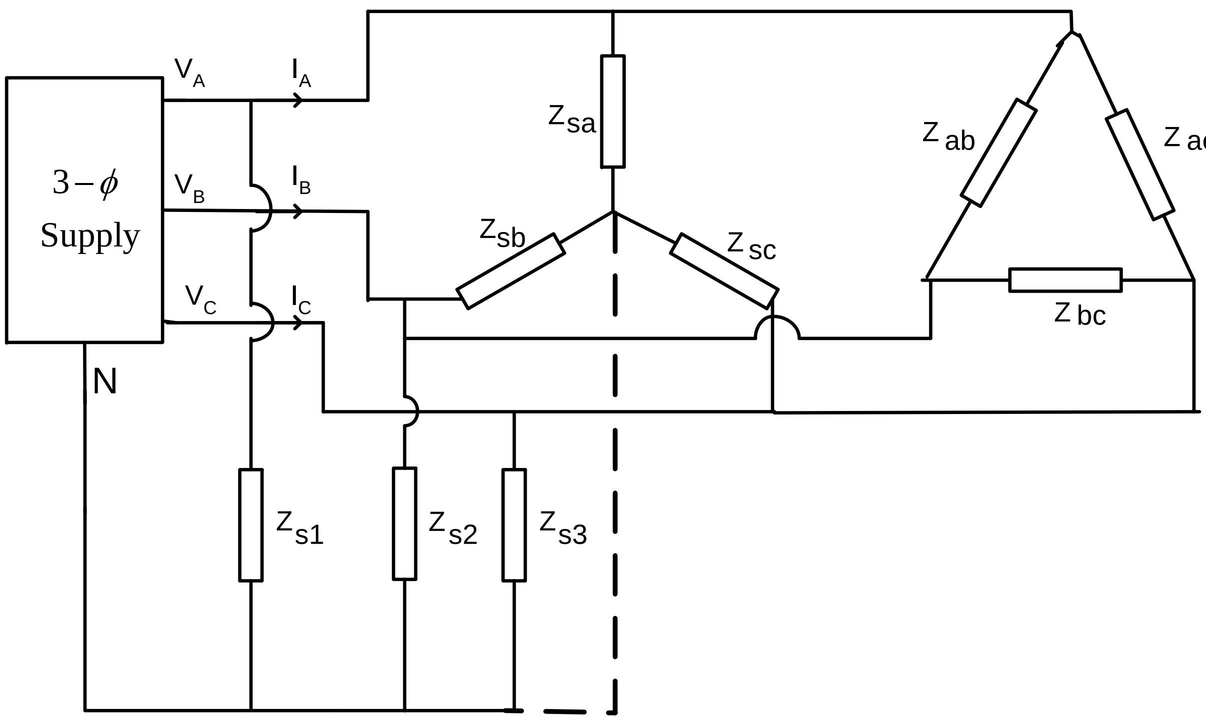
$$\bar{V}_{BC} = \sqrt{3}V_{ph}\angle -120^0$$

