* Used for compensator design (same as Root - locus)

* Older Method

1) System Identification (from real data)

2) Lead-lag design (SSE + rise time)

3) Comp. design for non-linear systems

4) Robustness design

b) What is the output of a linear system to a pure sinusoidal input?

\[ R(s) \rightarrow G(s) \rightarrow C(s) \]

\[ r(t) = A \cos \omega t + B \sin \omega t \]

\[ = \sqrt{A^2 + B^2} \cos \left[ \omega t - \tan^{-1} \frac{B}{A} \right] \]

\[ = M_i \cos \left[ \omega t + \phi_i \right] \]

\[ = M_i e^{j \phi_i} = A - jB = M_i / \phi_i \]

\[ C(s) = G_r(s) \frac{As + B\omega}{s^2 + \omega^2} = \frac{(As + B\omega)}{(s-j\omega)(s+j\omega)} \]

\[ = \frac{K_1}{s-j\omega} + \frac{K_2}{s+j\omega} + \text{Partial Frac. exp. of } G(s) \]

Assume \( G(s) \) is stable (All poles on open LHP)
Then \( C_{ss}(s) = \frac{k_1}{s-j\omega} + \frac{k_2}{s+j\omega} \)

{Calculate \( k_1, k_2 \) (let \( M_a = |c_e(i\omega)| \))

\[
\frac{M_i M_a e^{-j(\phi_i + \phi_a)}}{2(s+j\omega)} + \frac{M_i M_a e^{j(\phi_i + \phi_a)}}{2(s-j\omega)}
\]

\( C_{ss}(s) = M_i M_a \cos(\omega t + \phi_i + \phi_a) \)

\[
= \begin{bmatrix} M_a \frac{\phi_a}{|\phi_a|} \\ M_i \frac{\phi_i}{|\phi_i|} \end{bmatrix} \begin{bmatrix} M_i M_a \\ s 
\end{bmatrix}
\]

Freq. response = \( |G_e(i\omega)| / |G(i\omega)| \)

Q) How to plot \( |G_e(i\omega)| / |G(i\omega)| \)?

a) Separately \( \rightarrow \) Mag. plot \( \rightarrow \) \( |G_e(i\omega)| / |G(i\omega)| \) vs \( \omega \)

(Phase) Angle plot \( \rightarrow \) \( \theta \) vs \( \omega \)

\( \rightarrow \) For convenience, Mag plot is done with

\[
20 \log_{10} \frac{|G_e(i\omega)|}{|G(i\omega)|} \text{ vs } \log \omega
\]

\( \text{dB} \) vs \( \text{rad/sec} \)

\( \rightarrow \) Phase plot \( \phi = \frac{G_e(i\omega)}{G(i\omega)} \) vs \( \log \omega \)

\( \text{degrees} \) vs \( \text{rad/\omega} \)

b) Togethers \( \rightarrow \) As a sequence of complex nos on \( \text{E-plane} \) as \( \omega \) is varied.

Example: \( G(s) = \frac{1}{s+2} \)
a) Separate: Bode Plots

\[ G(j\omega) = \frac{1}{j\omega^2 + 1} \]

\[ |G(j\omega)| = \frac{1}{\sqrt{\omega^2 + 1}} \]

Phase plot: \(-\tan^{-1}(\frac{\omega}{2})\) vs \log \omega

b) Together: (Polar plot)

\[ \frac{1}{\sqrt{\omega^2 + 1}} \left( -\tan^{-1}(\frac{\omega}{2}) \right) \]

Exercise: Look up reason for "20°" in 20 log |G|.
\[ G(z) = \frac{K(z+z_1)(z+z_2)\ldots(z+z_K)}{z^n(z+p_1)(z+p_2)\ldots(z+p_n)} \]

\[ 20 \log |G(j\omega)| = 20 \log K + 20 \log |j\omega + z_1| \]
\[ + \ldots + 20 \log |j\omega + z_K| - 20 \log |(j\omega)^m| \]
\[ - 20 \log |j\omega + p_1| - \ldots - 20 \log |j\omega + p_n| \]

