Lecture 10: Bode Plots + Nyquist Diagrams

Note Title 28-03-2010

* Used for Compensator Design (same as Root-locus)
* Older Method
* Useful for $\longrightarrow$
  1) System Identification (from real data)
  2) Lead comp design (SSE + tr. resp)
  3) Comp. design for non-linear systems
  4) Robustness design

Q) What is the output of a linear system to a purely sinusoidal input?

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

$$r(t) = A\cos\omega t + B\sin\omega t$$

$$= \sqrt{A^2+B^2}\cos\left[\omega t - \tan^{-1}\frac{B}{A}\right]$$

$$= M_i \cos\left[\omega t + \Phi_i\right]$$

$$= M_i e^{j\Phi_i} = A - jB = M_i \angle \Phi_i$$

$$C(s) = G(s)\frac{As+B\omega}{s^2+\omega^2} = \frac{(As+B\omega)\,G(s)}{(s+j\omega)(s-j\omega)}$$

$$= \frac{K_1}{s-j\omega} + \frac{K_2}{s+j\omega} + \text{Partial frac exp. of } G(s)$$

Assume $G(s)$ is stable (All poles on open LHP)

Then $C_{ss}(s) = \dfrac{K_1}{s-j\omega} + \dfrac{K_2}{s+j\omega}$

$$\left\{ \text{Calculate } K_1, K_2 \quad \left(\text{Let } \begin{array}{l} M_G = |G(j\omega)| \\ \phi_G = \angle G(j\omega) \end{array} \right) \right\}$$

$$= \frac{\frac{M_i M_G}{2} e^{-j(\phi_i+\phi_G)}}{(s+j\omega)} + \frac{\frac{M_i M_G}{2} e^{j(\phi_i+\phi_G)}}{s-j\omega}$$

$$C_{ss}(t) = M_i M_G \cos(\omega t + \phi_i + \phi_G)$$

$$= \underbrace{\left[M_G \angle \phi_G \right]}_{\text{Freq. response } = |G(j\omega)| \angle G(j\omega)} \left[M_i \angle \phi_i \right]$$

Q) How to plot $|G(j\omega)| \angle G(j\omega)$?

a) Separately $\rightarrow$ Mag. plot $\rightarrow |G(j\omega)|$ vs ω
(Phase) Angle plot $\rightarrow \angle G(j\omega)$ vs ω

$\rightarrow$ For convenience, Mag plot is done with $20\log_{10}|G(j\omega)|$ vs $\log \omega$
$\hookrightarrow \boxed{dB}$ $\hookrightarrow \boxed{\text{rad/sec}}$

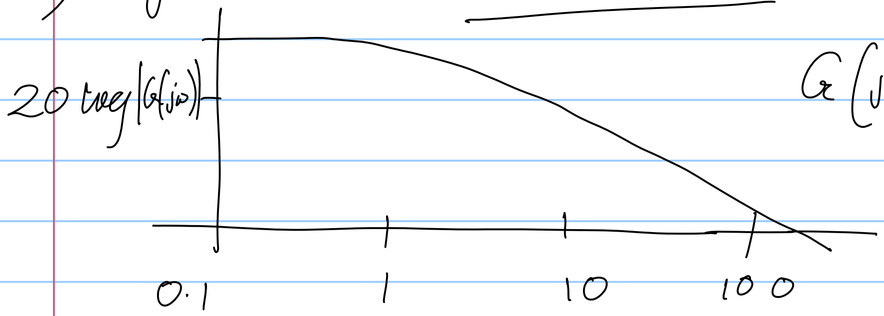
$\rightarrow$ Phase plot : $\underbrace{\angle G(j\omega)}_{\boxed{\text{degrees}}}$ vs $\log \omega$ $\hookrightarrow \boxed{\text{rad/sec}}$

b) Together $\rightarrow$ As a sequence of complex nos on $\mathbb{C}$-plane as ω is varied.

Example: $G(s) = \dfrac{1}{s+2}$

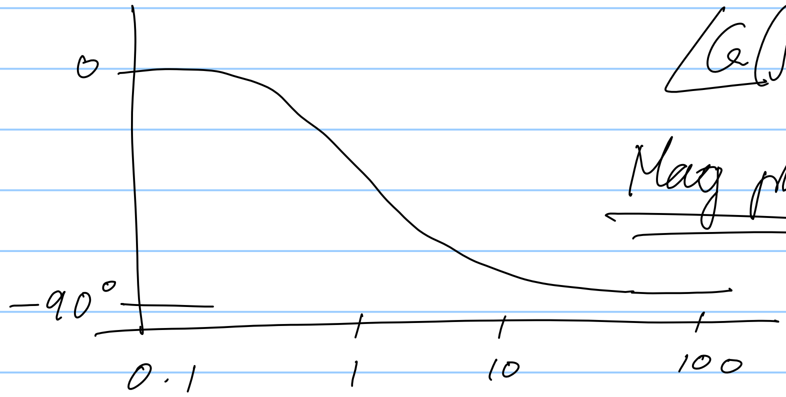
Hendrik Bode

a) Separate : Bode Plots



$$G(j\omega) = \frac{1}{j\omega + 2}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\omega^2+4}}$$

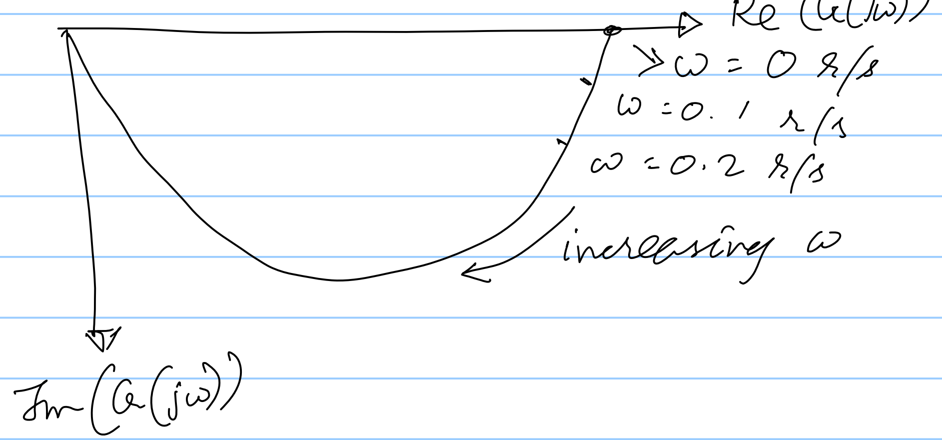


$$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{2}\right)$$

Mag plot : $20 \log \left| \frac{1}{\sqrt{\omega^2+4}} \right|$ vs $\log \omega$

Phase plot : $-\tan^{-1} \frac{\omega}{2}$ vs $\log \omega$

b) Together : (Polar plot) : $\frac{1}{\sqrt{\omega^2+4}} \angle -\tan^{-1}\frac{\omega}{2}$ for diff ω.



Exercise : Look up reason for "20" in $20 \log |\cdot|$.

Asymptotic Approximations of Bode Plots

$$G(s) = \frac{K(s+z_1)(s+z_2)\cdots(s+z_k)}{s^m(s+p_1)(s+p_2)\cdots(s+p_n)}$$

$$20\log|G(j\omega)| = 20\log K + 20\log|j\omega+z_1|$$
$$+ \cdots + 20\log|j\omega+z_k| - 20\log|(j\omega)^m|$$
$$- 20\log|j\omega+p_1| - \cdots - 20\log|j\omega+p_n|$$

* Advantage: Each term can be simply added to get a quick sketch.

* Similarly Total angle $= \angle j\omega+z_1 + \angle j\omega+z_2$
$+ \cdots \angle j\omega+z_k - \angle (j\omega)^m - \angle j\omega+p_1 - \cdots$
$\cdots - \angle j\omega+p_n$

Bode plot for $G(s) = (s+a)$

$G(j\omega) = (j\omega+a)$

* For $\omega \ll a$, $G(j\omega) \approx a$; $\angle G(j\omega) = 0°$

In dB, $20\log|G(j\omega)| = 20\log|a|$ $\longrightarrow$ (1)

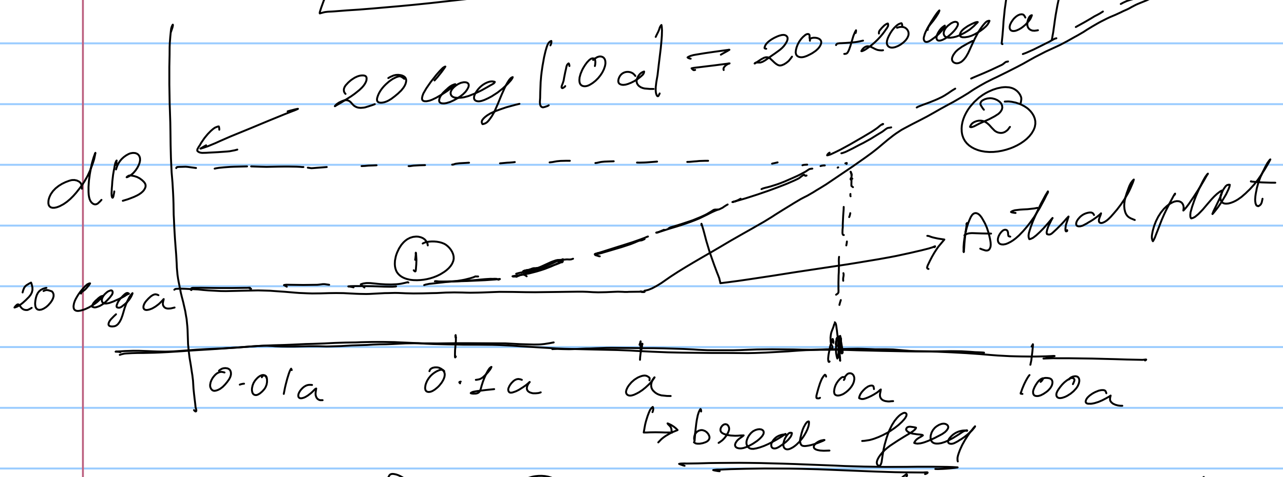
* For $\omega \gg a$, $G(j\omega) \approx j\omega$

$20\log|G(j\omega)| = 20\log\omega$ $\longrightarrow$ (2)

$\angle G(j\omega) = 90°$

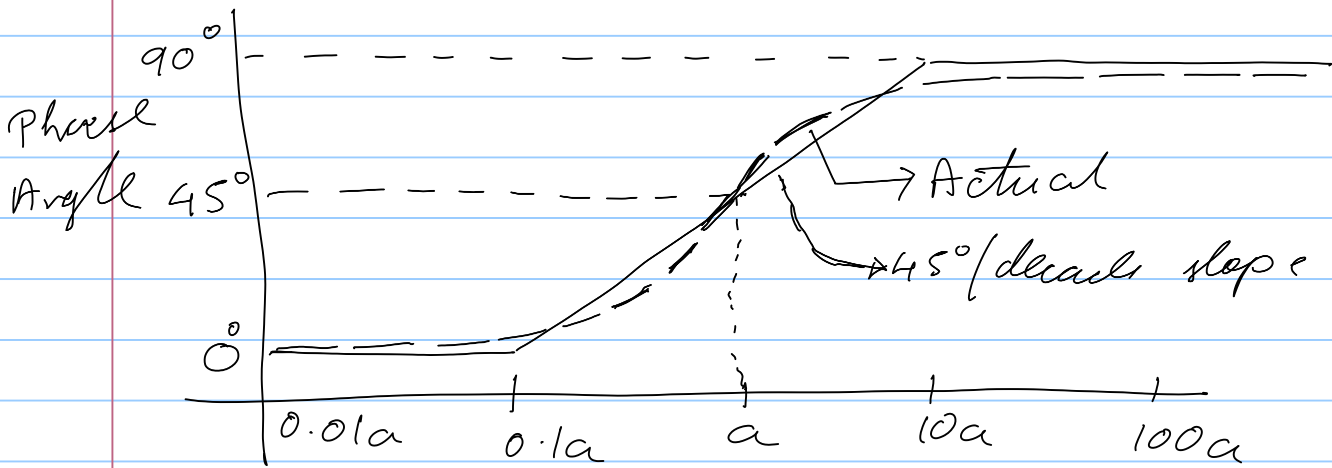
* For $\omega = a$, $G(j\omega) = (a + ja)$

$$\angle G(j\omega) = 45^\circ$$



NOTE: 1) (2) is a straight line with a slope 20 dB/decade
Equivalently 6 dB/octave

$$20 \log |2a| = 20 \log 2 + 20 \log a$$
$$= 6 + 20 \log a$$



Phase - plot

Exercise: Draw Bode mag/phase plot for $G(s) = s + 2$
$G(s) = s - 2$

Normalization & Freq Scaling

Aim: To get log-mag = 0 dB at break freq = 1.

$$(s+a) = a\left(\frac{s}{a}+1\right)$$
$$= a(s_1+1) \qquad \left[s_1 = \frac{s}{a}\right]$$

Plot: $20\log|j\omega_1+1|$ vs $\log\omega_1$

$\angle j\omega_1+1$ vs $\log\omega_1$

* Useful for comparison between different systems
* Also for hand plotting.

