Indian Institute of Technology Bombay

Dept of Electrical Engineering

Handout 11 **EE 101 Electrical & Electronic Circuits**

Tutorial 7 Sep 02, 2011

Question 1) An input of $V(t) = 230\sin(\omega_0 t + \frac{\pi}{4})\,Volts$ is applied across a circuit element. What is the effective voltage (also known as **rms**) voltage.

Solution: $\frac{230}{\sqrt{2}} = 162.6$ Watts.

If a sinusoidal current of $I_{rms} = 10A$ passes through this circuit element, what is the dissipated power for a lag (with respect to $V(t)$) of

1. $\theta = 30^{\text{deg}} \Rightarrow P = 1408$ Watts
2. $\theta = 45^{\text{deg}} \Rightarrow P = 1150$ Watts
3. $\theta = 90^{\text{deg}} \Rightarrow P = 0$ Watts

Question 2) One way to understand phasors is to treat them as complex numbers, with appropriate meanings (of magnitudes and phase angles) which correspond to sinusoids. In this consideration, it is sometimes useful to represent a sinusoid as a complex number $a + jb$, with magnitude

$$\sqrt{a^2 + b^2} = V_{rms}.$$

Notice the slight abuse of earlier notation, here the magnitude represents the rms value. This is useful in power computations. The phase angle ϕ represents the shift with respect to cosine waveform, which is our standard reference waveform.

$$\phi = \tan^{-1}\left(\frac{b}{a}\right)$$

a) Let $\tilde{V}$ and $\tilde{I}$ be complex numbers representing the voltage and current respectively (**rms** value), for a given circuit element. Show that the average active power is given by the formula

$$P = \text{Real }(\tilde{V}\tilde{I}^*).$$

Solution: We know that the average power delivered is $\frac{|V||I|}{2}\cos\theta$. If $\tilde{V} = V\underline{/\theta_1}$ and $\tilde{I} = I\underline{/\theta_2}$, the average active power is

$$P = VI\underline{/\theta_1 - \theta_2} = \text{ Real }(\tilde{V}\tilde{I}^*)$$

b) Compute the power factor, $\cos\theta$.

$$\cos\theta = \cos\tan^{-1}\left(\frac{\text{Imag }(\tilde{V}\tilde{I}^*)}{\text{Real }(\tilde{V}\tilde{I}^*)}\right) \tag{1}$$

Question 3) In the last question, the quantity $\tilde{V}\tilde{I}^*$ is known as **complex power**, denoted by S. As you have already shown, the real part of S gives the **active** power. The imaginary part of S is known as **reactive** power, which in some sense is not the useful power. Nevertheless, from basic energy conservation principles, the total complex power in a closed system is zero, implying that the used up reactive power is equal to that supplied. Verify this conservation by computing the complex power dissipated by each element in the following circuit.

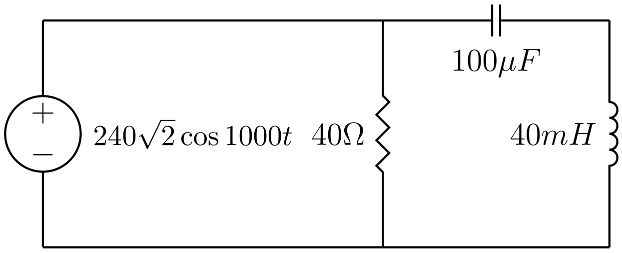


Figure 3

Solution Let the P_R be the power through the resistor, and similarly P_L and P_C for the inductor and capacitor.

$$P_R = \tilde{V}\angle 0 \tilde{I}_R^* = 240\angle 0 \frac{240\angle{-0}}{40\angle{-0}} = 1440W$$

$$P_C = \frac{\tilde{V}\angle 0}{j40 - j10}(-j10)\frac{\tilde{V}\angle 0}{j10 - j40} = j640W$$

$$P_L = \frac{\tilde{V}\angle 0}{j40 - j10}(j40)\frac{\tilde{V}\angle 0}{j10 - j40} = -j2560W$$

Let us now find the power delivered by the source.

$$P_S = \tilde{V}\angle 0 \left(6\angle 0 + \frac{\tilde{V}\angle{-0}}{j10 - j40}\right) = 1440 + j1920W$$

Thus the total active power and reactive power are conserved.

Question 4) The load in the circuit shown below is a $1000W$ motor. The input voltage is $V(t) = 200\cos 120\pi t$. Suppose the motor has a lagging power factor (p.f.) of 0.8. Find the complex power absorbed by each element in the circuit.