**Advantage:** Each term can be simply added to get a quick sketch.

Similarly, Total angle = \[ \frac{\log |j\omega + z_1| + \log |j\omega + z_2|}{\ldots} \]
\[ + \ldots + \log |j\omega + z_K| - \log |(j\omega)^m| - \log |j\omega + p_1| - \ldots \]
\[ - \log |j\omega + p_n| \]

**Bode plot for \( G(s) = (s+a) \)**
\[ G(j\omega) = (j\omega + a) \]

* For \( \omega << a \), \( G(j\omega) \approx \omega \) \; \text{and} \; |G(j\omega)| = 0^* \]
* For \( \omega >> a \), \( G(j\omega) \approx j\omega \) \; \text{and} \; 20 \log |G(j\omega)| = 20 \log |\omega| \rightarrow (1) \]
* \[ 20 \log |G(j\omega)| = 20 \log \omega \left( \frac{G(j\omega)}{G(j0^*)} = 90^\circ \right) \]
For \( \omega = \alpha \), \( A(j\omega) = (\alpha + j\alpha) \)

\[
\angle A(j\omega) = 45^\circ
\]

\[20 \log |10\alpha| = 20 + 20 \log |a|\] (2)

**NOTE:**

1. (2) is a straight line with a slope 20 dB/decade. Equivalently, 6 dB/decade.

\[
20 \log |2\alpha| = 20 \log 2 + 20 \log |a| = 6 + 20 \log |a|
\]

Phase Angle 45°  
45°/decade slope

**Exercise:** Draw Bode mag/phase plot for \( A(s) = s + 2 \)

\[
\begin{aligned}
A(s) &= s - 2 \\
\end{aligned}
\]
Normalization & Frequency Scaling

Aim: To get log-mag = 0 dB at break freq = 1.

\[(s+a) = a \left( \frac{s}{a} + 1 \right) = a(s_1 + 1) \quad \left[ s_1 = \frac{s}{a} \right] \]

Plot: 20 log \[\frac{\omega_1}{s_1 + 1}\] vs log \(\omega_1\),

\[\frac{\omega_1}{s_1 + 1}\] vs log \(\omega_1\).

- Useful for comparison between different systems
- Also for hand plotting.

\[G(s) = \frac{1}{s+a} \quad G(j\omega) = \frac{1}{j\omega+a} \]

For \(\omega < a\), \(G(j\omega) = \frac{1}{\omega} \)

\[20 \log|G(j\omega)| = -20 \log \omega \quad \angle G(j\omega) = 0^\circ \]

For \(\omega \gg a\), \(G(j\omega) = \frac{1}{j\omega} \)

\[20 \log|G(j\omega)| = 20 \log \left| \frac{1}{\omega} \right| = -20 \log \omega \quad \angle G(j\omega) = -90^\circ \]
\[ G(s) = \frac{1}{s} \quad G(j\omega) = \frac{1}{j\omega} \]

\[ 20 \log |G(j\omega)| = -20 \log |\omega| \]

\[ \angle G(j\omega) = -90^\circ \]
\[ G(s) = s^2 + 2 \omega_n s + \omega_n^2 \]
\[ G(j\omega) = (j\omega)^2 + 2\omega_n(j\omega) + \omega_n^2 \]

For \( \omega \ll \omega_n \) as \( \omega \to 0 \),
\[ \alpha(j\omega) = \omega_n^2 - 20 \log |G(j\omega)| = 20 \log (\omega_n^2) = 40 \log \omega_n \]
\[ \angle G(j\omega) = 0^\circ \]

For \( \omega \gg \omega_n \),
\[ \alpha(j\omega) = -\omega^2 \]
\[ 20 \log |G(j\omega)| = 20 \log (\omega^2) = 40 \log \omega \]
\[ \angle G(j\omega) = 180^\circ \]

These lines intersect at \( \omega = \omega_n \) (break freq)

Angle at break freq \( (\omega = \omega_n) \)
\[ G(j\omega) = (\omega_n^2 - \omega^2) + j2\omega \omega_n \omega \]