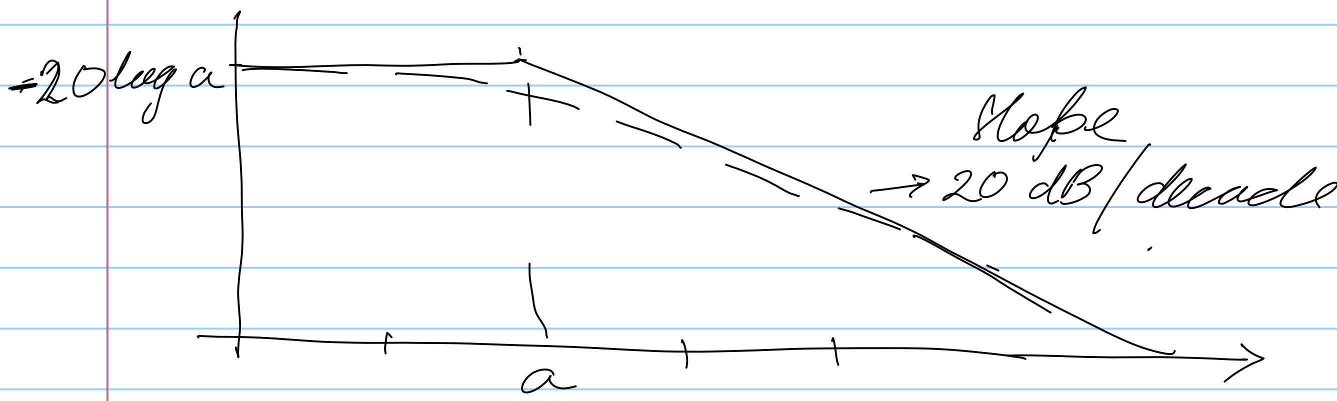
$$G(s) = \frac{1}{s+a} \qquad G(j\omega) = \frac{1}{j\omega+a}$$

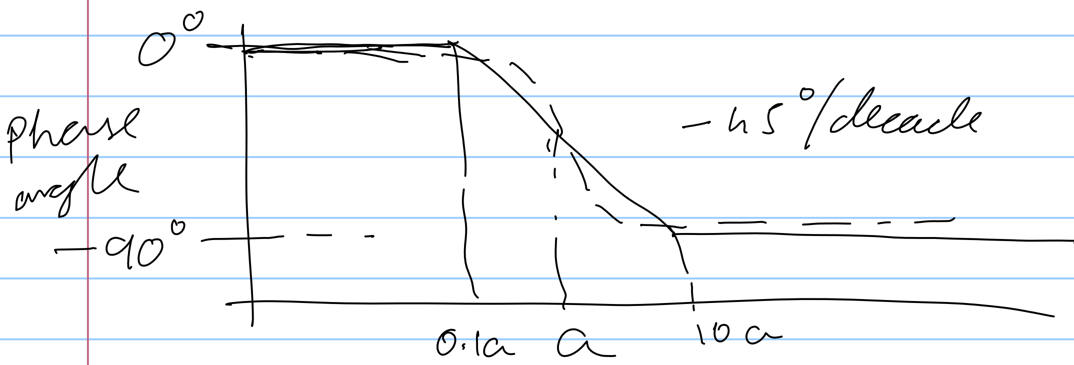
For $\omega \ll a$, $G(j\omega) = \frac{1}{a}$

$$20\log|G(j\omega)| = -20\log a$$
$$\angle G(j\omega) = 0^\circ$$

For $\omega \gg a$, $G(j\omega) = \frac{1}{j\omega}$

$$20\log|G(j\omega)| = 20\log\left|\frac{1}{\omega}\right| = -20\log\omega$$
$$\angle G(j\omega) = -90^\circ$$

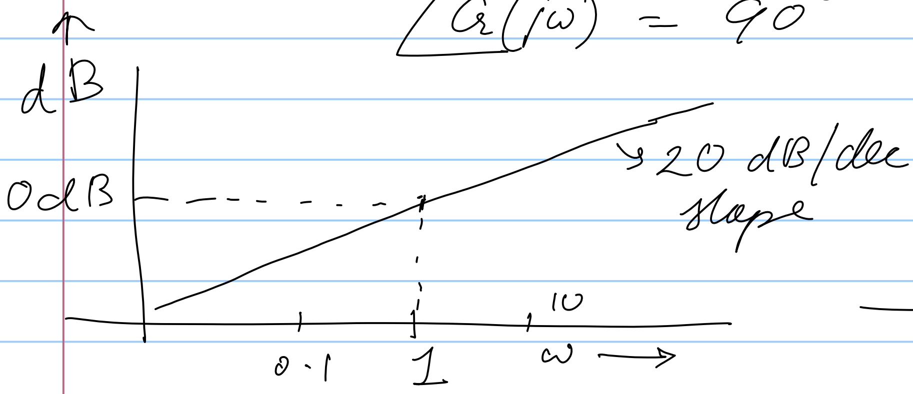


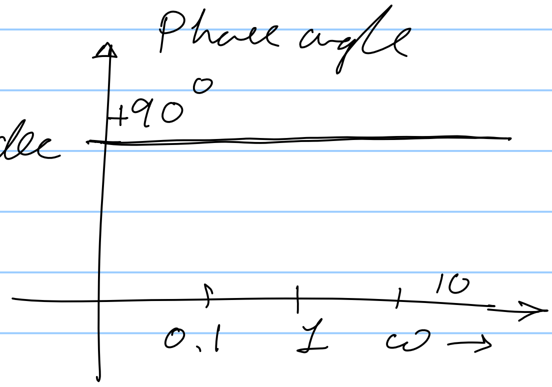


$G(s) = s$ $\qquad$ $G(j\omega) = j\omega$

$$20 \log |G(j\omega)| = 20 \log |\omega|$$

$$\angle G(j\omega) = 90^\circ$$

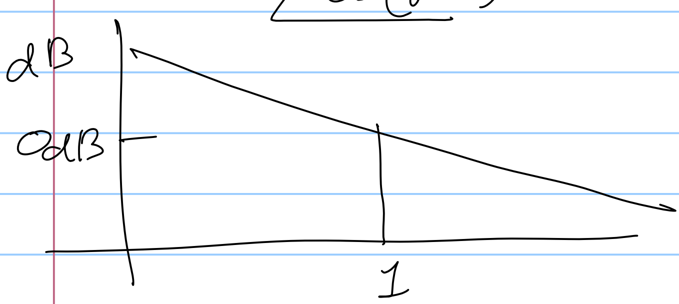


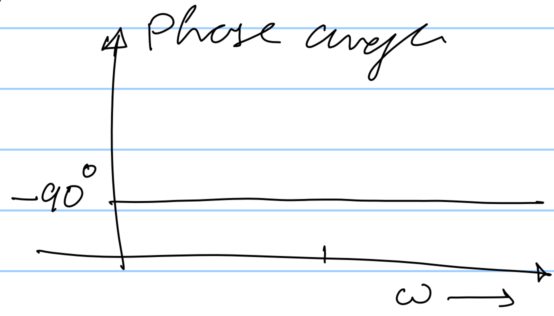


$G(s) = \frac{1}{s}$ $\qquad$ $G(j\omega) = \frac{1}{j\omega}$

$$20 \log |G(j\omega)| = -20 \log(\omega)$$

$$\angle G(j\omega) = -90^\circ$$





$G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$

$G(j\omega) = (j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2$

For $\omega \ll \omega_n$ or as $\omega \to 0$, $G(j\omega) = \omega_n^2$

$20\log|G(j\omega)| = 20\log(\omega_n^2) = 40\log\omega_n$

$\angle G(j\omega) = 0^\circ$

For $\omega \gg \omega_n$, $G(j\omega) = -\omega^2$

$20\log|G(j\omega)| = 20\log(\omega^2) = \underline{40\log\omega}$

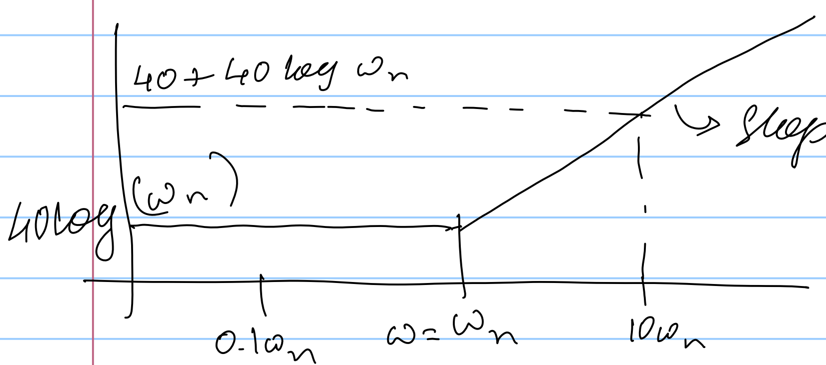
$\angle G(j\omega) = 180^\circ$

* These lines intersect at $\omega = \omega_n$ (break freq)

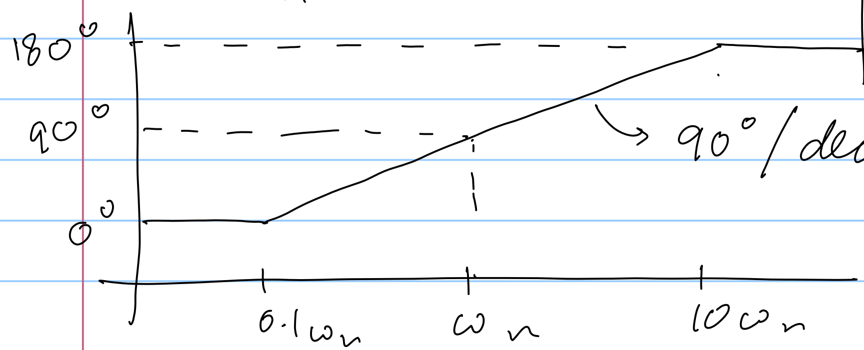
<u>Angle at break freq</u> $(\omega = \omega_n)$