$$\bar{V}_{CA} = \sqrt{3}V_{ph}\angle -240^0$$

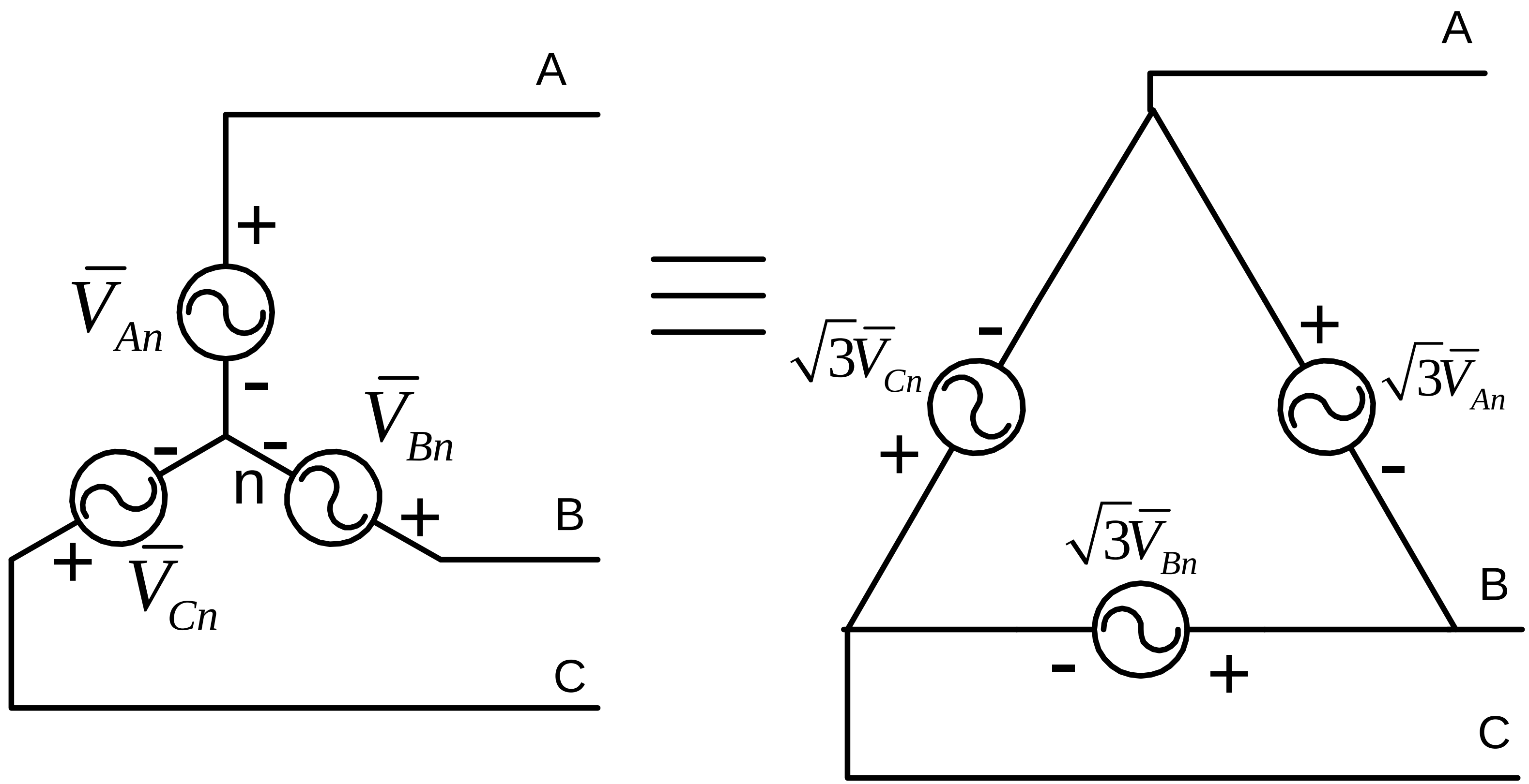
$$Z_{sa} = Z_{sb} = Z_{sc}$$

$$Z_{ab} = Z_{bc} = Z_{ca}$$

$$Z_{s1} = Z_{s2} = Z_{s3}$$

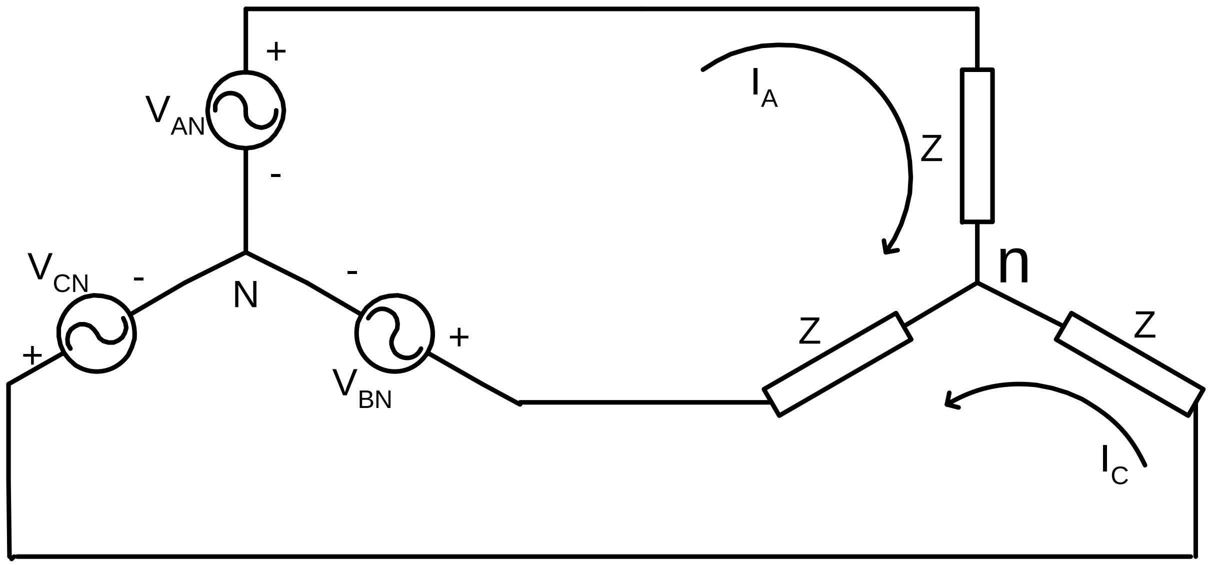


Equivalence of star and delta connected sources



Three phase three wire system

Balanced three phase system feeding a balanced load



$$-\overline{V}_{AN} + \overline{I}_A\overline{Z} + \left(\overline{I}_A + \overline{I}_C\right)\overline{Z} + \overline{V}_{BN} = 0$$

$$-\overline{V}_{CN} + \overline{I}_C\overline{Z} + \left(\overline{I}_A + \overline{I}_C\right)\overline{Z} + \overline{V}_{BN} = 0$$

or

$$2\overline{I}_A\overline{Z} + \overline{I}_C\overline{Z} = \overline{V}_{AN} - \overline{V}_{BN} \qquad (1)$$

$$2\overline{I}_C\overline{Z} + \overline{I}_A\overline{Z} = \overline{V}_{CN} - \overline{V}_{BN} \qquad (2)$$

$$2\bar{I}_A\bar{Z} + \bar{I}_C\bar{Z} = \bar{V}_{AN} - \bar{V}_{BN} \qquad (1)$$

$$2\bar{I}_C\bar{Z} + \bar{I}_A\bar{Z} = \bar{V}_{CN} - \bar{V}_{BN} \qquad (2)$$

