Indian Institute of Technology Bombay

Dept of Electrical Engineering

Handout 8 — **EE 101 Electrical & Electronic Circuits**

Tutorial 6 — Aug 26, 2011

Question 1) Let an input of $V(t) = 240\sin(\omega_0 t + \frac{\pi}{4})\,Volts$ is applied as input to the circuit below. At what value of ω_0 will the voltage sinusoid $v_R(t)$ achieve the minimal amplitude. (Recall that amplitude of a sine wave is the maximal level it can take, i.e. α in $\alpha\sin(\omega t)$)

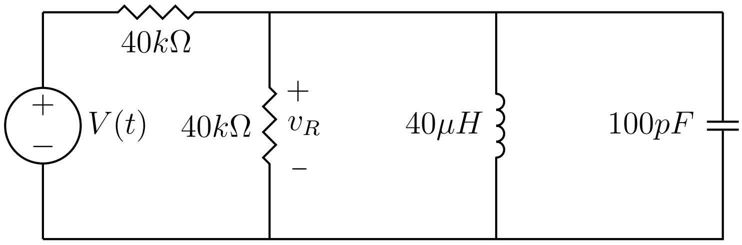

Figure 1

Solution: Converting the voltage source to a current source, we get a parallel RLC circuit, where $R = 20k\Omega$. Using phasor notation, we can write using KCL,

$$\begin{aligned} V &= I(\frac{1}{R} + j\omega C - \frac{j}{\omega L}) \\ &= I(G_R + jG_I) \end{aligned}$$

where $G_R = \frac{1}{R}$ and $G_I = \omega C - \frac{1}{\omega L}$. Notice that

$$|V| \geq IG_R$$

and equality happens when $G_I = 0$, which in turn implies that

$$j\omega C = \frac{j}{\omega L}$$

or

$$\omega^2 = \frac{1}{\sqrt{LC}} = 15.8 \times 10^6 Hz$$

This is known as the resonant frequency, because the admittance has the minimum magnitude. In a series RLC circuit, resonance occurs when the impedance is minimum, in either case the circuit becomes purely resistive.

*Can you justify the term **resonant frequency**, for the ω_0 you found in the question above.*

Question 2) Find the voltage across the inductor if $V(t) = e^{-5t}\cos(4t - 30°)u(t)\,Volts$.

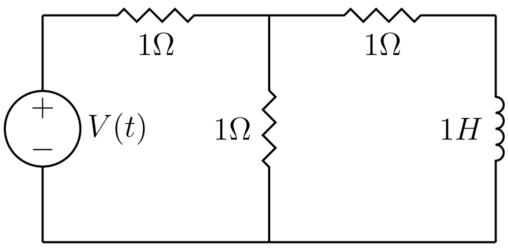

Figure 2

Solution: The purpose of the question was also to indicate that phasor analysis will not apply here. Notice that the input is not a sinusoid, rather a restricted sinusoid, which is not just a single frequency. So we fall back to good old Laplace Transform. By source transformations, we can replace all the resistances by a single one of $\frac{3}{2}\Omega$ in series with a voltage source of $\frac{1}{2}V(t)$ and an inductance of $1H$. Notice also that

$$\cos(4t + 30°) = \cos(4t)\cos(30) - \sin(4t)\sin(30)$$

By taking Laplace transforms,

$$\frac{1}{4}\frac{s+4}{(s+5)^2+16} - \frac{\sqrt{3}}{(s+5)^2+16} = (s+\frac{3}{2})I(s)$$

$$\begin{aligned} I &= \frac{1}{4}\frac{s+5-4\sqrt{3}}{(s+5)^2+16}\frac{1}{(s+1.5)} && (1)\\ &= \frac{1}{4}\left[\frac{1}{(s+5)^2+16} + \frac{3.5-4\sqrt{3}}{((s+5)^2+16)(s+1.5)}\right] && (2)\end{aligned}$$

Notice that

$$\frac{1}{((s+5)^2+16)(s+1.5)} = \frac{1}{(41-8.5)\times 1.5}\left(\frac{1}{s+1.5} - \frac{s+8.5}{(s+5)^2+16}\right)$$

Taking inverse Laplace transform, we get

$$i(t) = \frac{1}{16}e^{-5t}\cos(4t) + (3.5-4\sqrt{3})\times 0.005(e^{-1.5t} + e^{-5t}\cos(4t) + 0.875e^{-5t}\sin(4t))\,,\ t > 0.$$

Question 3) If $V(t) = 10\cos(2t - \frac{\pi}{3})$, find the Thevenin equivalent between A and B.