$G(j\omega) = (\omega_n^2 - \omega^2) + j2\zeta\omega_n\omega$

For $\omega = \omega_n$, $\angle G(j\omega) = 90^\circ$



$40\log 10\omega_n$
$= 40\log 10 + 40\log\omega_n$
$= 40 + 40\log\omega_n$

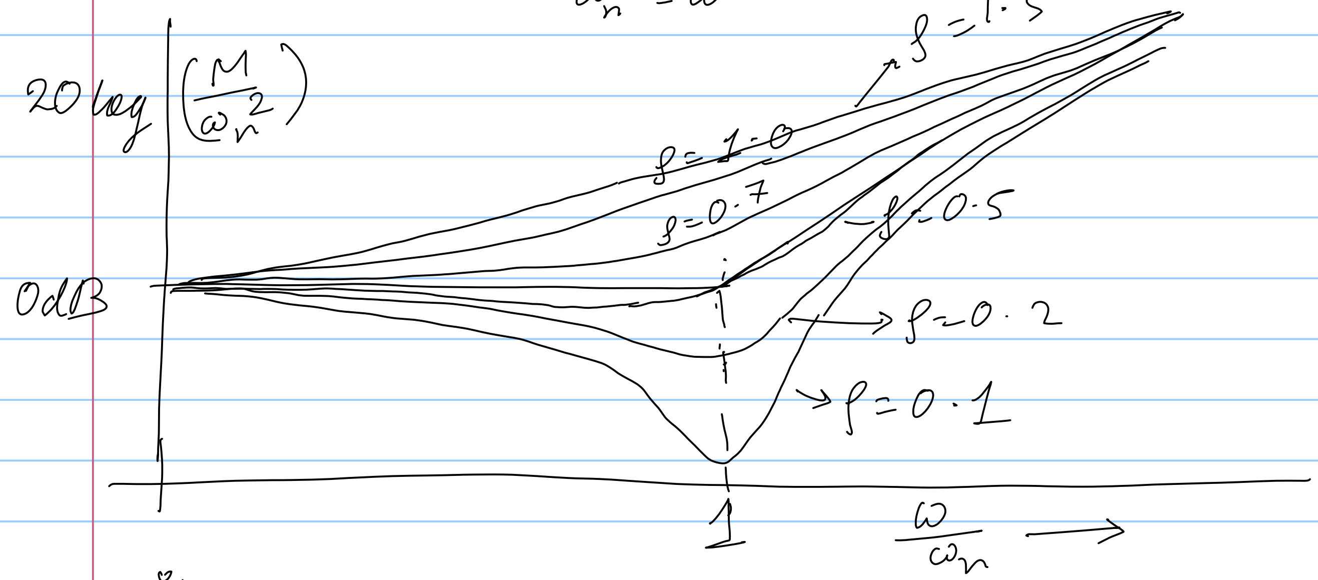


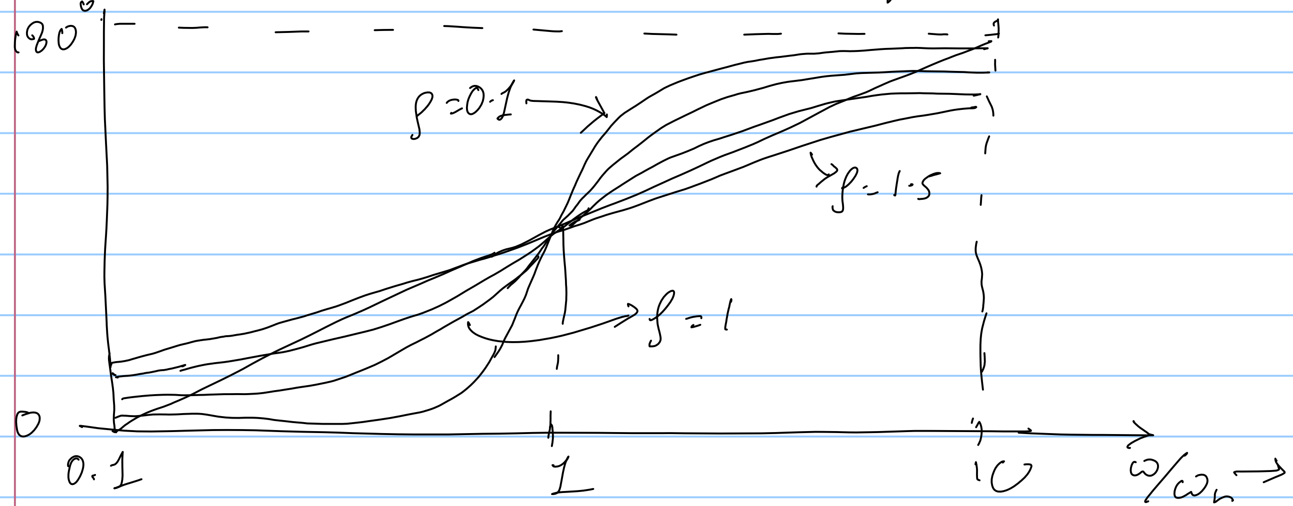
Unlike 1^{st} order terms: Actual and Asymptotic plots may differ widely.

→ <u>Disparity depends on ζ</u>

$$M = \sqrt{(\omega_n^2-\omega^2) + (2\zeta\omega_n\omega)^2}$$

$$\text{Phase} = \tan^{-1}\frac{2\zeta\omega_n\omega}{\omega_n^2-\omega^2}$$





$$G(s) = \frac{1}{s^2+2\zeta\omega_n s+\omega_n^2} \qquad G(j\omega) = \frac{1}{(\omega_n^2-\omega^2)+j(2\zeta\omega_n\omega)}$$

For $\omega \to 0$, $G(j\omega) = \frac{1}{\omega_n^2}$

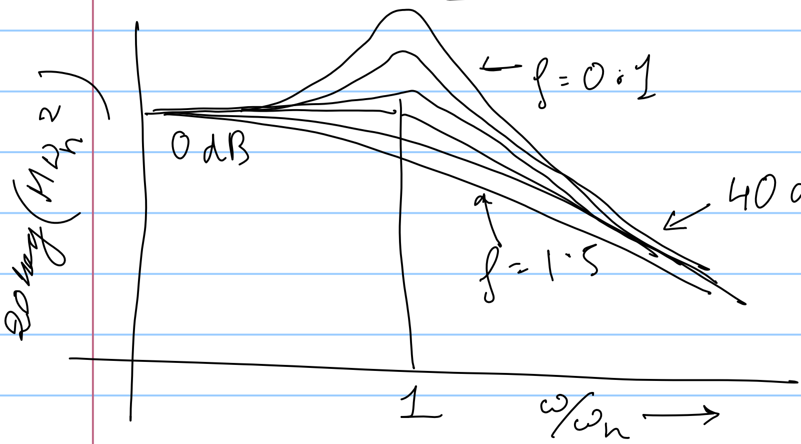
$$20 \log |G(j\omega)| = -40 \log \omega_n$$

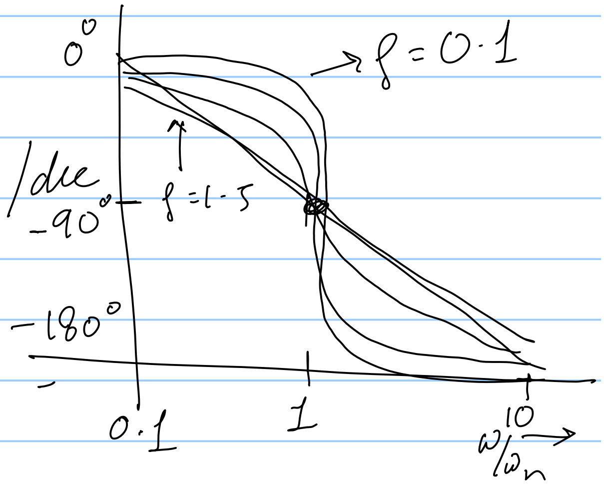
$$\angle G(j\omega) = 0^\circ$$

For $\omega \gg \omega_n$, $G(j\omega) \simeq \frac{1}{\omega^2}$

$$20 \log |G(j\omega)| = -40 \log \omega$$

$$\angle G(j\omega) = -180^\circ$$





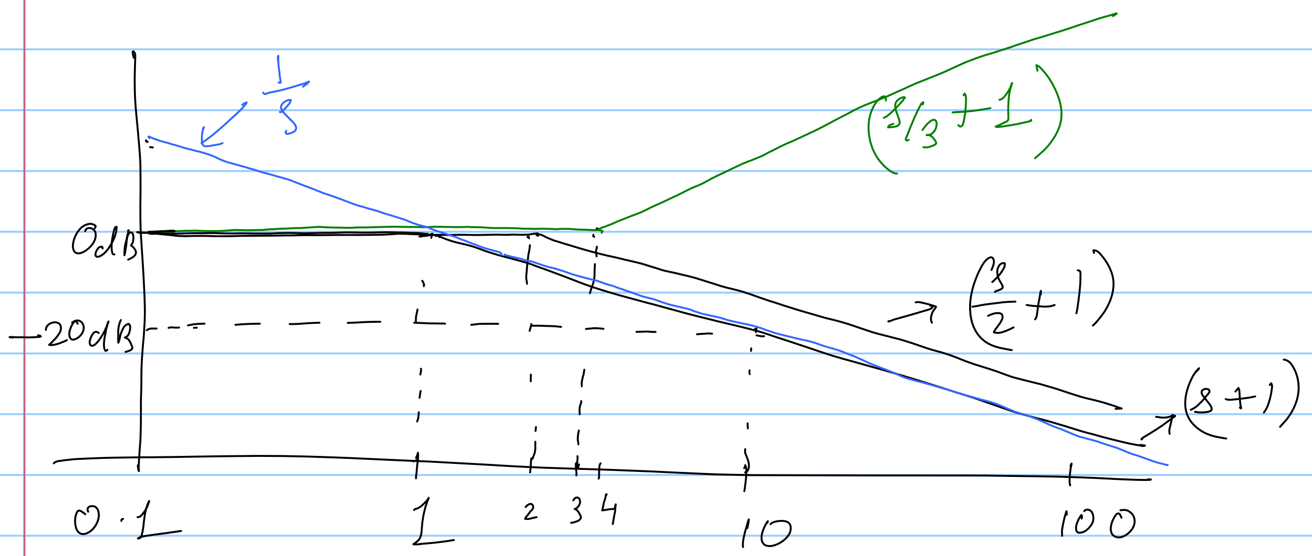
<u>Example</u>: $G(s) = \dfrac{K(s+3)}{s(s+1)(s+2)}$

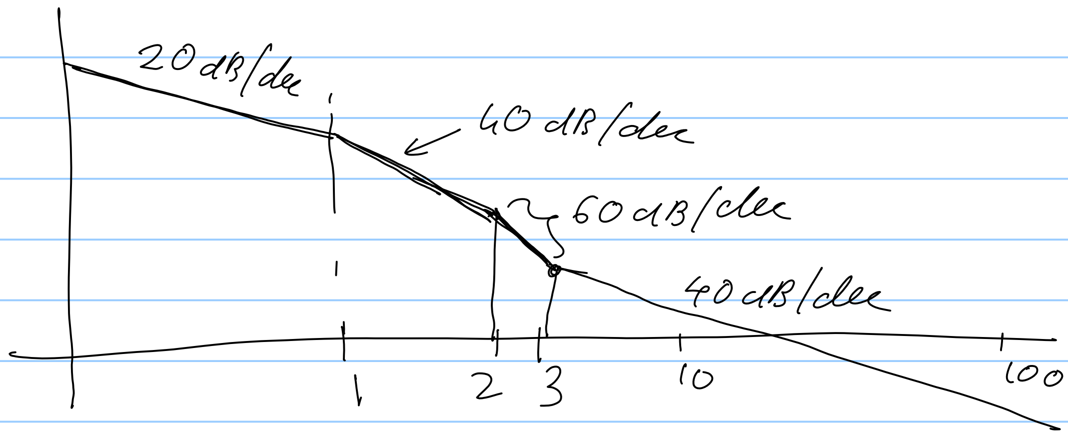
Choose $K = 1$

$$= \frac{\frac{3K}{2}\left(\frac{s}{3}+1\right)}{s(s+1)\left(\frac{s}{2}+1\right)}$$

* Break freq. are 1, 2, 3.
* Mag. plot should extend from one decade below lowest break freq to one decade beyond largest br. freq.

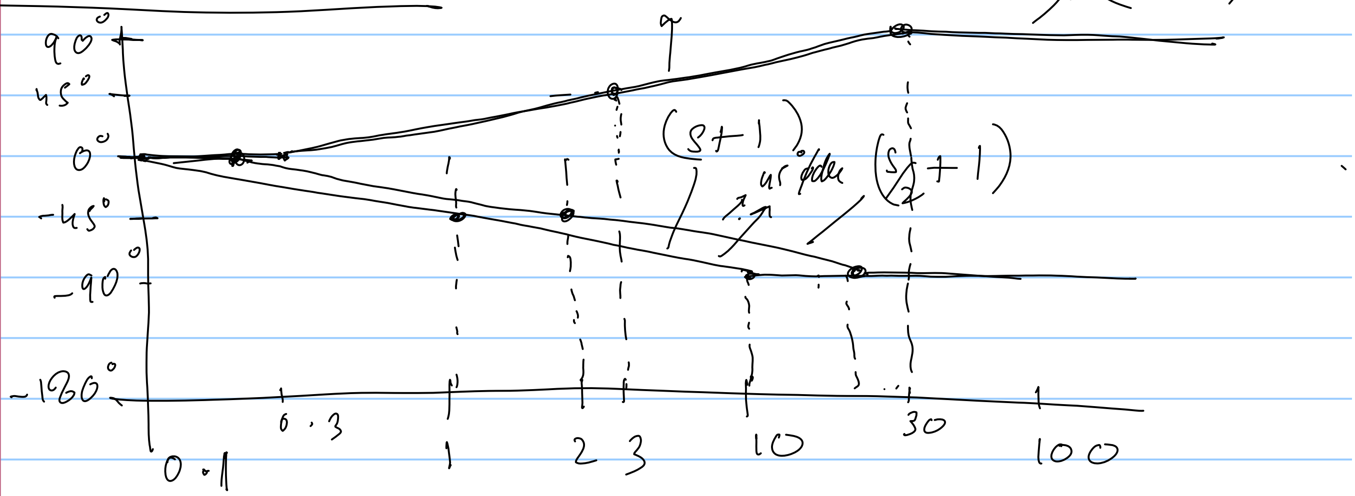
$\longrightarrow$ Range 0.1 rad to 100 rad.





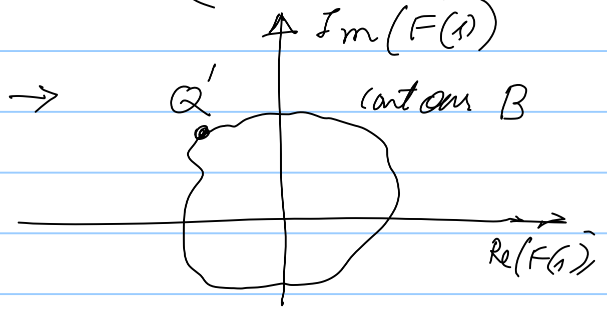
* For getting the actual plot $20 \log \left| \frac{3K}{2} \right|$ should be added to this.