$$(1) - 2\times(2)$$

$$2\bar{I}_A\bar{Z} + \bar{I}_C\bar{Z} = \bar{V}_{AN} - \bar{V}_{BN}$$

$$4\bar{I}_C\bar{Z} + 2\bar{I}_A\bar{Z} = 2\bar{V}_{CN} - 2\bar{V}_{BN}$$

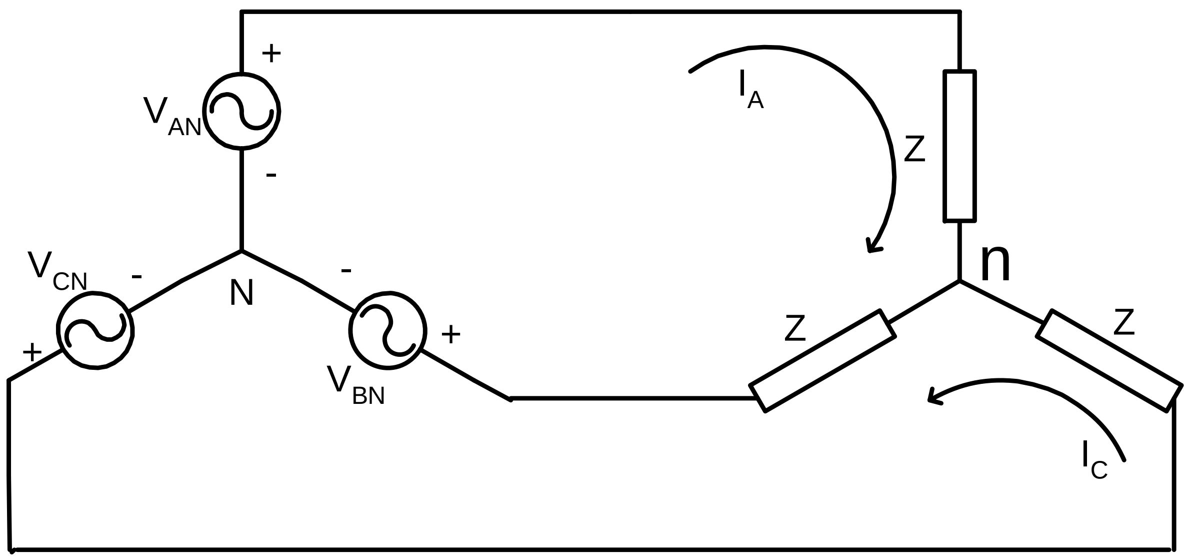
$$- \qquad - \qquad - \qquad +$$

$$-3\bar{I}_C\bar{Z} = \bar{V}_{AN} - 2\bar{V}_{CN} + \bar{V}_{BN}$$

or

$$-3\bar{I}_C\bar{Z} = -3\bar{V}_{CN}$$

$$\bar{I}_C = \frac{\bar{V}_{CN}}{\bar{Z}} \qquad \bar{I}_B = \frac{\bar{V}_{BN}}{\bar{Z}} \qquad \bar{I}_A = \frac{\bar{V}_{AN}}{\bar{Z}}$$



$$\overline{I}_A = \frac{\overline{V}_{AN}}{\overline{Z}}$$

$$\overline{I}_B = \frac{\overline{V}_{BN}}{\overline{Z}}$$

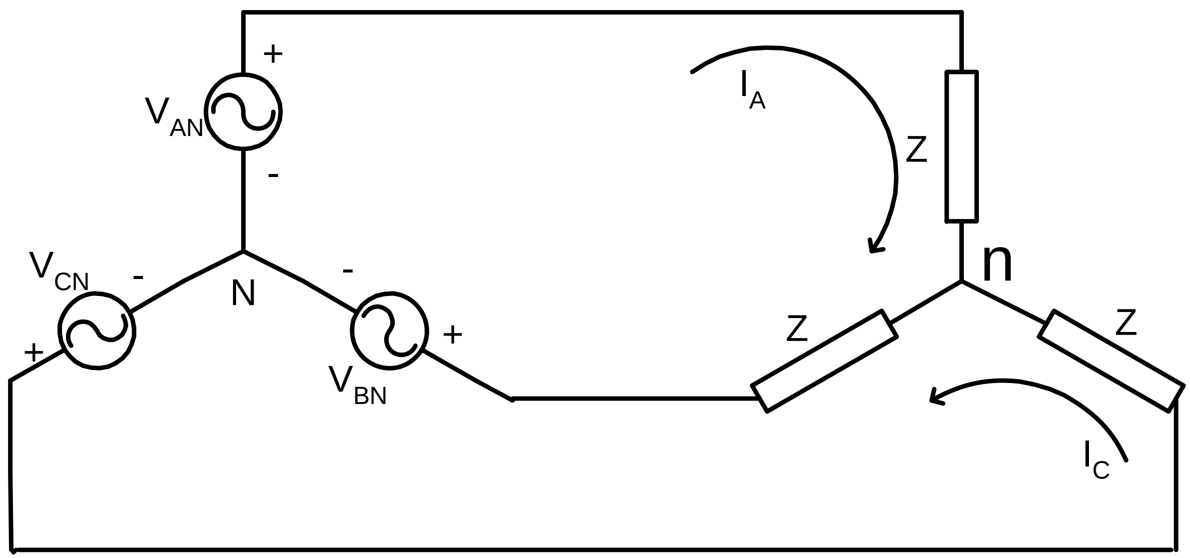
$$\overline{I}_C = \frac{\overline{V}_{CN}}{\overline{Z}}$$