For \( \omega = \omega_n \),
\[ \angle G(i\omega) = 90^\circ \]

\[ 40 + 40 \log \omega_n \]

\[ 40 \log (\omega_n) \]

\[ 0 \to 0.1\omega_n \to \omega_n \to 10\omega_n \]

\[ 180^\circ \]

\[ 90^\circ \]

\[ 90^\circ / \text{decade} \]

\[ \text{shape} = 40 \text{ dB/decade} \]

\[ 40 \log 10 \omega_n \]

\[ = 40 \log 10 + 40 \log \omega_n \]

\[ = 40 + 40 \log \omega_n \]
Unlike 1st order terms, actual and asymptotic plots may differ widely.

\[ h = \sqrt{(\omega_n^2 - \omega^2) + (2\omega_n \omega)^2} \]

Phase = \( \tan^{-1} \frac{2\omega_n \omega}{\omega_n^2 - \omega^2} \)

\[ 20 \log \left( \frac{M}{\omega_n^2} \right) \]

\[ 0 \text{ dB} \]

\[ 0^\circ \]

\[ 0 \text{ dB} \]

\[ \frac{\omega}{\omega_n} \rightarrow \]

\[ \frac{\omega}{\omega_n} \rightarrow \]

\[ \frac{\omega}{\omega_n} \rightarrow \]

\[ \text{For } \omega \rightarrow 0, \quad G(\omega) = \frac{1}{\omega_n^2} \]
\[ 20 \log |A(j\omega)| = -40 \log \omega \quad \underline{\theta_A(j\omega) = 0^\circ} \]

For \( \omega \gg \omega_n \), \( A(j\omega) \approx \frac{1}{\omega^2} \)

\[ 20 \log |A(j\omega)| = -40 \log \omega \quad \underline{\theta_A(j\omega) = -180^\circ} \]

Example: \( G(s) = \frac{K(s+3)}{s(s+1)(s+2)} \)

Closest \( K \approx 1 \)

\[ = \frac{3K}{2} \frac{\left( \frac{s}{3} + 1 \right)}{s(s+1)\left( \frac{s}{2} + 1 \right)} \]

- Break freq. are 1, 2, 3
- May plot should extend from one decade below lowest break freq. to one decade beyond largest break freq.
- Range 0.1 rad to 100 rad.
For getting the actual plot

\[ 20 \log \left| \frac{3K}{2} \right| \] should be added to this.
Contour Mapping: Let \( F(s) = \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)} \ldots \)

1. Plane \( A \) \( \rightarrow \) \( F(s) \rightarrow \) \( A' \)
   - Contour \( B \)

Examples:

Assume \( A \) is clockwise

1) \( F(s) = (s-z_1) \)
   - Direction of contour \( B \) mapping is clockwise

2) \( F(s) = \frac{1}{s-p_1} \)
   - Direction of \( B \) is clockwise

3) \( F(s) = s - z_1 \)
   - \( R = V \)
4) Encircles origin in clockwise direction

\[ F(s) = \frac{1}{s-p_1} \]

5) Encircles origin in anti-clockwise direction

\[ F(s) = \frac{s-z_1}{s-p_1} \]

\[ R = \frac{V_1}{V_2} \]

\[ \text{DOES NOT ENCIRCLE ORIGIN} \]

(Direction depends on relative position of poles/zeros)

FACT: No of counter-clockwise encirclements of the origin (assuming \(A\) is clockwise)

= No of poles of \(F(s)\) in \(A\)

- No of zeroes of \(F(s)\) in \(A\)

[Implicit assumption:

\[ F(s) = \frac{(s-z_1)(s-z_2) \cdots}{(s-p_1)(s-p_2) \cdots} \] ]
\[ F(s) = \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)(s-p_3)} \]

- \( V_1, V_2, V_3 \) rotate 360° along \( A \)
- \( V_4, V_5 \) rotate 0° along \( A \)

1) \( V_1, V_2 \) contribute \( (+360° \times 2) \) of rotation in \( R \).
2) \( V_3 \) contribute \(-360°\) of rotation in \( R \).
3) \( V_4, V_5 \) contribute 0° of rotation in \( R \).

\[ (V_1, ..., V_5) \text{ contribute } (2 \times 360° - 360°) = 360° \text{ of rotation in } R \]

\[ = -1 \text{ clockwise enclosure of the origin} \]

\[ = -1 \text{ counter-clockwise enclosure of the origin} \]

So our **FACT** holds.
\[
\frac{1}{A} - \frac{2}{B} = -1
\]

Stability of closed loop from open loop i.e.