Phase plot



Contour Mapping: Let $F(s) = \dfrac{(s-z_1)(s-z_2)\cdots}{(s-p_1)(s-p_2)\cdots}$

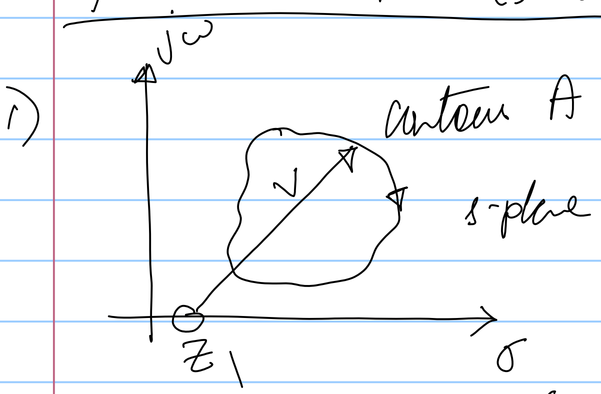
$\rightarrow F(s) \rightarrow$

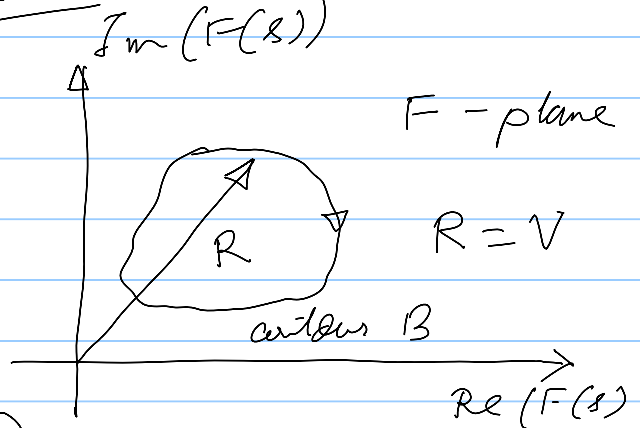


Examples:

Assume A is clockwise

1)

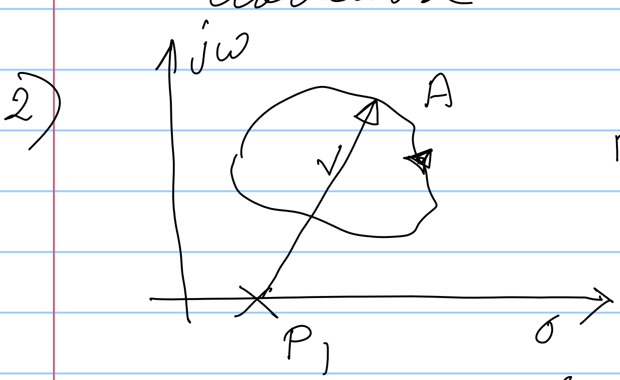




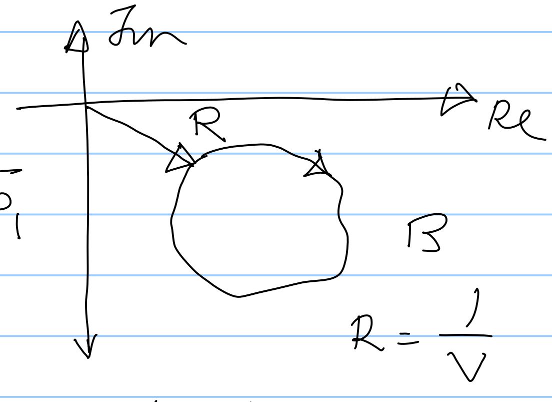
$F(s) = (s - z_1)$

* Direction of contour B mapping is clockwise

2)

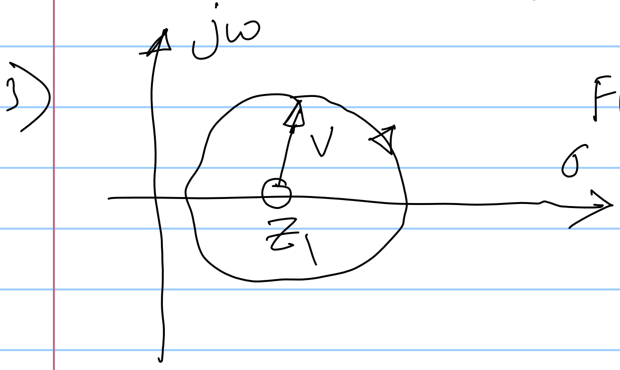


$F(s) = \dfrac{1}{s - p_1}$



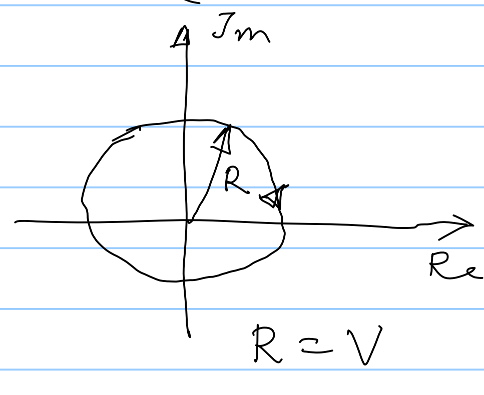
* Direction of B is clockwise

3)



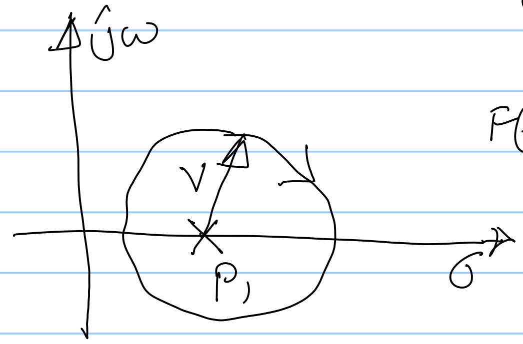
F(s)

$F(s) = s - z_1$

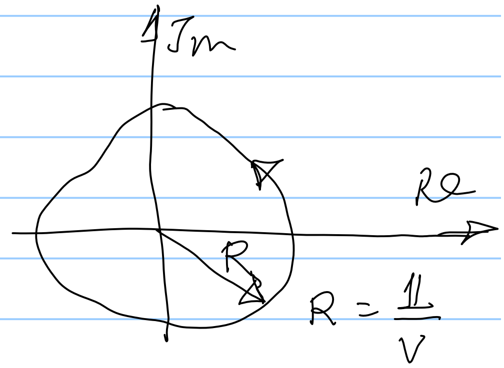


* Encircles origin in clockwise direction

4)

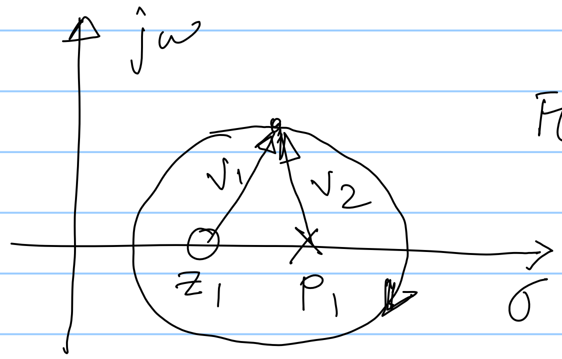


$$F(s) = \frac{1}{s - p_1}$$

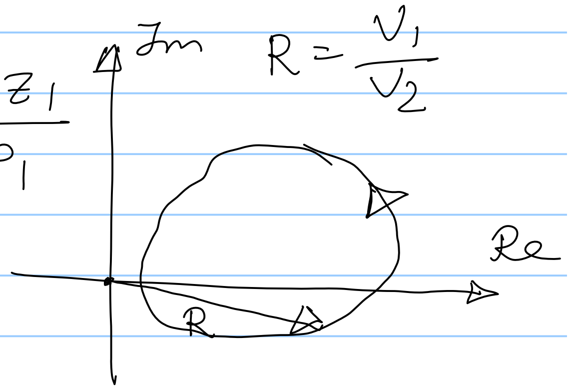


* Encircles origin in <u>anti</u>-clockwise direction

5)



$$F(s) = \frac{s - z_1}{s - p_1}$$



* DOES NOT ENCIRCLE ORIGIN
(Direction depends on relative position of poles/zeros)

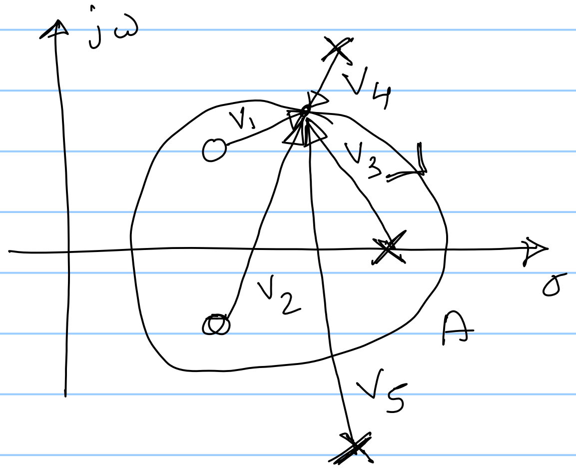
<u>FACT</u>: No of counter-clockwise encirclements of the origin (assuming A is clockwise)

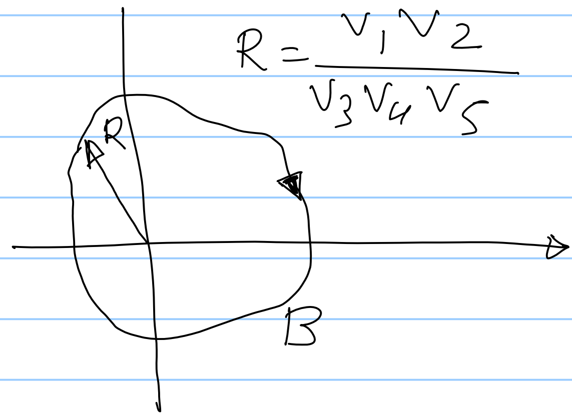
= No of poles of F(s) in A

− No of zeros of F(s) in A.

[Implicit assumption:

$$F(s) = \frac{(s - z_1)(s - z_2) \cdots}{(s - p_1)(s - p_2) \cdots}$$
]





$$R = \frac{V_1 V_2}{V_3 V_4 V_5}$$

$$F(s) = \frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)(s-p_3)}$$

$\circ$ V_1, V_2, V_3 rotate 360° along A
$\times$ V_4, V_5 rotate 0° along A

$\Downarrow$

1) V_1, V_2 contribute $(+360^\circ \times 2)$ of rotation in R.
2) V_3 contribute -360° of rotation in R
3) V_4, V_5 contribute 0° of rotation in R.

$\Downarrow$

$(V_1, \dots, V_5)$ contribute $(2 \times 360^\circ - 360^\circ)$

$= 360^\circ$ of rotation in R

$\equiv$ One clockwise encirclement of the origin

$= -1$ counterclockwise encirclement of the origin

So our <u>FACT</u> holds!

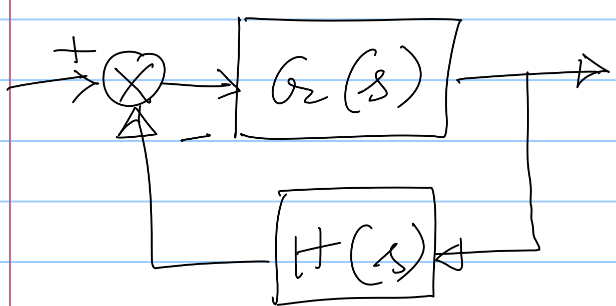
$$\underset{(\text{pole inside } A)}{1} \; - \; \underset{(\text{zeroes inside } A)}{2} \; = \; -1$$

$\downarrow$ counterclockwise encirclements of the origin by B.