- N and n are at the same potential

- If N and n are connected no current will flow through that wire

This is also corroborated by:

$$\overline{I}_A + \overline{I}_B + \overline{I}_C = 0$$



$$\bar{Z} = Z\angle\phi$$

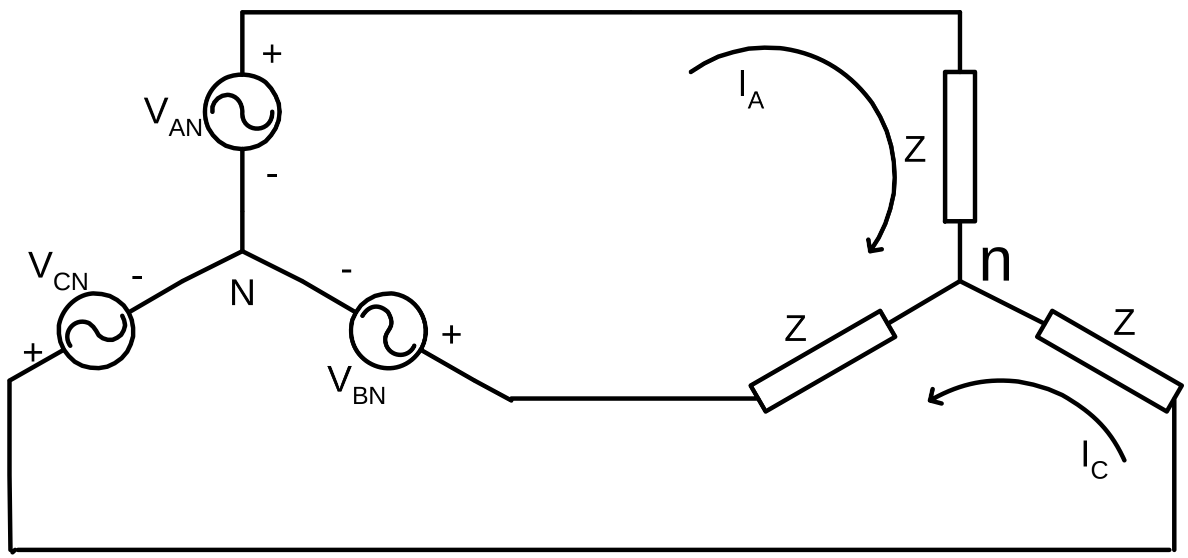
$$\bar{I}_A = \frac{\bar{V}_{AN}}{Z}\angle -\phi$$

For star connected balanced load:

$$\left|I_{ph}\right| = \left|I_L\right| \qquad \left|V_{ph}\right| = \left|\frac{1}{\sqrt{3}}V_{LL}\right|$$

Power consumed in Phase-A of the balanced system

$$= V_{An} I_A \cos\phi \qquad = V_{AN} I_A \cos\phi \qquad = V_{ph} I_{ph} \cos\phi$$