[We already know 2 different methods of doing this]

Nyquist Criterion

\[
G(s) = \frac{N_a(s)}{D_a(s)} \\
H(s) = \frac{N_H(s)}{D_H(s)}
\]

\[
G(s)H(s) = \frac{N_a(s)N_H(s)}{D_a(s)D_H(s)}
\]

\[
1 + G(s)H(s) = \frac{D_a(s)D_H(s) + N_a(s)N_H(s)}{D_a(s)D_H(s)}
\]

\[
T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{N_a(s)N_H(s)}{D_a(s)D_H(s) + N_a(s)N_H(s)}
\]

\(\text{POLES of } G(s)H(s) = \text{POLES } 1 + G(s)H(s)\)
2) ZEROS of \(1 + G(s)H(s)\) = POLES of \(T(s)\) (closed loop poles)

In FACT, use (i) \(F(s) = 1 + G(s)H(s)\)

(ii) Contour \(A = \text{entire RHP}\)

\[ \begin{array}{c}
\int_{\gamma} F(s) ds \\
\text{contour } B
\end{array} \]

\[ 1 + G(s)H(s) \]

**FACT**: NO of counterclockwise encirclements of origin by \(B\) \(= N\)

\(= \) NO of poles of \([1 + G(s)H(s)]\) on RHP

\(-\) NO of zeros of \([1 + G(s)H(s)]\) on RHP

\(=\) NO of open loop poles on RHP (known) \(= P\)

\(-\) NO of closed loop poles on RHP (unknown) \(= Z\)

So \(N = P - Z\) or \(Z = P - N\)

Simplification: Take \(F(s) = G(s)H(s)\)

Everything else same.

FACT: NO of counterclockwise encirclement
of \((-1+j0)\) by \(B (=: N)\)

\[= \text{No of open loop poles on RHP (=: P)} - \text{No of closed loop poles on RHP (=: Z)}\]

[This is known as Nyquist Criterion and the plot of \(G(s)H(s)\) contour is called Nyquist plot]

(2) Why is Nyquist Criteria/plots classified as freq. response?

Examples:

1) \(s\)-plane

2) \(s\)-plane

\[P = 0\]

\[Z = P - N = 2 \Rightarrow 2 \text{ closed loop poles on RHP}\]
Sketching Nyquist Diagrams

\[ G(s) = \frac{500}{(s+1)(s+3)(s+10)} \]

Plot from A to C

\[ B' = \frac{500}{V_1V_2V_3} \]

From A to C

\[ Q + O_2 + O_3 \quad \text{ga} \]

for 0° to 270°

\( \Rightarrow B' \) goes from 0° to -270°

For A to C, Nyquist plot = Polar plot of \( G(j\omega)H(j\omega) \)

Exercise: Derive analytical expression for \( G(j\omega) \) (\( \omega = 0 \) to \( \infty \))

Plot for 0 to D° Does not matter here since

the magnitude = 0.

\( G, H \) plan

Angle change +3 × 180°

Radius = 0.

Plot from D to A

\[ \text{Re} \left[ G(j\omega)H(j\omega) \right] = \text{even function} \]

\[ \text{Im} \left[ G(j\omega)H(j\omega) \right] = \text{odd function} \]
Exercise: Prove:

So the Nyquist plot for $G(s)$ is a mirror image (about Re axis) of the $N-$ plot for $AC$.

Nyquist Plots for $G(s)$ with open loop poles on Imaginary axes

−1

Problem

Detour radius

$\approx 0$

Example: $G(s) = \frac{s+2}{s^2}$
An ellipse with labeled points and vectors. The text includes:

- A total angle \( \theta \) at \( B \) is calculated as \(-2 \times 90° + 0°\), resulting in a magnitude of \(-180°\).

- The angle change for \( AB \) is calculated as \(90° - 2 \times 90° = -90°\).

