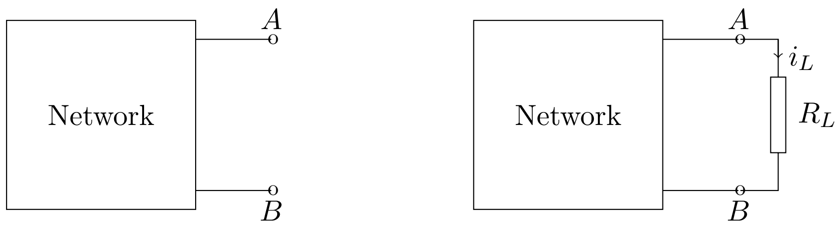
Indian Institute of Technology Bombay
Dept of Electrical Engineering

Handout 16 **EE 101 Electrical & Electronic Circuits**
Notes on Thevenin Sep 13, 2011

1 Thevenin's and Norton's Theorems

Thevenin's theorem is a very handy result in circuit analysis. We describe the theorem for resistive circuits. Extensions to reactive circuits are also possible, and will be explained later. In short, Thevenin's theorem states that a network of sources and resistive elements can be equivalently replaced by an appropriate voltage source in series with a resistance. Furthermore, the theorem specifies the value of the source voltage as well as the series resistance, in terms of the network parameters. Let us look at this in some more detail.



(a) no load (b) with load

Figure 1: Electrical Network as a black box

The effort put in learning Thevenin's (or Norton's) theorem is really worth it. This can be seen by considering a particular example. In Figure 1a we have shown an electrical network, where our primary interest is concerning the terminals or **ports** marked as A and B. We will call this port AB. Suppose we know everything about the network inside the box, i.e. the sources and corresponding currents/voltages, when no load is connected at the port AB. Imagine that a load of R_L is now connected across AB as in Figure 1b. Do we need to neglect our original idea of the circuit and compute the currents/voltages again in the presence of R_L. This seems too naive, as every time R_L is changed, we need to re-calculate the circuit parameters. Perhaps a clever thing is to retain the relevant parameters from the original circuit(without load). Naturally, the relevant parameters retained should not dependent on R_L, otherwise we may need to change it with R_L.

An assumption that we require before proceeding is that the network is not an ideal current source or an ideal voltage source for the load. If the network is an ideal source element, there is nothing to be done, as the circuit cannot get any simpler. With this assumption, let us start writing the Kirchoff's laws for the network. For example, let us write KCL for the nodes. To this end, assume that $V_A > V_B$ and node B is at zero potential, no generality is lost by these assumptions. Thus the current i_L between A and B is also positive. We make the following observations, and state these as facts.

Fact1
Once the node voltage V_A is specified, there exists a sufficient set of KCL(node) equations without any term having R_L. i.e. the dependency on R_L is completely contained in the term V_A.

Fact2
Once the load current i_L is specified, Kirchoff's law equations can be made to depend on R_L only through i_L, i.e. they can be made independent of R_L, if i_L is given.

Fact3
By our assumption that $V_A > 0$ (and $V_B = 0$), the node voltage V_A is an increasing function of R_L, which implies that V_A decreases as the load current i_L increases.

Fact 1 can be easily validated by writing the KCL equations. Once V_A is known, there is no need for node equations at A, and R_L enters the equations only through the nodal analysis of A (recall that $V_B = 0$). Fact 3 implicitly uses the idea that for a given network, the load current is changed only by a change in load resistance, with all other elements of the network unchanged.

Using Fact 1, from the point of view of the load, the entire network can be replaced by any circuit whose dependency on R_L is just in providing a node voltage of $V_A(R_L)$[1]. With the correct V_A value, no parameter inside the network is disturbed as the KCL equations are unchanged. So the question boils down to, is there a simple circuit which can generate a node voltage of $V_A(R_L)$?.