Stability of closed loop from Open loop t.f

[We already know 2 different methods of doing this]

Nyquist Criterion



$$G(s) = \frac{N_G(s)}{D_G(s)}$$

$$H(s) = \frac{N_H(s)}{D_H(s)}$$

$$G(s)H(s) = \frac{N_G(s)\,N_H(s)}{D_G(s)D_H(s)}$$

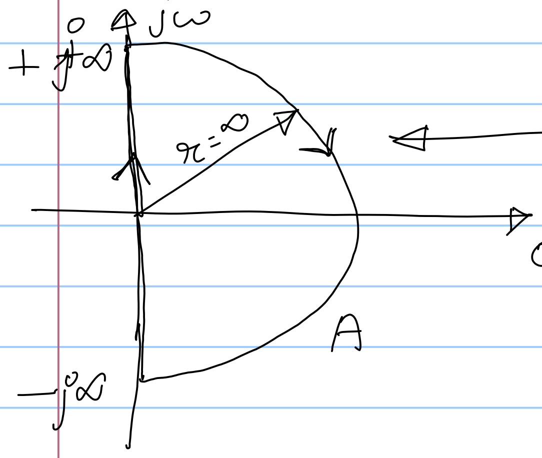
$$1 + G(s)H(s) = \frac{D_G(s)D_H(s) + N_G(s)\,N_H(s)}{D_G(s)\,D_H(s)}$$

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{N_G(s)\,N_H(s)}{D_G(s)D_H(s) + N_G(s)\,N_H(s)}$$

i) POLES of $G(s)H(s)$ = POLES $1 + G(s)H(s)$

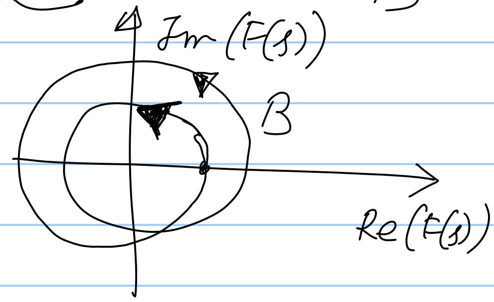
2) ZEROS of $1+G(s)H(s)$ = POLES of $T(s)$
(closed loop poles)

In FACT, use (i) $F(s) = 1+G(s)H(s)$
(ii) Contour A = entire RHP



$\rightarrow F(s) \sim$ contour B

$$\underline{\underline{F(s) = 1+G(s)H(s)}}$$



FACT: No of counterclockwise encirclements of origin by B ($=: N$)

= No of poles of $[1+G(s)H(s)]$ on RHP
− No of zeros of $[1+G(s)H(s)]$ on RHP

= No of open loop poles on RHP (KNOWN) ($=: P$)
− No of closed loop poles on RHP (UNKNOWN) ($=: Z$)

So $N = P - Z$ or $Z = P - N$

Simplification: Take $F(s) = G(s)H(s)$
Everything else same.

FACT: No of counterclockwise encirclements of $(-1+j0)$ by B $(=: N)$

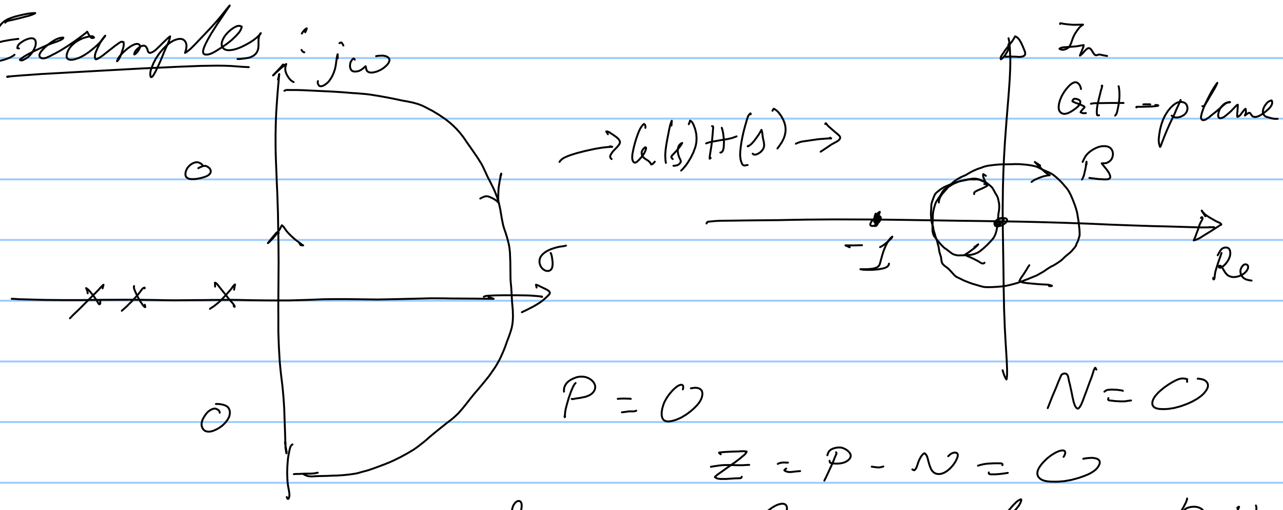
$=$ No of open loop poles on RHP $(=: P)$
$-$ No of closed loop poles on RHP $(=: Z)$

[This is known as Nyquist Criterion and the plot of $G(s)H(s)$ contour is called Nyquist plot]

Q) Why is Nyquist Criteria/plots classified as freq. response?

Examples:

1)

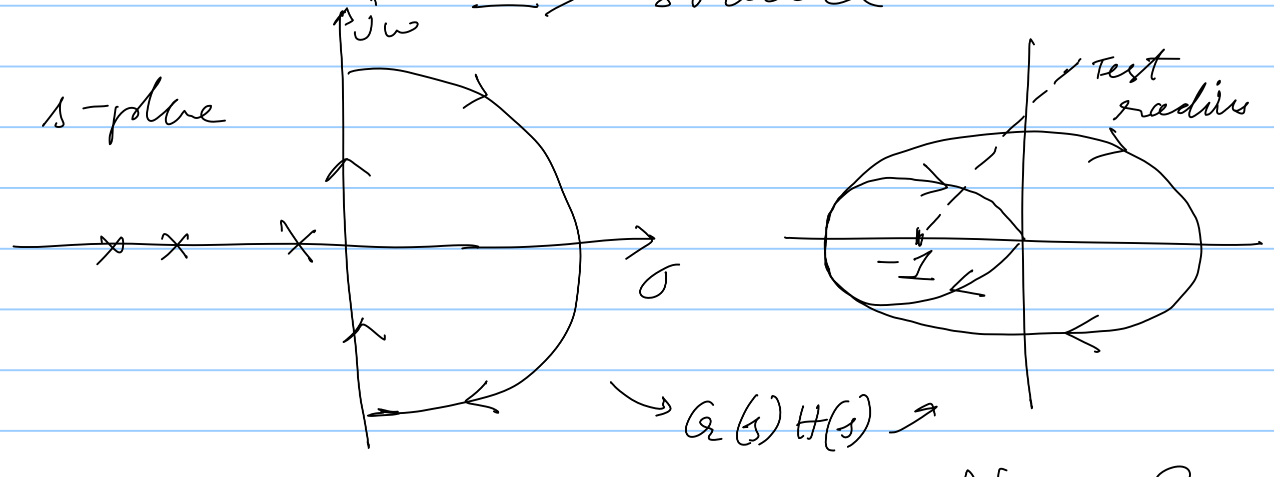


$P = 0$ $\quad$ $N = 0$

$Z = P - N = 0$

So no C.L. poles on RHP
$\Longrightarrow$ Stable

2)



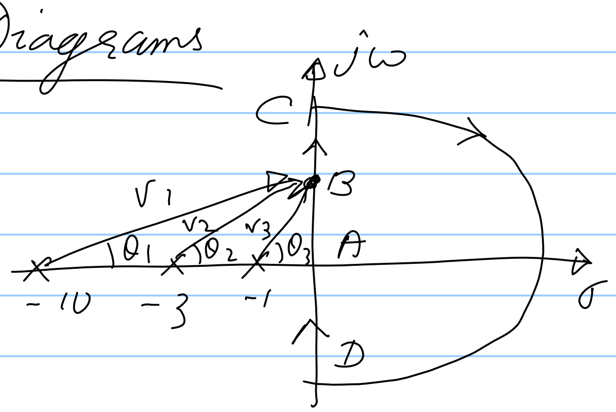
$N = -2$

$P = 0$

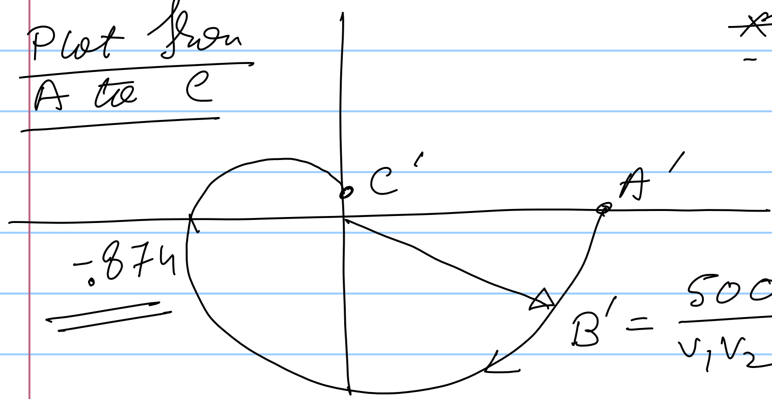
$Z = P - N = 2 \Rightarrow$ 2 closed loop poles on RHP

Sketching Nyquist Diagrams

$$G(s) = \frac{500}{(s+1)(s+3)(s+10)}$$



Plot from A to C



$$B' = \frac{500}{V_1 V_2 V_3}$$

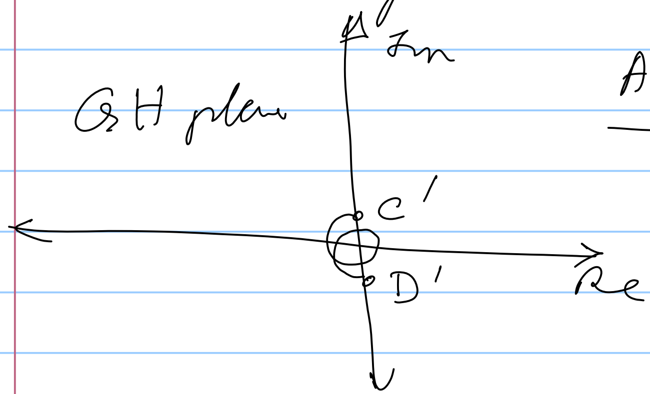
From A to C, $\theta_1 + \theta_2 + \theta_3$ go from $0°$ to $270°$

$\Rightarrow$ B' goes from $0°$ to $-270°$

From A to C, Nyquist Plot $=$ Polar plot of $G(j\omega)H(j\omega)$

Exercise: Derive analytical Expression for $G(j\omega)$ ($\omega = 0$ to ∞)

Plot from C to D: Does not matter here since the magnitude $= 0$.



Angle change $+3 \times 180°$

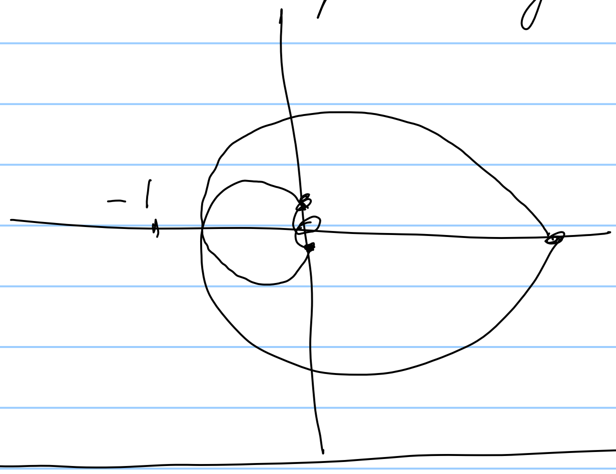
Radius $= 0$.

Plot from D to A

- $Re[G(j\omega)H(j\omega)]$ = even function
- $Im[G(j\omega)H(j\omega)]$ = odd function

Exercise : Prove :

So the Nyquist plot for DA is a mirror image (about Re axis) of ~~the~~ N.-plot for AC.