- The magnitude at \( B = 0 \) is indicated with a limit expression: \( \lim_{p \to 0} \frac{V_2}{V_1} = 0 \).

- The angle \( BC2 \) is indicated as \( 0° \) (one zero length / 2 pole length).

- The total angle change for \( AB \) is \(-180° + 2 \times 180° = 360°\).

- The image \( DE \) is the mirror image of \( AB' \).

- The magnitude at \( EFA \) is indicated as \( \infty \) (pole length finite / 2 zero pole length).

- The zero angle does not change. Total angle change is \(-2 \times 180° = -360°\).

- The test radius \( N = 0 \) and \( P = 0 \).

- The pole \( Z = P - N = 0 \) (No zero, 4 poles in RH region).

- The rule \( T(8) = \frac{s + 2}{s^2 + s + 2} \) is simplified as \( \frac{s + 2}{s^2 + 3s + 2} \).
\( c_i \text{ pole } = -1 \pm j \sqrt{7} \frac{2}{2} \)

**Stability via Nyquist Dig.**

\[
\begin{array}{c}
R(s) \quad \bullet \quad \frac{K(s+3)(s+5)}{(s-2)(s-4)} \quad C(s)
\end{array}
\]

Q) For what values of \( K \) C.L. is stable?

A1) \( \rightarrow \) Using Routh-Hurwitz criterion

A2) \( \rightarrow \) Using Root-Locus

A3) \( \rightarrow \) Unity Nyquist Diagram

**Steps:**

\( \quad \) Draw Nyquist Dig with \( K=1 \)

\[
\begin{array}{c}
\text{Draw Nyquist Dig with } K=1
\end{array}
\]

\[
N = 2, \quad P = 2
\]

\[
\Rightarrow Z = P - N = 0 \quad \Rightarrow \text{C.L. stable for } K = 1.
\]

For \( K < \frac{1}{1.33} \), \( N = 0 \), \( P = 2 \), \( Z = 2 \)

\[
K = \frac{1}{1.33} \quad \Rightarrow \text{marginally unstable}
\]

\( \quad \) Alternatively, one can keep Nyq. Dig.

\( \quad \) fixed and imagine the critical pt at \( \frac{1}{K} \).
*NOTE: If the Nyquist plot intersects the real axis at \(-1 + j0\)

\[ \Rightarrow G(j\omega) H(j\omega) = -1 \]

\[ \Rightarrow \text{Root-locus crosses the } j\omega \text{-axis.} \]

\[ \Rightarrow \text{Marginal stability} \]

\[ \text{Stability via mapping alg. onto } j\omega \text{-axis} \]

* Simple forms of Nyquist criterion

1) System is stable for those values of gain \(K\), for which

\[ |G(j\omega) H(j\omega)| < 1 \text{ for that value of } \omega \], where \[\angle G(j\omega) H(j\omega) = \pm 180^\circ\]
System is stable for these values of gain \( K \), for which \(|a(j\omega)h(j\omega)| > 1\) for that value of \( \omega \), where \( |a(j\omega)h(j\omega)| = \pm 180^\circ \)

**Example:**

\[
\begin{array}{c}
\text{Find range of } K \text{ for stability, marginal stability, instability.}
\end{array}
\]

No RHP poles \((0,1)\)

\( \Rightarrow \) For stability \( \Rightarrow \) no encirclements of \((-1+j0)\)

\( \Rightarrow \ |a(j\omega)| < 1 \) for these \( \omega \) where \(|a(j\omega)| = \pm 180^\circ\)

Let \( K=1 \) initially and draw \( AB' \).

\[
\begin{array}{c}
A + A \rightarrow \\
\text{Mag} \rightarrow \frac{1}{2\sqrt{2}} - \frac{1}{4}
\end{array}
\]

\[
\begin{array}{c}
\text{At} \ A \rightarrow -\left[0^\circ - 45^\circ + 45^\circ\right] = 0^\circ
\end{array}
\]
At \( B \rightarrow -\left[ 90^\circ + 90^\circ + 90^\circ \right] = -270^\circ \)
\( \text{Mag} \rightarrow 0 \)

To find \( P \): \( \text{Re}(j\omega) = \frac{1}{(s^2 + 2s + 2)(s+2)} \bigg|_{s=j\omega} \)

Putting \( \text{Im}(\text{Re}(j\omega)) = 0 \) we get \( \omega = \sqrt{6} \)

Putting \( \omega = \sqrt{6} \) in \( \text{Re}(\text{Re}(j\omega)) = -\frac{1}{20} \)

Hence if \( K < 20 \), system stable.
\( K = 20 \), marginally stable.
\( K > 20 \), unstable.