Let $V_1, V_2, \cdots, V_k$ be the rest of the independent node voltages. Using KCL or nodal analysis at the above nodes, we get

$$\underset{k \times k+1}{\begin{bmatrix} & & \\ & \mathbf{T} & \\ & & \end{bmatrix}} \begin{bmatrix} V_1 \\ \cdot \\ \cdot \\ V_k \\ V_A \end{bmatrix} = \underset{k \times 1}{\begin{bmatrix} f_1 \\ \cdot \\ \cdot \\ f_k \end{bmatrix}} \tag{1}$$

where $f_1, f_2, \cdots, f_k$ are functions (in amperes) of the sources(voltage or current) in the network, which are completely known in advance. Notice that we wrote only k node equations, i.e. node equation at A is not included in the set of equations above. Let us now write the equation for node A, where we will assume that i_L is the current flowing through the load.

$$\sum_{i=1}^{k} \alpha_i V_i + \beta V_A = i_L + \sum_{i=1}^{k} \gamma_i f_i,$$

where $\alpha_i, \gamma_i, \beta$ are functions of the given network parameters. The last term in the RHS represents the total current contribution from the sources which are connected to node A, let us denote this term by Δ_s. Thus, the KCL equations are

$$\begin{bmatrix} & & & & \\ & \mathbf{T} & & & \\ & & & & \\ \alpha_1 & \cdot & \cdot & \alpha_k & \beta \end{bmatrix} \begin{bmatrix} V_1 \\ \cdot \\ \cdot \\ V_k \\ V_A \end{bmatrix} = \begin{bmatrix} f_1 \\ \cdot \\ \cdot \\ f_k \\ i_L + \Delta_s \end{bmatrix} \tag{2}$$

Since this is a solvable set of equations, by taking the inverse,

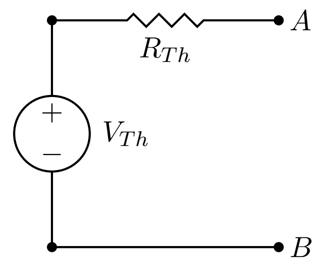
$$V_A = R' i_L + \sum_{i=1}^{k} a_i f_i$$

[1] we use $V_A(R_L)$ to explicitly show the dependency on R_L, but will shorten this as V_A to save space

where the constants $a_i, 1 \leq i \leq k$ and R' are appropriately chosen. From our stated Fact 3, $R' \leq 0$, so we rename $R_{Th} = -R'$. Let us also give a name to $\sum_{i=1}^{k} a_i f_i$, since it has the units of voltage, let us call it V_{Th}.

$$V_A = V_{Th} - i_L R_{Th}$$

Thus we obtained an equivalent representation of the network from the point of view of the load, which is depicted in figure. Notice that V_{Th} and R_{Th} has nothing to do with the load R_L (observe that there is no R_L in Equation (2)).



When we open-circuit the load $V_A = V_{oc}$, and we can identify $V_{Th} = V_{oc}$. On the other hand, while short-circuiting the load,

$$R_{Th} = \frac{V_{Th}}{I_{sc}},$$

where I_{sc} is the short-circuit current.

Proposition 1 $\frac{V_{oc}}{I_{sc}}$ *is the equivalent resistance of the network from* A *to* B*.*

Recall that the equivalent resistance can be found by setting all independent sources to zero, and then applying a test voltage. Let us apply a test voltage of $V_{Th} = V_{oc}$. First assume that the independent sources are left as it is. In this case, there will be no effective current entering/leaving the test source through node A or B. This is because there is no potential difference between terminals at either side of the port AB. Applying the superposition principle, the current driven by the test source into the network is exactly canceled by that driven by the sources inside the network through the load terminals. But what is the current driven by sources of the network through AB. By superposition, we can find this current by short-circuiting AB, i.e I_{sc} is the current supplied by the sources inside the network. Thus V_{oc} will drive a current equal to the negative of I_{sc} into the network, when all other independent sources are set to zero. Hence $\frac{V_{oc}}{I_{sc}}$ is nothing but the effective resistance from A to B, which is computed by applying V_{oc} as the test voltage.

Norton's theorem can be showed merely as a corollary of Thevenin's by employing source transformations. You should however, observe that if the circuit is effectively a current source, only Norton's representation exists. Similarly an equivalent voltage source network will have only Thevenin's representation.