

**Question 1)** Which of the following series RLC circuits are (i) under-damped, (ii) critically damped and (iii) over-damped? The capacitors and inductors were de-energized at  $t < 0$ . If a step voltage of  $2V$  ( $v = 0$  for  $t < 0$  and  $v = 2V$  for  $t \geq 0$ ) is applied to the circuits, determine the current in each case.

1.  $R = 4\Omega, L = 2H, C = \frac{1}{2}F$  – **critically damped**
2.  $R = 2\Omega, L = 1H, C = \frac{1}{2}F$  – **under damped**
3.  $R = 5\Omega, L = 3H, C = \frac{1}{3}F$  – **under damped**

**Question 2)** The initial conditions are zero for the circuit in Figure 2. A step voltage of  $V_{in}$  Volts is applied at time  $t = 7s$ . This means that the applied voltage is zero before time  $7s$  and the level changes to  $V_{in}$  there after. You are required to find the voltage  $v_c(t)$  across the capacitor.

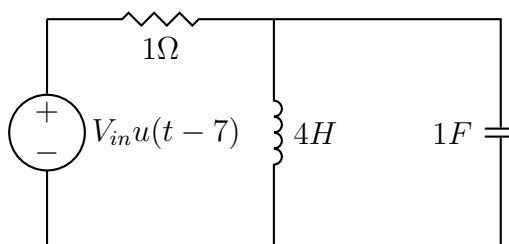


Figure 2

*Hint: This can be solved faster, if you know the current solution of the series RLC circuit. In particular, this circuit is in some sense a dual of the RLC circuit we extensively discussed in class.*

**Solution**

$$v_c(t) = V_{in} \cdot (t - 7) e^{-\frac{t-7}{2}} u(t - 7) \text{ Volts}$$

**Question 3)** The switch in Figure 3 is closed at time  $t = 0$ .

1. What is the voltage across the capacitor at  $t = 0$ ? **ans:** 80 Volts.
2. What are the values of  $i_1(t)$  and  $i_2(t)$ .

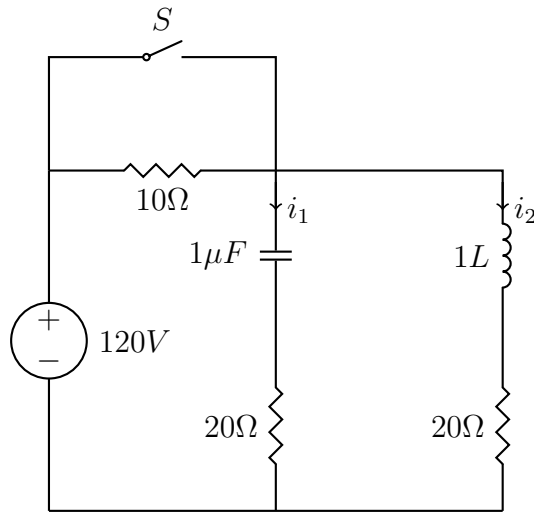


Figure 3.

**Solution**

$$i_1(t) = 2e^{-5 \times 10^4 t} u(t) \text{ Amps} \quad (1)$$

$$i_2(t) = (6 - 2e^{-20t})u(t) \text{ Amps} \quad (2)$$

**Question 4)** The switch in Figure 4 is at position *a* for a long time. At  $t = 5 : 00pm$  today, the switch was moved to position *b*. Find the voltage  $v_c(t)$  across the capacitor for all time.

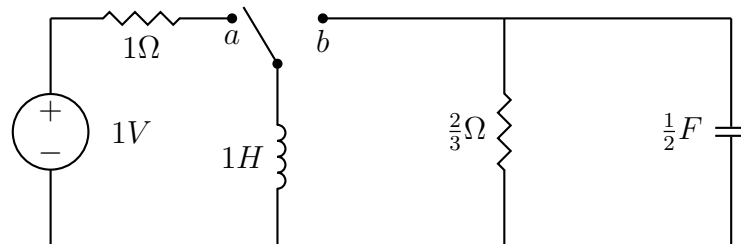


Figure 4

**Solution**

$$v_c(t) = (2e^{-2(t-t_0)} - e^{-(t-t_0)})u(t - t_0) \quad (3)$$

where  $t_0$  is the time when switch was moved.

**Question 5)** Find  $v_1(t)$  and  $v_2(t)$  if the voltage is  $6(1 - u(t))$ . (This is to compensate for Problem 5 of Tutorial 4, where a switch was present, which renders the circuit currents to zero after time zero. So you can ignore Problem 5 from Tutorial 4).

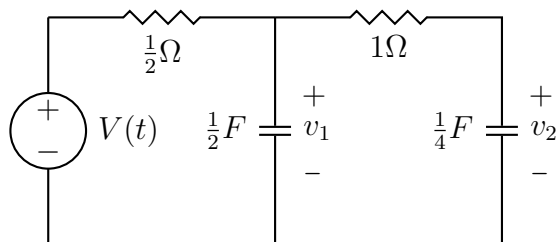


Figure 5

**Solution**

$$v_1(t) = \begin{cases} 6V, & \text{if } t < 0 \\ 8e^{-2t} - 2e^{-8t}V & \text{otherwise} \end{cases} \quad (4)$$

$$v_1(t) = \begin{cases} 6V, & \text{if } t < 0 \\ 4e^{-2t} + 2e^{-8t}V & \text{otherwise} \end{cases} \quad (5)$$

**Question 6)** Find  $v(t)$  and  $i(t)$  if the input  $V(t) = 12(1 - u(t))$ .

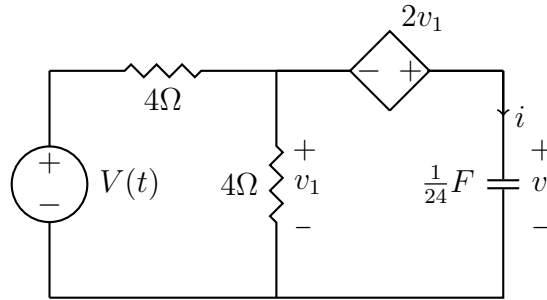


Figure 6

**Solution**

$$v(t) = 18e^{-4t}V, t > 0$$

$$i(t) = -3e^{-4t}A, t > 0$$

**Question 7)** If  $K_1 = -3$ , find the voltage across the capacitor  $v_c(t)$ .

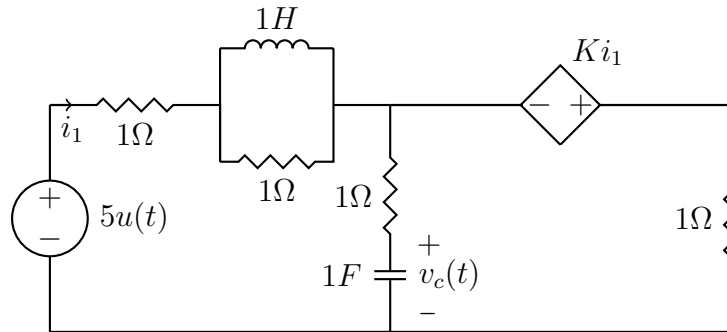


Figure 7

**Solution**

$$v_c(t) = 4 - e^{-\frac{3}{4}t} \left( 2 \sin \frac{t}{4} + 4 \cos \frac{t}{4} \right) u(t) \text{ Volts}$$