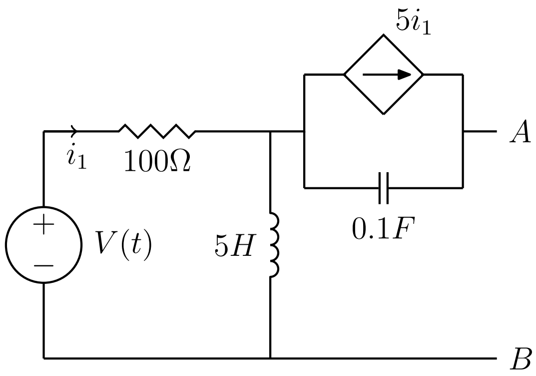

Figure 3

Solution: $V_{Th} = 3.48\angle 84.3°$, $Z_{Th} = 5.8\angle 53.3°$.

Question 4) In Figure 4, let $V(t) = 24\sqrt{2}\cos(2t)\, Volts$. Find the voltage across terminals A and B.

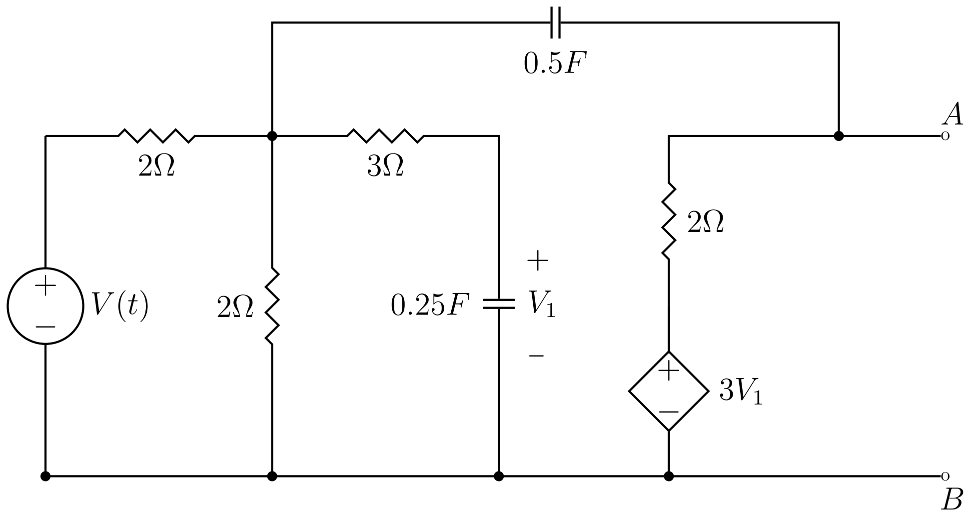

Figure 4

Solution: $4.78\sqrt{2}\cos(2t + 66°)$.

Question 5) If $V_1(t) = \cos 4t$ and $V_2(t) = 2\sin 8t$, find the current through the capacitor.

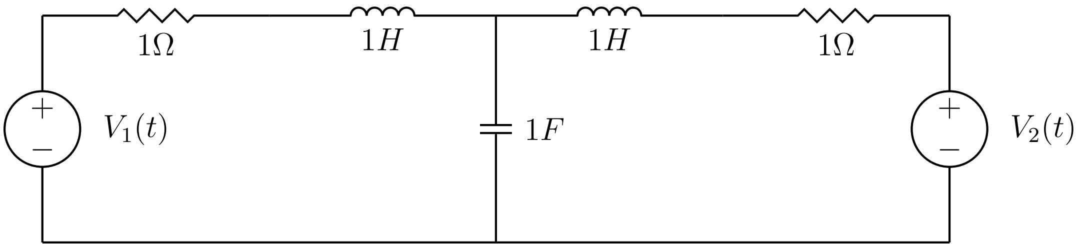

Figure 5

Solution: $0.275\cos(4t - 74°) + 0.256\cos(8t - 82°)$