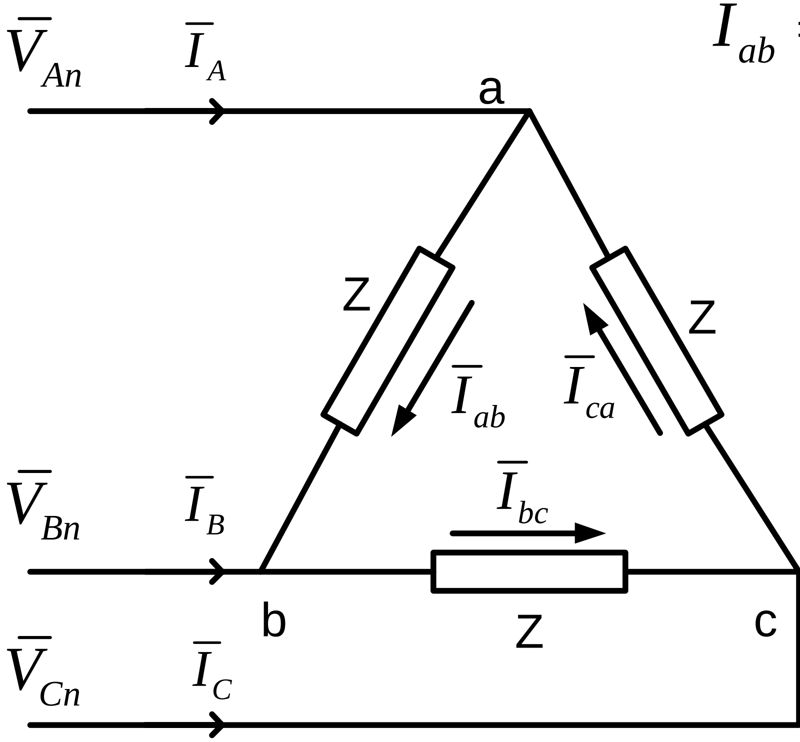
Total Power consumed in the star connected balanced load

$$= 3V_{ph} I_{ph} \cos\phi$$

$$= 3\frac{V_{LL}}{\sqrt{3}} I_L \cos\phi$$

$$= \sqrt{3} V_{LL} I_L \cos\phi$$

Three phase system feeding a balanced delta connected load

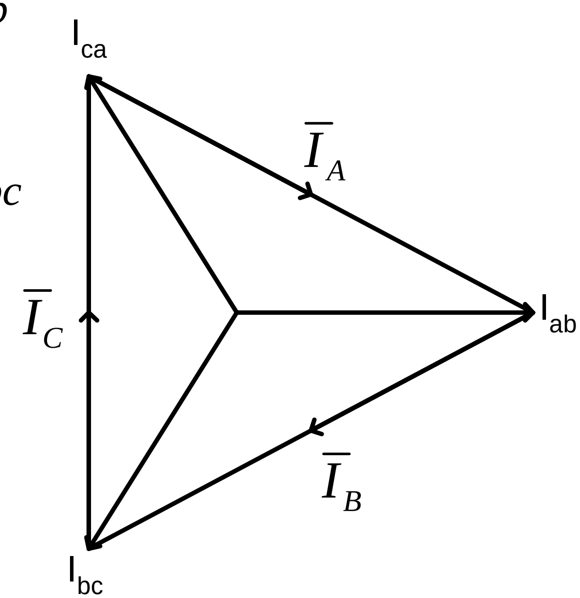


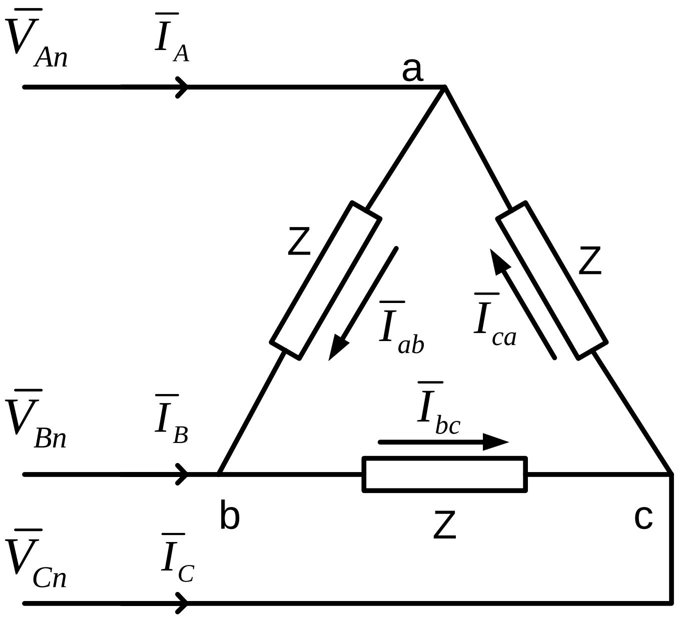
$$\overline{I}_{ab} = \frac{\overline{V}_{AB}}{\overline{Z}} \qquad \overline{I}_{bc} = \frac{\overline{V}_{BC}}{\overline{Z}} \qquad \overline{I}_{ca} = \frac{\overline{V}_{CA}}{\overline{Z}}$$

