**Question 1)** Let an input of \( V(t) = 240 \sin(\omega_0 t + \frac{\pi}{4}) \) Volts is applied as input to the circuit below. At what value of \( \omega_0 \) will the voltage sinusoid \( v_R(t) \) achieve the minimal amplitude. (Recall that amplitude of a sine wave is the maximal level it can take, i.e. \( \alpha \) in \( \alpha \sin(\omega t) \) )

![Figure 1](image1)

**Solution:** Converting the voltage source to a current source, we get a parallel RLC circuit, where \( R = 20k\Omega \). Using phasor notation, we can write using \( KCL \),

\[
V = I\left( \frac{1}{R} + j\omega C - \frac{j}{\omega L} \right) = I(G_R + jG_I)
\]

where \( G_R = \frac{1}{R} \) and \( G_I = \omega C - \frac{1}{\omega L} \). Notice that

\[
|V| \geq IG_R
\]

and equality happens when \( G_I = 0 \), which in turn implies that

\[
j\omega C = \frac{j}{\omega L}
\]

or

\[
\omega^2 = \frac{1}{\sqrt{LC}} = 15.8 \times 10^6 \text{Hz}
\]

This is known as the resonant frequency, because the admittance has the minimum magnitude. In a series RLC circuit, resonance occurs when the impedance is minimum, in either case the circuit becomes purely resistive.

*Can you justify the term resonant frequency, for the \( \omega_0 \) you found in the question above.*

**Question 2)** Find the voltage across the inductor if \( V(t) = e^{-5t} \cos(4t - 30^\circ)u(t) \) Volts.

![Figure 2](image2)
Solution: The purpose of the question was also to indicate that phasor analysis will not apply here. Notice that the input is not a sinusoid, rather a restricted sinusoid, which is not just a single frequency. So we fall back to good old Laplace Transform. By source transformations, we can replace all the resistances by a single one of $\frac{3}{2}\Omega$ in series with a voltage source of $\frac{1}{2}V(t)$ and an inductance of $1H$. Notice also that

$$\cos(4t + 30^\circ) = \cos(4t)\cos(30) - \sin(4t)\sin(30)$$

By taking Laplace transforms,

$$I = \frac{1}{4} \frac{s + 4}{(s + 5)^2 + 16} - \frac{\sqrt{3}}{(s + 5)^2 + 16} = (s + \frac{3}{2})I(s)$$

$$I = \frac{1}{4} \frac{s + 5 - 4\sqrt{3}}{(s + 5)^2 + 16} \frac{1}{s + 1.5}$$

$$= \frac{1}{4} \left[ \frac{1}{(s + 5)^2 + 16} + \frac{3.5 - 4\sqrt{3}}{((s + 5)^2 + 16)(s + 1.5)} \right]$$

Notice that

$$\frac{1}{((s + 5)^2 + 16)(s + 1.5)} = \frac{1}{(41 - 8.5) \times 1.5} \left( \frac{1}{s + 1.5} - \frac{s + 8.5}{(s + 5)^2 + 16} \right)$$

Taking inverse Laplace transform, we get

$$i(t) = \frac{1}{16} e^{-5t} \cos(4t) + (3.5 - 4\sqrt{3}) \times 0.005(e^{-1.5t} + e^{-5t} \cos(4t) + 0.875e^{-5t} \sin(4t)), \ t > 0.$$  

Question 3) If $V(t) = 10 \cos(2t - \frac{\pi}{3})$, find the Thevenin equivalent between $A$ and $B$.

**Figure 3**

Solution: $V_{Th} = 3.48/84.3^\circ$, $Z_{Th} = 5.8/53.3^\circ$. 

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Question 4) In Figure 4, let \( V(t) = 24\sqrt{2}\cos(2t)\) Volts. Find the voltage across terminals \( A \) and \( B \).

Solution: 4.78\( \sqrt{2}\)\( \cos(2t + 66^\circ) \).

Question 5) If \( V_1(t) = \cos 4t \) and \( V_2(t) = 2\sin 8t \), find the current through the capacitor.

Solution: 0.275\( \cos(4t - 74^\circ) \) + 0.256\( \cos(8t - 82^\circ) \)