Indian Institute of Technology Bombay

Dept of Electrical Engineering

Handout 9 **EE 101 Electrical & Electronic Circuits**

Quiz Ib Aug 26,2011

Question 1) The switch in Figure is closed at time $t = -10s$.

1. What is the voltage across the capacitor at $t = -11s$.
2. What are the values of $i_1(t)$ and $i_2(t)$.

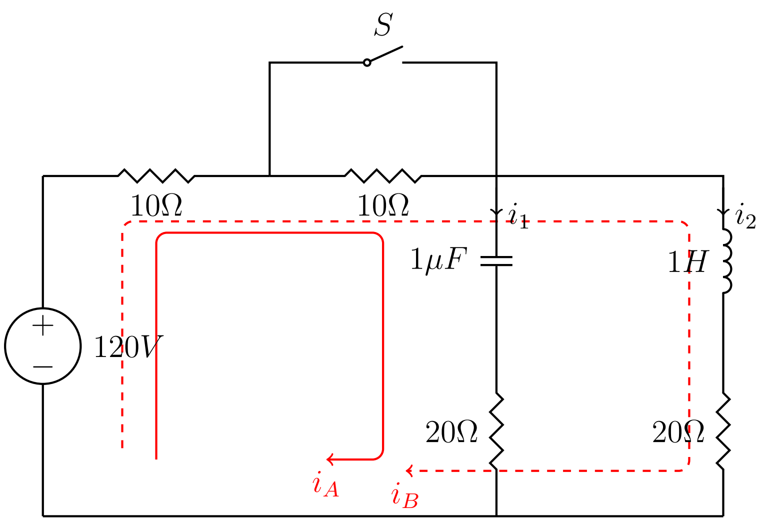

Figure 1.

Solution: I apologize that you found the computations a bit harder than expected. The intention was not to stretch you on computations, rather the form of the answer is more important. Notice also that, this problem was part of Tutorial 5, except for an additional 10Ω resistance.

As we discussed in class, let us assume that the switch was closed at time $t = 0$. Then, by writing loop equations for i_A and i_B shown above for $t > 0$,

$$V(t) = 10(i_A + i_B) + 20i_A + \frac{1}{C}\int_{-\infty}^{t} i_A(\tau)d\tau \tag{1}$$

$$V(t) = 10(i_A + i_B) + 20i_B + \frac{di_B}{dt} \tag{2}$$

It is simple to notice that the initial current through the inductor is $3A$ and the capacitor voltage is $60V$. Taking Laplace transform and denoting $d = \frac{1}{C} = 10^6$, we will get

$$\begin{bmatrix} \frac{V}{s} - \frac{60}{s} \\ \frac{V}{s} + 3 \end{bmatrix} = \begin{bmatrix} (30 + \frac{d}{s}) & 10 \\ 10 & (30 + s) \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} \tag{3}$$

where I_A and I_B are the transforms of $i_A(t)$ and $i_B(t)$ respectively. The solution to I_A and I_B can now be computed.

$$\begin{bmatrix} I_A \\ I_B \end{bmatrix} = \frac{s}{(30s + d)(30 + s) - 100s} \begin{bmatrix} 30 + s & -10 \\ -10 & 30 + \frac{d}{s} \end{bmatrix} \begin{bmatrix} \frac{60}{s} \\ \frac{3s+120}{s} \end{bmatrix} \tag{4}$$

Notice that the polynomial shown in the denominator is

$$(30s+d)(30+s)-100s = 30(s^2+33360s+10^6) = 30(s+\alpha)(s+\beta),$$

where $\alpha \approx 30$ and $\beta \approx 33330$. (I re-iterate that finding α, β is not the important thing, you can simply leave it as α and β, however, it is important to know that both α and β are positive numbers, which is straight forward to check.)

Let us find I_B first.

$$I_B = \frac{-600s+(30s+d)(3s+120)}{30s(s^2+33360s+d)} = \frac{3s+(10^5+100)+4d/s}{(s+\alpha)(s+\beta)} \tag{5}$$

From this we can find $i_B(t)$ as,

$$i_B(t) = [3(\beta e^{-\beta t}-\alpha e^{-\alpha t})+(10^5+100)(e^{-\alpha t}-e^{-\beta t})+4d\left(\frac{1}{\alpha}(1-e^{-\alpha t})-\frac{1}{\beta}(1-e^{-\beta t})\right)]\frac{u(t)}{\beta-\alpha}$$

Needless to say that for $t>0$, the current is of the form

$$c_1+c_2e^{-\alpha t}+c_3e^{-\beta t}.$$

As for I_A, we can write

$$I_A = \frac{s-10}{s^2+33360s+d}.$$

From this,

$$i_A(t) = ((\beta+10)e^{-\beta t}-(\alpha+10)e^{-\alpha t})\frac{u(t)}{\beta-\alpha}$$

To get the response in question,

$$i_1(t) = i_A(t+10) \tag{6}$$
$$i_2(t) = i_B(t+10) \tag{7}$$

whenever $t \geq -10$. Before this time, just put the initial conditions.