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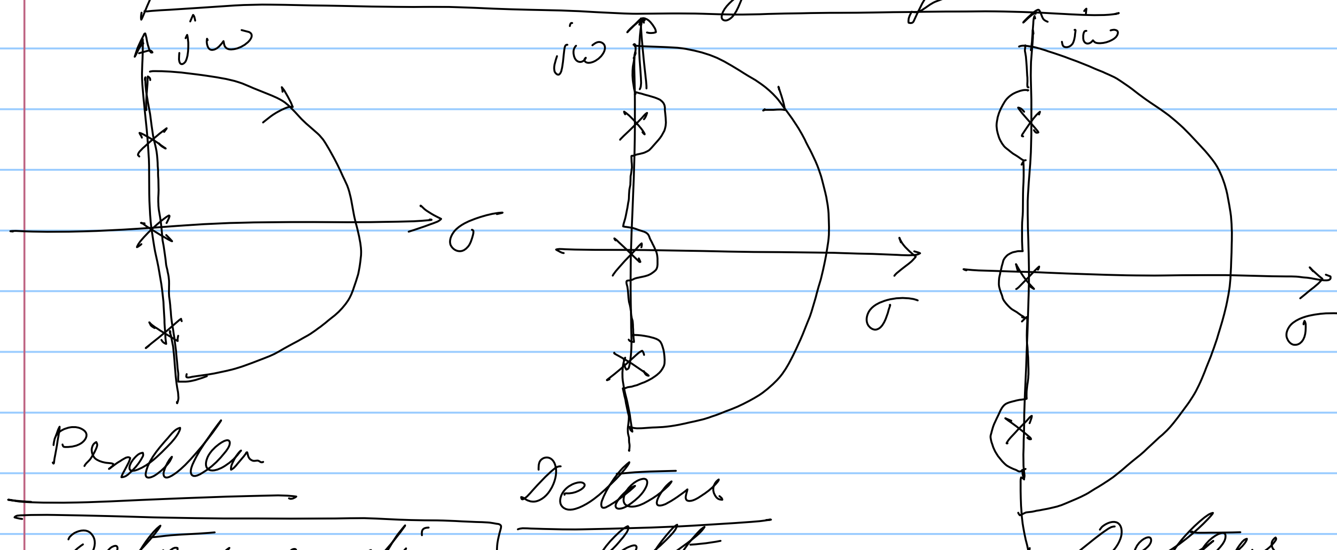
* Nyquist Plots for GH with open loop poles on Imaginary axis



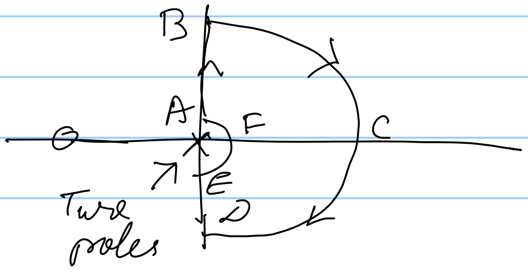
Problem

Detour left

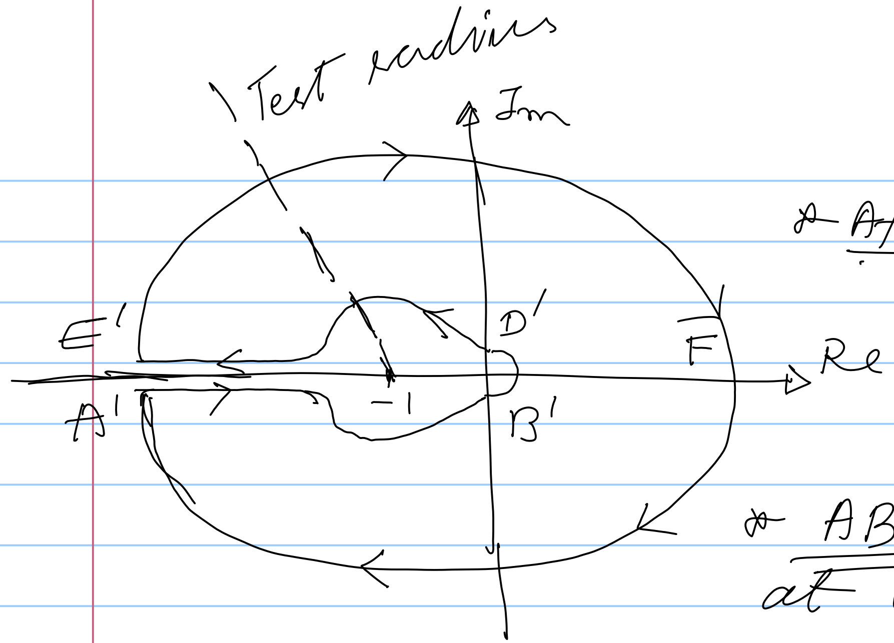
Detour Right

* $\boxed{\text{Detour radius} \approx 0}$

Example: $G(s) = \dfrac{s+2}{s^2}$



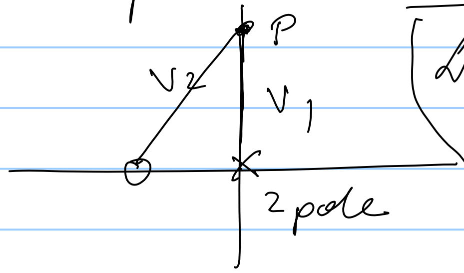
Detour Right



* At A total angle $= -2\times 90^\circ + 0^\circ$
$= -180^\circ$
Magnitude $= \infty$

* AB → Total angle at B $= 90^\circ - 2\times 90^\circ$
$= -90^\circ$

Magnitude at B $= 0$



$$\boxed{\lim_{P\to j\infty} \frac{V_2}{V_1^2} = 0}$$

BCD: Magnitude ≈ 0 $\left(\dfrac{\text{one zero length}}{\text{2 pole lengths}}\right)$

Total angle change $= -180^\circ + 2\times 180^\circ = +180^\circ$

D'E' is mirror image of A'B'

EFA: Magnitude $\approx \infty$ $\left(\dfrac{\text{pole length finite}}{\text{2 zero pole length}}\right)$

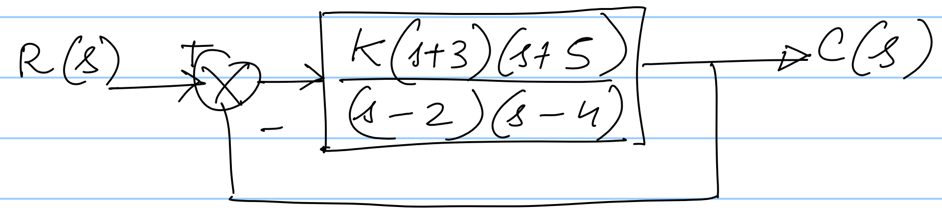
Zero angle do not change
Total angle change $= -2\times 180^\circ$
$= -360^\circ$

Test radius → $N = 0$
$P = 0$
$Z = P - N = 0$ (No C.L. poles in RHP)

check
$$T(s) = \frac{\frac{s+2}{s^2}}{1 + \frac{s+2}{s^2}} = \frac{s+2}{s^2+s+2}$$

C.L. poles $= \dfrac{-1 \pm j\sqrt{7}}{2}$

Stability via Nyquist Diag.



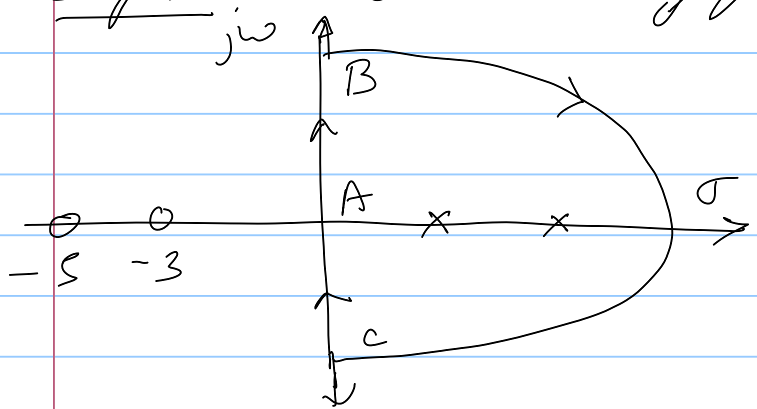
Q) For what values of K C.L. is stable?

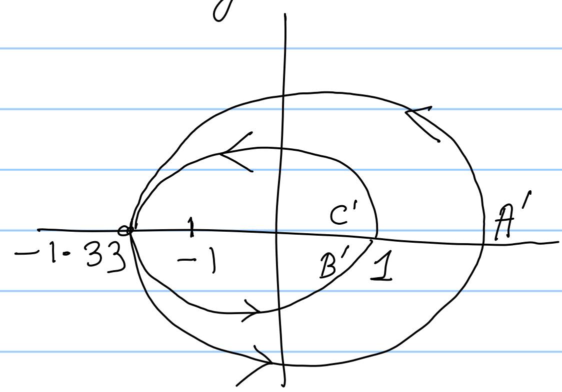
A1) → Using Routh-Hurwitz criteria

A2) → Using Root-locus

A3) → Using Nyquist Diagram

Steps: Draw Nyquist Diag with $K=1$.





$N = 2$, $P = 2$

$\Rightarrow Z = P - N = 0 \Rightarrow$ C.L. stable for $K = 1$.

For $K < \dfrac{1}{1\cdot33}$, $N = 0$, $P = 2$, $Z = 2$ $\Rightarrow$ unstable

$K = 1/1\cdot33 \longrightarrow$ marginally stable.

* Alternatively, one can keep Nyq. Diag. fixed and imagine the critical pt at $-\frac{1}{K}$.

* NOTE: If the Nyquist plot intersects the real axis at $-1+j0$

$\Leftrightarrow$ $G(j\omega)H(j\omega) = -1$

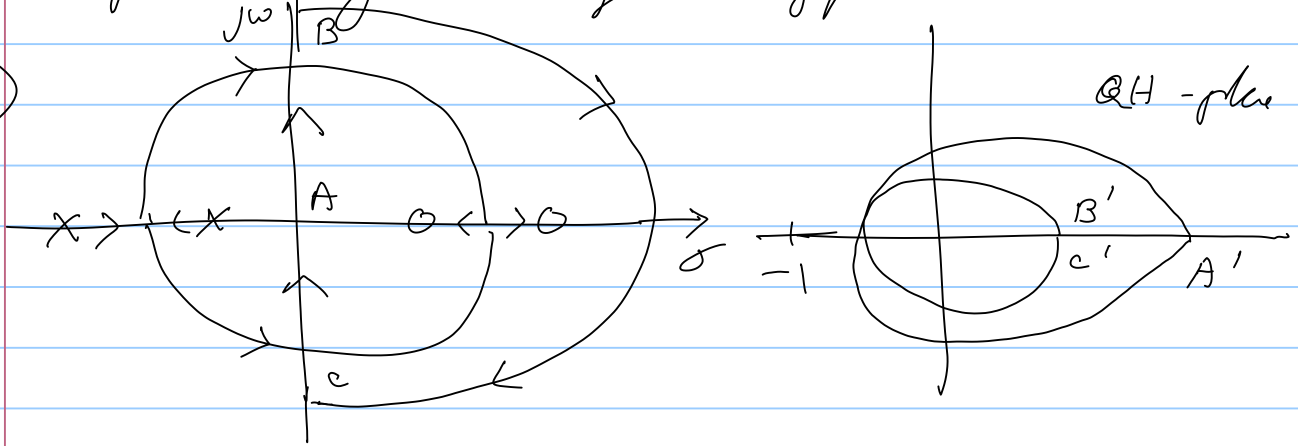
$\Leftrightarrow$ Root-locus crosses the $j\omega$-axis.

$\Leftrightarrow$ Marginal stability

Stability via mapping only the $j\omega$-axis

* Simple forms of Nyquist criterion

1)

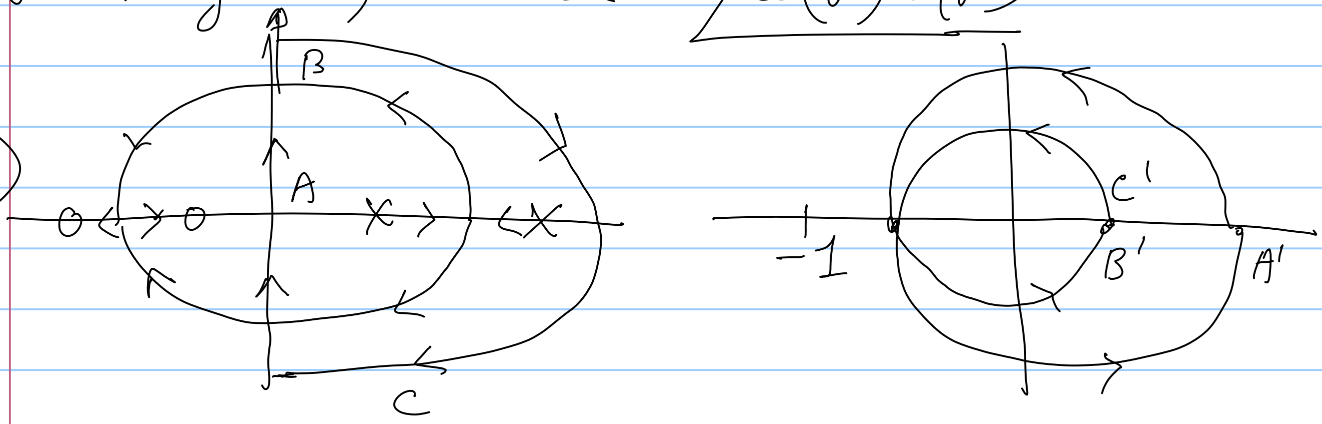


System is stable for those values of gain K, for which

$$|G(j\omega)H(j\omega)| < 1 \text{ for that}$$

value of ω, where $\angle G(j\omega)H(j\omega) = \pm 180^\circ$

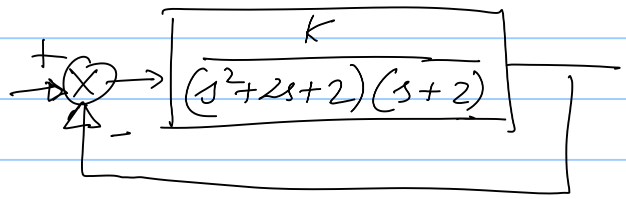
2)



