Indian Institute of Technology Bombay

Dept of Electrical Engineering

Handout 11 | **EE 101 Electrical & Electronic Circuits**
Tutorial 7 | Sep 02, 2011

Question 1) An input of $V(t) = 230\sin(\omega_0 t + \frac{\pi}{4})\,Volts$ is applied across a circuit element. What is the effective voltage (also known as **rms**) voltage.
If a sinusoidal current of $I_{rms} = 10A$ passes through this circuit element, what is the dissipated power for a lag (with respect to $V(t)$) of

1. $\theta = 30^{\text{deg}}$
2. $\theta = 45^{\text{deg}}$
3. $\theta = 90^{\text{deg}}$

Question 2) One way to understand phasors is to treat them as complex numbers, with appropriate meanings (of magnitudes and phase angles) which correspond to sinusoids. In this consideration, it is always useful to consider a sinusoid as a complex number $a + jb$, with magnitude

$$\sqrt{a^2 + b^2} = V_{rms}$$

The phase angle ϕ represents the shift with respect to cosine waveform, which is our standard reference waveform.

$$\phi = \tan^{-1}\left(\frac{b}{a}\right)$$

a) Let $\tilde{V}$ and $\tilde{I}$ be complex numbers representing the voltage and current respectively, for a given circuit element. Show that the average power is given by the formula

$$P = \text{Real}\ (\tilde{V}\tilde{I}^*).$$

b) Compute the power factor, $\cos\theta$.

Question 3) In the last question, the quantity $\tilde{V}\tilde{I}^*$ is known as **complex power**, denoted by S. As you have already shown, the real part of S gives the **active** power. The imaginary part of S is known as **reactive** power, which in some sense is not the useful power. Nevertheless, from basic energy conservation principles, the total complex power in a closed system is zero, implying that the used up reactive power is equal to that supplied. Verify this conservation by computing the complex power dissipated by each element in the following circuit.

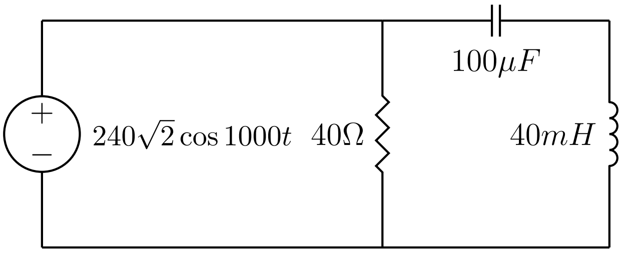


Figure 3

Question 4) The load in the circuit shown below is a $1000W$ motor. The input voltage is $V(t) = 200\cos 120\pi t$. Suppose the motor has a lagging power factor (p.f.) of 0.8. Find the complex power absorbed by each element in the circuit.

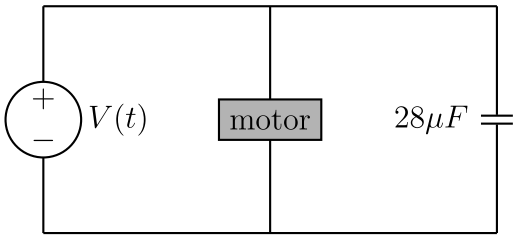


Figure 4

b) What is the use of the capacitor in the above circuit. (Think what will happen if we remove the capacitor)

Question 5) Consider the following series circuit, where Z_L is a complex load. Let I_{rms} be the effective current passing through the circuit. Justify each step in the expressions shown below.

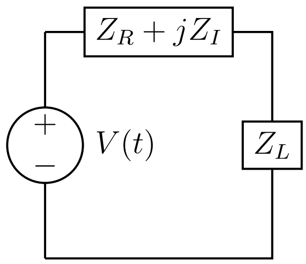


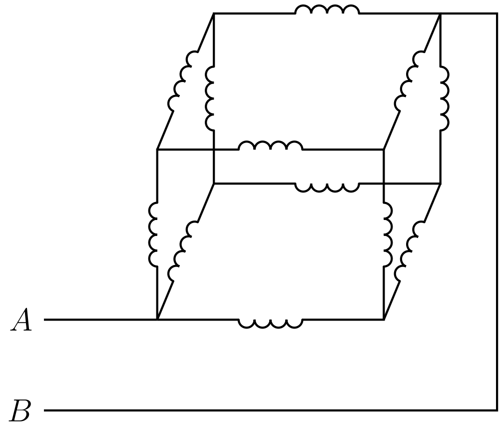
Figure 5

$$\begin{aligned}
P &= V_{rms} I_{rms} \cos(\theta_L) && (1)\\
&\leq V_{rms} I_{rms} && (2)\\
&= V_{rms} \frac{V_{rms}}{\sqrt{(Z_R + \text{ Re } (Z_L))^2 + (Z_I + \text{ Im } (Z_L))^2}} && (3)\\
&\leq \frac{V_{rms}^2}{Z_R + \text{ Re } (Z_L)} && (4)\\
&\leq \frac{V_{rms}^2}{Z_R} && (5)
\end{aligned}$$

There was a typo in the last inequality. This is not quite the inequality that I was hoping to get. So let us take an alternate approach to obtain what is known as the maximum power transfer theorem.

b) Deduce the Z_L that has to be connected to deliver the maximal load power.

Question 6) If each inductance is $1H$, show that the effective inductance is $\frac{5}{6}H$



A
B