Q. What is the freq of oscillation when \( K = 20 \)?

Gain Margin & Phase Margin

Assume: system is stable if there are no encirclements of \( -1 \).

Phase diff before instability.

Gain diff before instability.

# Two ways for this system to become unstable:
At phase $\pm 180^\circ$, gain $\frac{1}{a}$ should increase to 1.

At gain $1$, phase $\pm 180^\circ + \alpha$ should reduce to $\pm 180^\circ$.

Gain Margin is the change in O.I. gain (in dB) required at $180^\circ$ of phase shift to make C.L. system unstable.

Phase margin is the change in O.I. phase shift required at unity gain to make the C.L. system unstable.

G.M. & P.M. are measures of robustness = Quantitative measures of how much stable = How much change in system parameters it can withstand before becoming unstable.

In Root Locus technique, same information is provided by the distance of the C.L. poles from imaginary axis.


\[ GM = 20 \log \frac{20}{6} = 10.45 \text{ dB} \]

\[ PM = \alpha = 67.7^\circ \]
A.M. & P.M. are difficult to find (computationally) on the Nyquist plot.

→ We use BODE PLOTS instead.

→ Subsets of Nyquist Diagram

→ Easy to draw (unlike Nyquist d. R-L)

**BODE PLOTS to Stability**

Example: \[ \frac{K}{(s+2)(s+4)(s+5)} \]

Determine range of \( K \) for which C.L. system is stable:

∀ All O.L. poles are on C.H.P. → No encirclements of \((-1+jo)\) by Nyq.Plot, for

stability \[ |\tilde{G}(jo)| < 1 \] for \( \omega \)

satisfying \[ \tilde{G}(jo) = \pm 180^\circ \]

Plot BODE mag & phase:

\[ G_o(s) = \frac{K}{(s+2)(s+4)(s+5)} = \frac{K}{40} \frac{\left( \frac{s}{2} + 1 \right) \left( \frac{s}{4} + 1 \right) \left( \frac{s}{5} + 1 \right)}{(s+2)(s+4)(s+5)} \]

Let \( K = 40 \) to start off:

\[ 0 \text{dB/dec} \]

\[ 20 \text{dB/dec} \]

\[ 40 \text{dB/dec} \]

\[ 60 \text{dB/dec} \]

0.1 1 2 4 5 10 100
\[ \angle G(j\omega) = -180^\circ \text{ at } \omega = 7 \text{ rad/sec} \]

At \( \omega = 7 \text{ rad/sec} \), \( |G(j\omega)| = -20 \text{ dB} \)

Hence increase of +20 dB is possible before instability.

\[ 20 \text{ dB} = 20 \log_{10} K_1 \]

\[ K_1 = 10 \]

But \( K = 40 \) was already chosen.

So the system will remain stable up to \( K = 40 \times 10 = 400 \).

Hence stable \( \rightarrow 0 < K < 400 \).

* This is approximate since asymptotes were used.

**Gain Margin & Phase Margin**

Gain Margin & Phase Margin from Bode plots.

**Gain plot**

\( 0 \text{ dB} \)

\( K_M \rightarrow \text{Gain margin} \)
Example: In the example above
\[ G_M = 20 \text{ dB} \]
\[ \Phi_M = 180^\circ + \text{Phase at 2 rad/sec} \]

**IMPORTANT NOTE**:
A BODE plot (Nyq. S) is magnitude & phase of O.C.L.
While P.M. a M an amplitude is of the O.C.L. system.

* There has been no need of O.C.L. freq response until now.