$$\overline{I}_A = \overline{I}_{ab} - \overline{I}_{ca}$$

$$\overline{I}_B = \overline{I}_{bc} - \overline{I}_{ab}$$

$$\overline{I}_C = \overline{I}_{ca} - \overline{I}_{bc}$$



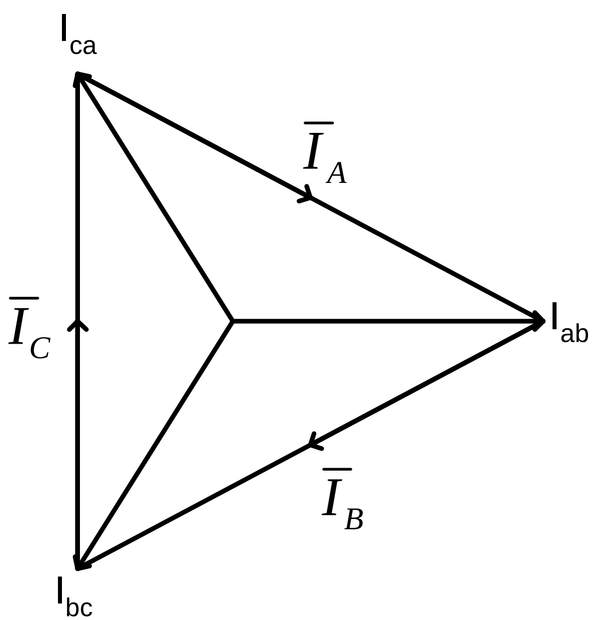


$$\overline{I}_A = \overline{I}_{ab} - \overline{I}_{ca}$$

$$= I_{ph}\left[1 - \left\{-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right\}\right]$$

$$= I_{ph}\left[\frac{3}{2} - j\frac{\sqrt{3}}{2}\right]$$

$$= \sqrt{3} I_{ph} \angle -30^0$$

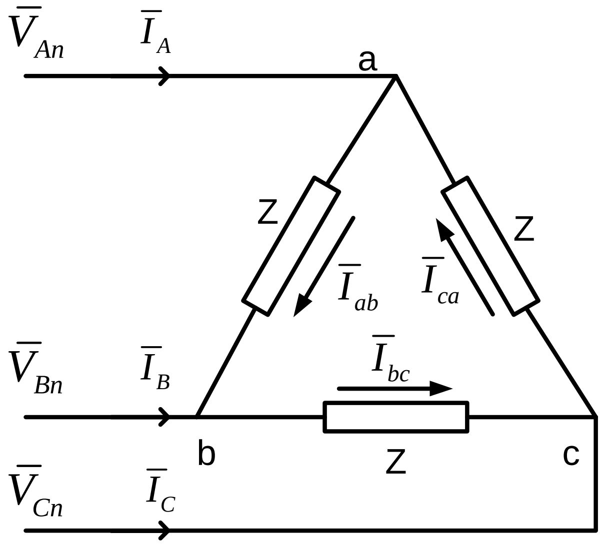


$$\overline{I}_B = \sqrt{3} I_{ph} \angle -150^0$$

$$\left|I_{ph}\right| = \left|\frac{I_L}{\sqrt{3}}\right|$$

$$\overline{I}_C = \sqrt{3} I_{ph} \angle +90^0$$

$$\left|V_{ph}\right| = \left|V_{LL}\right|$$



Power consumed in each phase

$$= V_{ph} I_{ph} \cos\phi$$

Total power consumed

$$= 3 V_{ph} I_{ph} \cos\phi$$

$$= 3 V_{LL} \frac{I_L}{\sqrt{3}} \cos\phi$$

$$= \sqrt{3} V_{LL} I_L \cos\phi$$