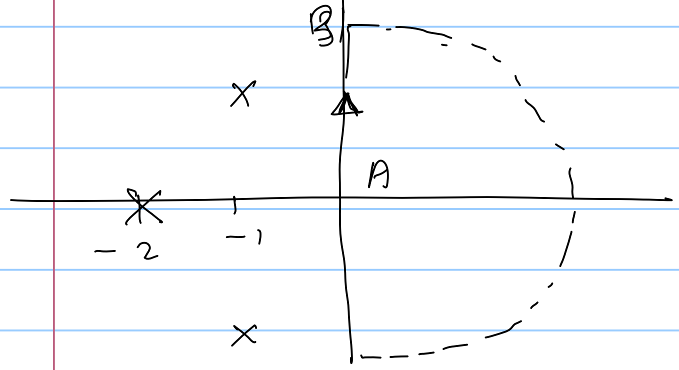
System is stable for those values of gain K, for which

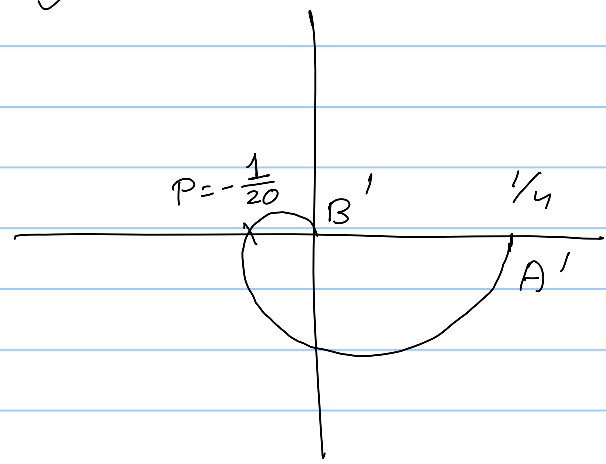
$$|G(j\omega)H(j\omega)| > 1$$ for that value of ω, where $\angle G(j\omega)H(j\omega) = \pm 180^\circ$

Example:



Find range of K for stability, marginal stability, instability





No RHP poles (O.L.)

$\Rightarrow$ For stability $\rightarrow$ no encirclements of $(-1+j0)$

$\Rightarrow |G(j\omega)| < 1$ for those ω where $\angle G(j\omega) = \pm 180^\circ$

Let $K = 1$ initially and draw $A'B'$.

At $A \rightarrow -[0^\circ - 45^\circ + 45^\circ] = 0^\circ$

Mag $\longrightarrow \dfrac{1}{2\sqrt{2}\sqrt{2}} = \dfrac{1}{4}$

At B → $-[90^\circ + 90^\circ + 90^\circ] = -270^\circ$

Mag → 0

<u>To find P:</u> $G(j\omega) = \left.\dfrac{1}{(s^2+2s+2)(s+2)}\right|_{s=j\omega}$

Putting $\text{Im}(G(j\omega)) = 0$ we get $\omega = \sqrt{6}$

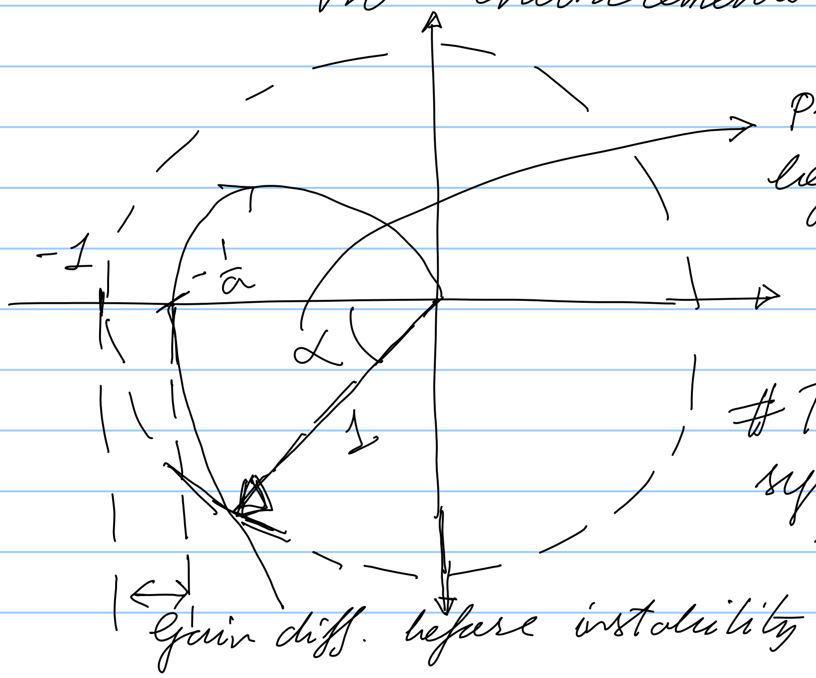
Putting $\omega = \sqrt{6}$ in $\text{Re}(G(j\omega)) = -\frac{1}{20}$

Hence if $K < 20$, system stable.
$K = 20$, marginally stable
$K > 20$; unstable

Q. What is the freq of oscillation when $K = 20$?

Gain Margin & Phase Margin

ASSUME: system is stable if there are no encirclements of -1.



Two ways for this system to become unstable:

1) At phase $\pm 180°$, gain $\frac{1}{a}$ should increase to 1.
2) At gain 1, phase $\pm 180° + \alpha$ should reduce to $\pm 180°$

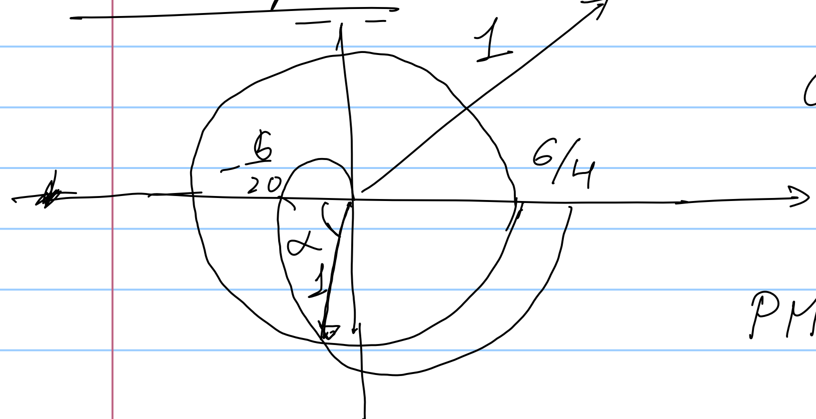
Gain Margin (in dB) is the change in O.L. gain required at 180° of phase shift to make C.L. system unstable.

Phase margin is change in O.L. phase shift required at unity gain to make the C.L. system unstable.

G.M. & P.M. are measures of ROBUSTNESS. ≡ Quantitative measures of how much stable
≡ How much change in system parameters it can withstand before becoming unstable.

⟷ In Root Locus → same information is provided by the distance of the C.L. poles from imaginary axis

Example



Find G.M & P.M. for K=6.

$$GM = 20 \log \frac{20}{6}$$
$$= 10.45 \, dB$$

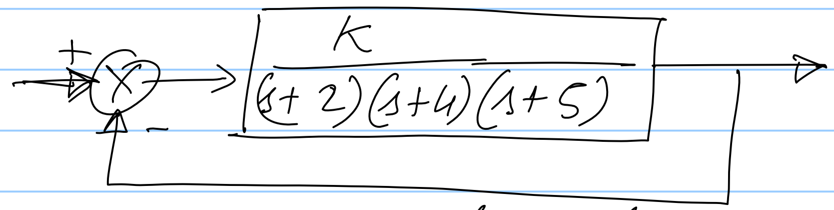
$$PM = \alpha = 67.7°$$

* G.M. & P.M. are difficult to find (computationally) on the Nyquist plot.

→ We use BODE PLOTS instead.
→ Subsets of Nyquist Diag
→ Easy to draw (unlike Nyquist d. or R-L)

BODE PLOTS to Stability

Example:



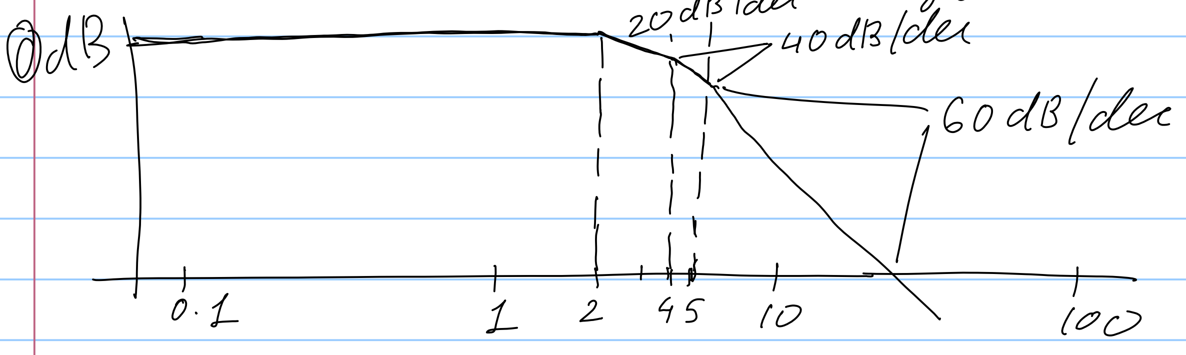
Determine range of K for which C.L. system is stable:

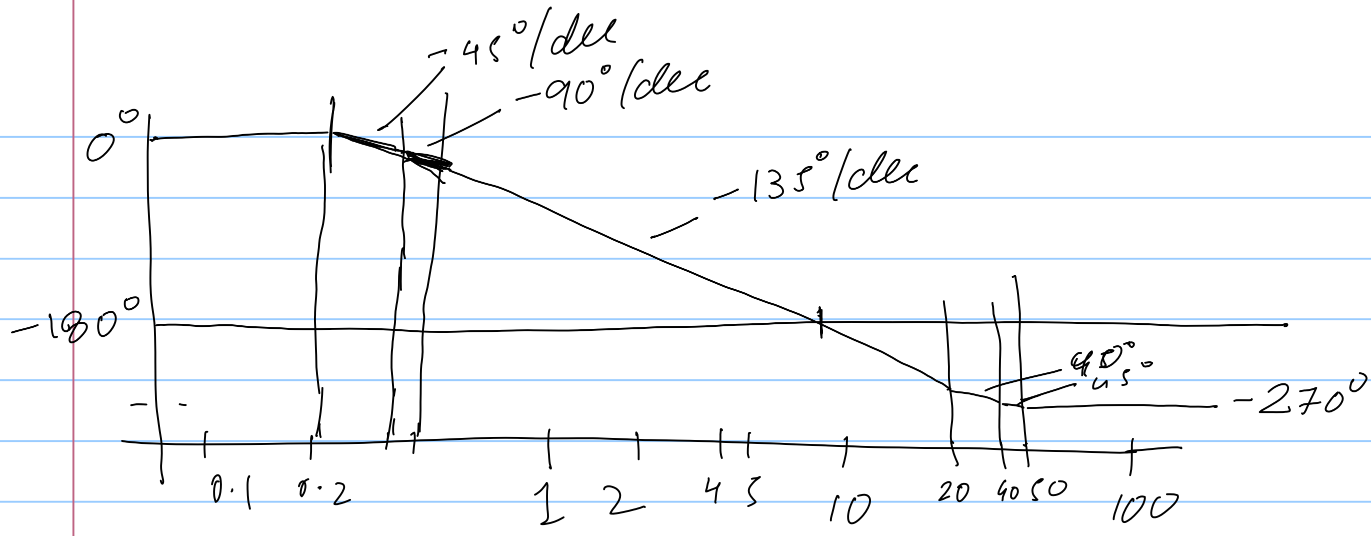
* All O.L. poles are in LHP. ⇒ No encirclements of $(-1+j0)$ by Nyq.Pl. for stability ⇒ $|G(j\omega)| < 1$ for ω satisfying $\angle G(j\omega) = \pm 180°$

Plot BODE mag & phase:

$$G(s) = \frac{K}{(s+2)(s+4)(s+5)} = \frac{K/40}{\left(\frac{s}{2}+1\right)\left(\frac{s}{4}+1\right)\left(\frac{s}{5}+1\right)}$$

Let $K = 40$ to start off:





$\angle G(j\omega) = -180^\circ$ at $\omega = 7$ rad/sec

At $\omega = 7$ rad/sec, $|G(j\omega)| = -20\,dB$

Hence increase of +20 dB is possible before instability.