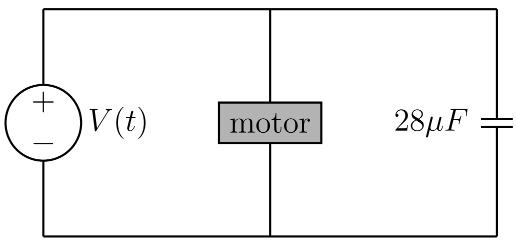


Figure 4

Solution: Given that the power is 1000W, with a power factor of 0.8, the current(magnitude) through the motor is

$$|I| = \frac{P}{\tilde{V}\cos\theta} = \frac{1000\sqrt{2}}{200 \times 0.8} = 8.84A$$

From this the current through the motor is

$$i_m = 8.84 \times 0.8 - j8.84 \times 0.6 = 7.07 - j5.3$$

The complex power aborbed by the motor is

$$P_m = P_{real} + j|V||I|\sin\theta = 1000 + j750W$$

The complex power absorbed by the capacitor is

$$P_c = \frac{200 * 200}{2}(-j120\pi.28 \times 10^{-6}) = -j211W$$

The complex power supplied by the source is

$$P_s = \frac{200}{\sqrt{2}}(i_m^* - j\frac{200}{\sqrt{2}}.120\pi.28 \times 10^{-6}) = 1000 - j539W$$

b) What is the use of the capacitor in the above circuit. (Think what will happen if we remove the capacitor) Removing the capacitor will decrease the power factor of the source and thus the effective rms current will increase. Having a higher current value will generate more losses in practical circuits. The presence of the capacitor provides a better match (in terms of phase) between the voltage and current, thus lower currents(effective values) are required to deliver the same active power. Thus the capacitor improves the energy utilization in practical circuits, this is known as power factor correction.

Question 5) Consider the following series circuit, where Z_L is a complex load. Let I_{rms} be the effective current passing through the circuit. Justify each step in the expressions shown below.

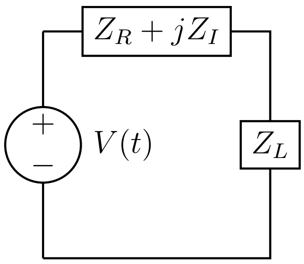


Figure 5

$$\begin{aligned}
P &= V_{rms} I_{rms} \cos(\theta_L) && (2)\\
&\leq V_{rms} I_{rms} && (3)\\
&= V_{rms} \frac{V_{rms}}{\sqrt{(Z_R + \text{ Re }(Z_L))^2 + (Z_I + \text{ Im }(Z_L))^2}} && (4)\\
&\leq \frac{V_{rms}^2}{Z_R + \text{ Re }(Z_L)} && (5)\\
&\leq \frac{V_{rms}^2}{Z_R} && (6)
\end{aligned}$$

There was a typo in the last inequality. This is not quite the inequality that I was hoping to get. So let us take an alternate approach to obtain what is known as the maximum power transfer theorem.

b) Deduce the Z_L that has to be connected to deliver the maximal load power.

Solution: We know that the active powered delivered to the load is the real part of $\tilde{V}\tilde{I}^*$, where $\tilde{V}$ is the rms voltage and $\tilde{I}$ the rms current. Let $Z_L = X_R + jX_I$.

$$\begin{aligned}
V_L I_L^* &= \frac{V\angle 0}{Z_R + X_R + j(Z_I + X_I)}(X_R + jX_I)\tilde{I}^* && (7)\\
&= \frac{V^2}{(Z_R + X_R + j(Z_I + X_I))} \frac{X_R + jX_I}{(Z_R + X_R - j(Z_I + X_I))} && (8)\\
&= \frac{V^2}{(Z_R + X_R)^2 + (Z_I + X_I)^2}(X_R + jX_I) && (9)
\end{aligned}$$

By taking the real part,

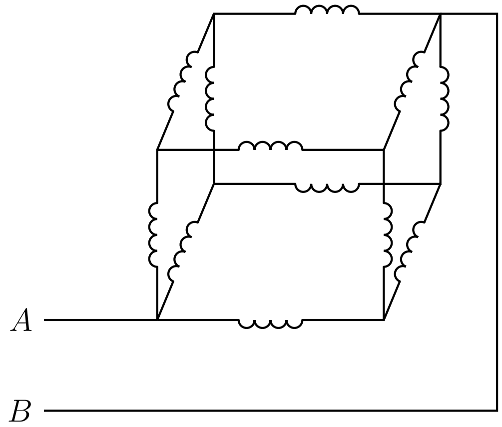
$$P = \frac{V^2}{(Z_R + X_R)^2 + (Z_I + X_I)^2} X_R \tag{10}$$

$$\leq \frac{V^2}{(Z_R + X_R)^2} X_R \tag{11}$$

$$\leq \frac{V^2}{4X_R} \tag{12}$$

where the last step is obtained by maximizing over all X_R. Clearly, the maximal power is transferred when $X_R + jX_I = Z_R - jZ_I$.

Question 6) If each inductance is $1H$, show that the effective inductance is $\frac{5}{6}H$



Solution: While one can use star-delta conversions to do this job, the easiest way is to use the symmettry in the problem. In particular, draw each inductance connected to the terminals A and B as two parallel inductances of value $2H$ each. The notice that the circuit separates out to 6 parallel current paths, each of value $5H$.