C.L. Transient Resp. from C.L. Freq. Response
(2nd order systems)

\[ \omega_n^2 \frac{1}{s(s + 2p_0\omega_n)} \quad \Rightarrow \quad T(s) = \frac{\omega_n^2}{s^2 + 2p_0\omega_n s + p_0^2} \]

\[ M = |T(j\omega)| = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + 4p_0^2\omega_n^2 \omega^2}} \]
1) YOS from BODE PLOT (\(M_p\))

Hence one can measure \(M_p\) from BODE PLOT

\[ M_p \rightarrow \text{Calculate } s \rightarrow \text{Calculate } YOS \]

Peak occurs only for \(s < 0.707\)

Recall YOS in step response occurs for \(0 < s < 1\)

2) \(T_s\) and \(T_p\) from BODE PLOT (\(\omega_B\))

Bandwidth of C.L. freq response is the freq at which the magnitude curve is 3 dB below its value at zero freq. \(\rightarrow \omega_B\)

Putting \(M = \frac{1}{\sqrt{2}}\) in (1) & \(T_s = \frac{4}{8\omega_n}\)

\[ \omega_B = \frac{4}{T_s\omega} \sqrt{(1-2s^2)+\sqrt{4s^4-4s^2+2}} \]

Using, \(\omega_n = \frac{T}{T_p\sqrt{1-s^2}}\)
\[ \omega_{BW} = \frac{T_p}{k_p} \sqrt{1 - \frac{\delta^2}{\omega_n^2}} \left( \sqrt{4 \delta^4 - 4 \delta^2} + \sqrt{1 - 2 \delta^2} \right) \]

C.L. Transient Resp from O.L. Frequency Resp.

Damping Ratio from P.M.

\[ \frac{\omega_n^2}{\sqrt{\delta (\delta + 2 \delta \omega_n)}} \]

Put:

\[ |\alpha(j\omega)| = \frac{\omega_n^2}{\sqrt{-\omega^2 + \delta^2 \omega^2}} = 1 \quad \text{②} \]

Solving ② for \( \omega \):

\[ \omega_1 = \omega_n \sqrt{-2 \delta^2 + \sqrt{1 + 4 \delta^4}} \]

\[ \angle \alpha(j\omega) = -90^\circ - \tan^{-1} \left[ \frac{\sqrt{-2 \delta^2 + \sqrt{1 + 4 \delta^4}}}{2 \delta} \right] \]

Phase margin: \( \phi_M = 180^\circ + \angle \alpha(j\omega) \)

\[ = 90^\circ - \tan^{-1} \left[ \frac{\sqrt{-2 \delta^2 + \sqrt{1 + 4 \delta^4}}}{2 \delta} \right] \]

\[ = \tan^{-1} \left[ \frac{2 \delta}{\sqrt{-2 \delta^2 + \sqrt{1 + 4 \delta^4}}} \right] \]

SSE (\( k_p, k_v \) and \( k_a \)) from (O.L.) Bode Plots

\[ \alpha(s) = \frac{1}{\sqrt{\pi}} \frac{k_p}{(s+Zi)} \quad \text{Type zero} \]
In Bode magnitude plot

\[ 20 \log K_p = 20 \log k \frac{\bar{N} z_i}{\bar{N} p_i^*} \]

"Type 1" \[ G_c(s) = K \frac{\bar{N}(s+z_i)}{s \bar{N}(s+p_i)} \]

The first part eqn:

\[ k \frac{\bar{N} z_i}{\omega \bar{N} p_i^*} \]

Find where \( \omega \) intersects 0 dB line:

\[ K \frac{\bar{N} z_i}{w \bar{N} p_i} = 1 \Rightarrow \omega_1 = k \frac{\bar{N} z_i}{\bar{N} p_i} = K \]

"Type 2" \[ G_c(s) = K \frac{\bar{N}(s+z_i)}{s^2 \bar{N}(s+p_i)} \]

Eqn for 1st part:

\[ k \frac{\bar{N} z_i}{w_1 \bar{N} p_i} = 1 \]

\[ \omega_1 = \sqrt{\frac{k \bar{N} z_i}{\bar{N} p_i}} = \sqrt{K_a} \]