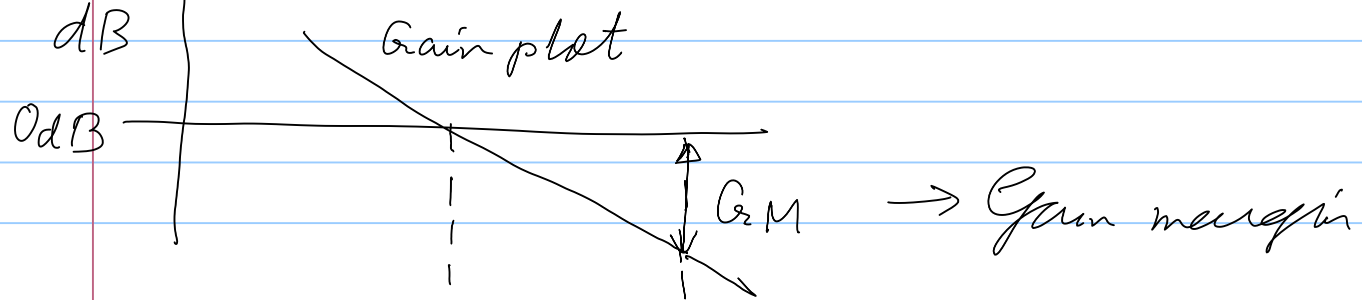
$$20\,dB = 20\log_{10} K_1$$
$$K_1 = 10$$

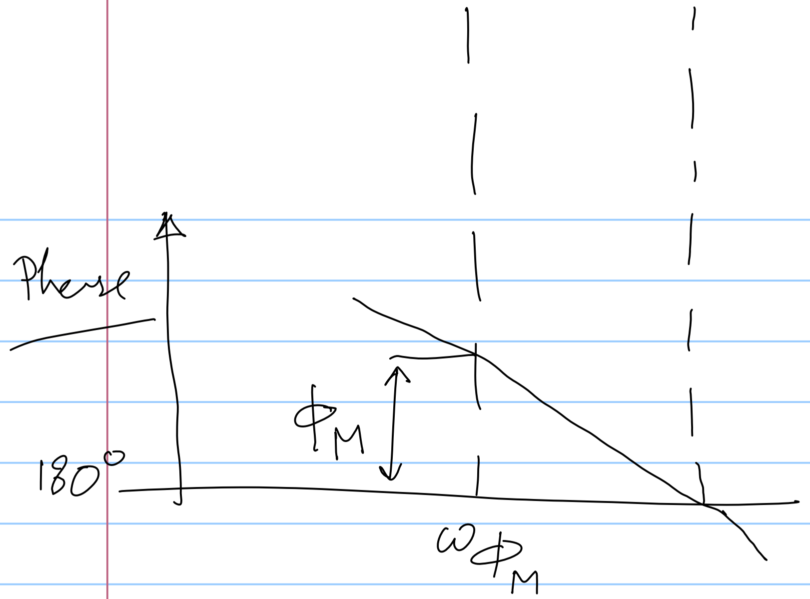
But $K = 40$ was already chosen so the system will remain stable upto $K = 40 \times 10 = 400$.

Hence stable $\rightarrow$ $0 < K < 400$.

* This is approx. since Asymptotes were used.

Gain Margin & Phase Margin from BODE Plots





$\Phi_M \to$ phase margin

Example : In the example above

$$G_M = 20\,dB$$

$$\Phi_M = 180^\circ + \text{Phase at } 2\,rad/sec$$

★★★ SIMULATION ★★★

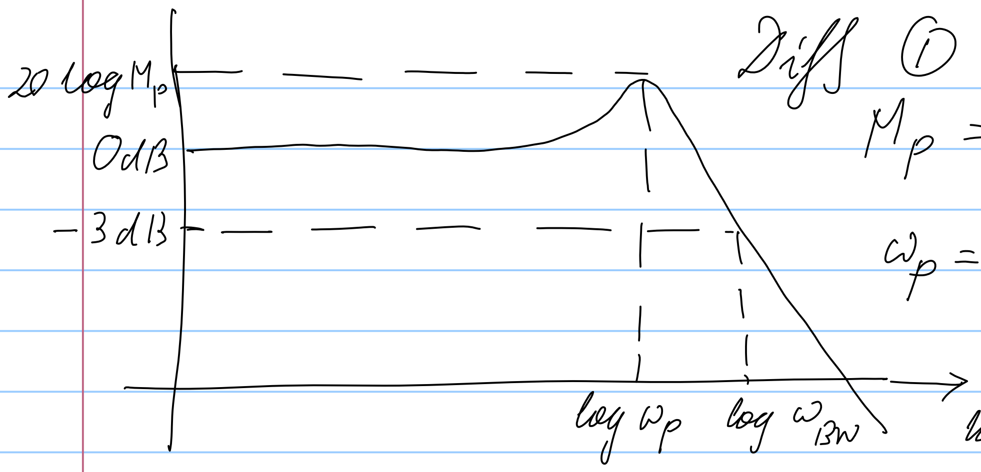
IMPORTANT NOTE : BODE plot/Nyq. D. is magnitude & phase of O.L. WHILE P.M., G.M an stability is of the C.L. system.

★ There has been no need of C.L. freq response until now.

C.L. Transient Resp. from C.L. Freq. Response (2nd order systems)

$$\frac{\omega_n^2}{s(s+2\zeta\omega_n)} \text{ (unity feedback)} \quad \text{or} \quad T(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$M = |T(j\omega)| = \frac{\omega_n^2}{\sqrt{(\omega_n^2-\omega^2)^2+4\zeta^2\omega_n^2\omega^2}} \quad \text{--- (1)}$$



Diff (1) & eq. to zero,

$$M_p = \frac{\;}{2\zeta\sqrt{1-\zeta^2}}$$

$$\omega_p = \omega_n\sqrt{1-2\zeta^2}$$

1) **% OS from BODE PLOT** (M_p)

Hence one can measure M_p from BODE PLOT

$M_p \longrightarrow$ Calculate $\zeta \longrightarrow$ Calculate % OS

* Peak occurs only for $\zeta < 0.707$
* Recall % OS in step response occurs for $0 < \zeta < 1$

2) **T_s and T_p from BODE PLOT** (ω_B)

Bandwidth: of C.L. freq response is the freq at which the magnitude curve is 3 dB below its value at zero freq. $\rightarrow \omega_{BW}$

Putting $M = \frac{1}{\sqrt{2}}$ in (1), & $T_s = \frac{4}{\zeta\omega_n}$

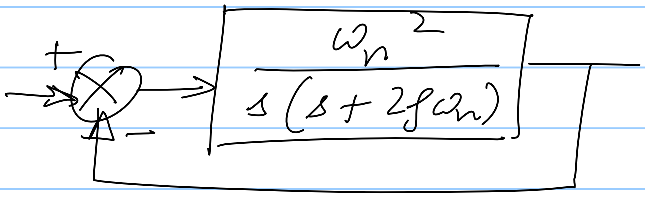
$$\boxed{\omega_{BW} = \frac{4}{T_s\zeta}\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}}}$$

Using, $\omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}}$

$$\boxed{\omega_{BW} = \frac{\pi}{T_p\sqrt{1-\zeta^2}}\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}}}$$

<u>C.L. Transient Resp from O.L. Freq. Resp.</u>

<u>Damping Ratio from P.M.</u>



Put:

$$|G(j\omega)| = \frac{\omega_n^2}{|-\omega^2+j2\zeta\omega_n\omega|} = 1 \quad \text{—} \quad (*)$$

Solving (*) for ω: $\omega_1 = \omega_n\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}$

$$\angle G(j\omega_1) = -90^\circ - \tan^{-1}\left[\frac{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}{2\zeta}\right]$$

Phase margin: $\phi_M = 180^\circ + \angle G(j\omega_1)$

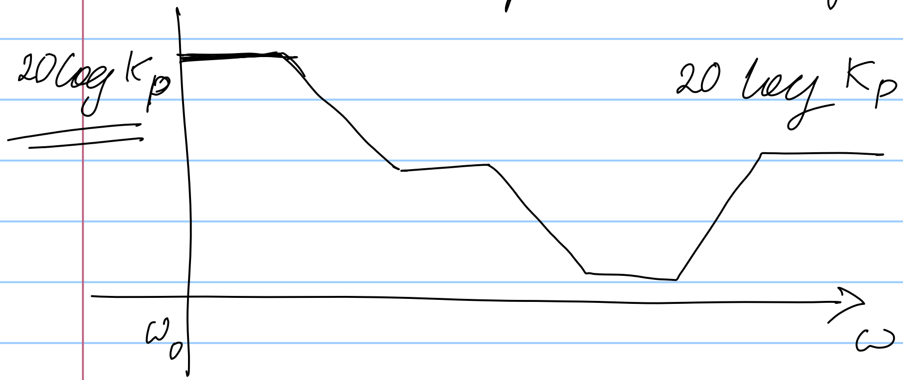
$$= 90^\circ - \tan^{-1}\frac{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}{2\zeta}$$

$$= \tan^{-1}\left[\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}\right]$$

<u>SSE (K_p, K_v and K_a) from (O.L.) Bode Plots</u>

$$G(s) = K\,\frac{\prod(s+z_i)}{\prod(s+p_i)} \qquad \text{"Type zero"}$$

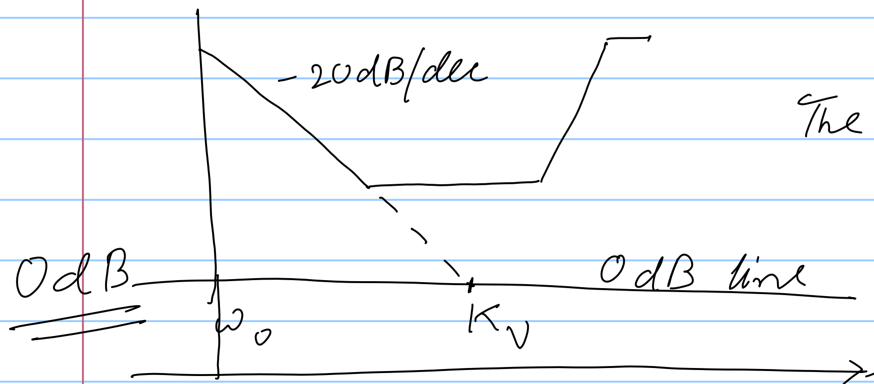
In Bode magnitude plot



$$20 \log K_p = 20 \log K \frac{\prod z_i}{\prod p_i}$$

"Type 1"

$$G(s) = K \frac{\prod (s+z_i)}{s \prod (s+p_i)}$$



The first part eqn:

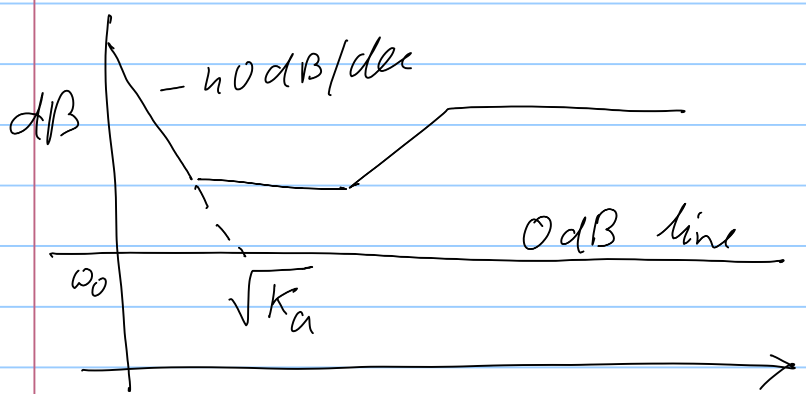
$$K \frac{\prod z_i}{\omega \prod p_i}$$

Find where it intersects 0dB line:

$$K \frac{\prod z_i}{\omega_1 \prod p_i} = 1 \Rightarrow \omega_1 = K \frac{\prod z_i}{\prod p_i} = K_v$$

"Type 2":

$$G(s) = K \frac{\prod (s+z_i)}{s^2 \prod (s+p_i)}$$



Eqn for 1st part

$$K \frac{\prod z_i}{\omega_1^2 \prod p_i} = 1$$

$$\omega_1 = \sqrt{\frac{K \prod z_i}{\prod p_i}}$$

$$= \sqrt{K_a}$$