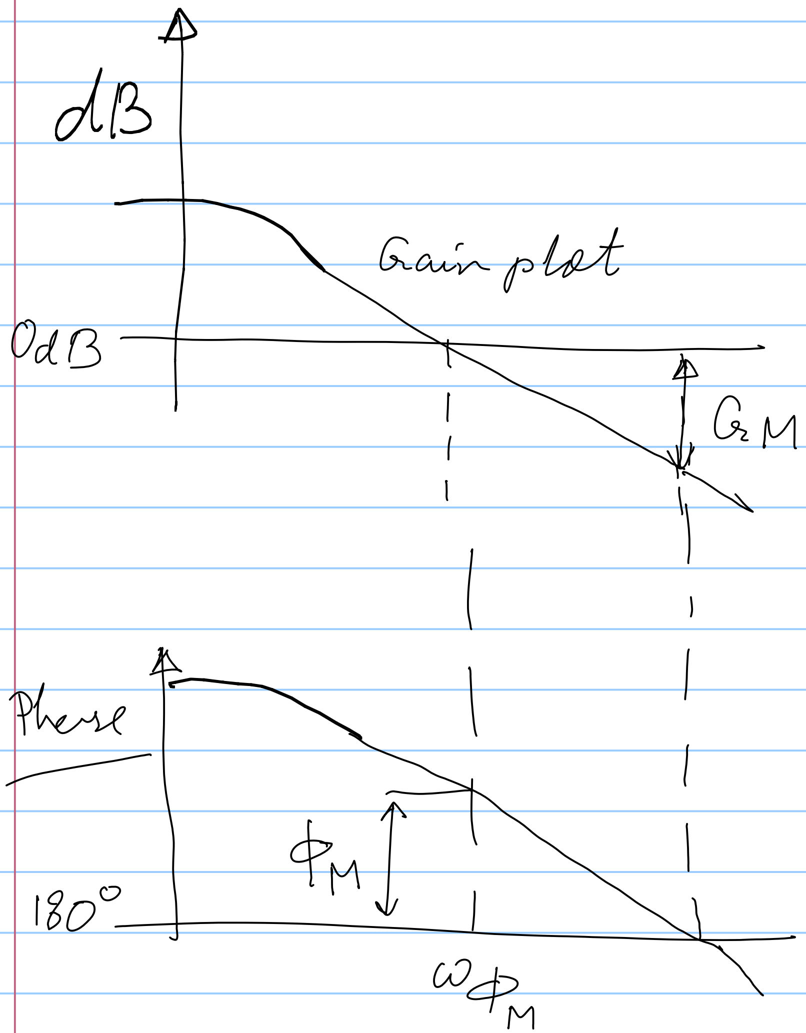
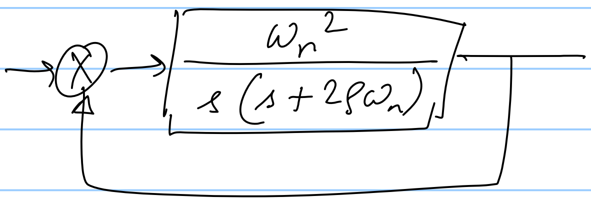
Lecture 12: Design via Bode Plots



$G_M \rightarrow$ Gain margin

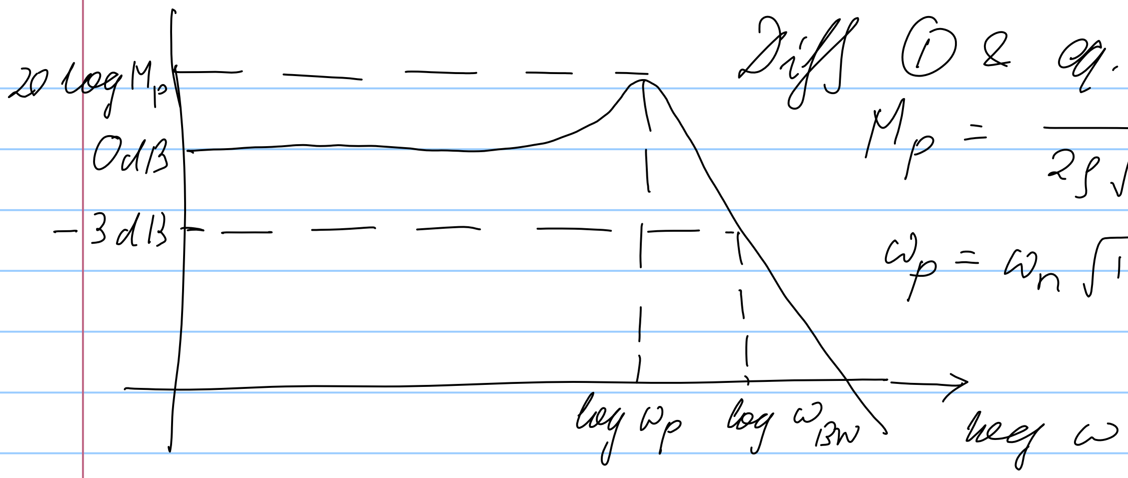
$\phi_M \rightarrow$ phase margin

C.L. Transient Resp. from C.L. Freq. Response (2nd order systems)



or $T(s) = \dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$$M = |T(j\omega)| = \frac{\omega_n^2}{\sqrt{(\omega_n^2-\omega^2)^2+4\zeta^2\omega_n^2\omega^2}} \quad \text{---} \quad (1)$$



Diff (1) & eq. to zero,

$$M_p = \frac{}{2\zeta\sqrt{1-\zeta^2}}$$

$$\omega_p = \omega_n\sqrt{1-2\zeta^2}$$

1) <u>% OS from BODE PLOT</u> (M_p)

Hence one can measure M_p from BODE PLOT

$M_p \longrightarrow$ Calculate $\zeta \longrightarrow$ Calculate % OS

* Peak occurs only for $\zeta < 0.707$
* Recall % OS in step response occurs for $0 < \zeta < 1$

2) <u>T_s and T_p from BODE PLOT</u> (ω_B)

<u>Bandwidth</u>: of C.L. freq response is the freq at which the magnitude curve is 3 dB below its value at zero freq. $\rightarrow \omega_{BW}$

Putting $M = \frac{1}{\sqrt{2}}$ in (1), & $T_s = \frac{4}{\zeta\omega_n}$

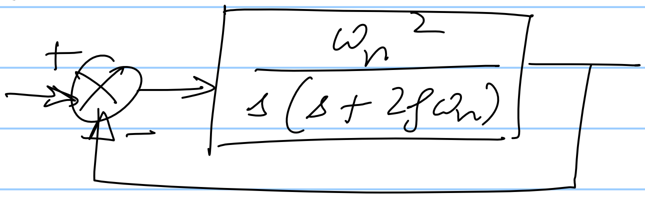
$$\boxed{\omega_{BW} = \frac{4}{T_s\zeta}\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}}}$$

Using, $\omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}}$

$$\boxed{\omega_{BW} = \frac{\pi}{T_p\sqrt{1-\zeta^2}}\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}}}$$

C.L. Transient Resp from O.L. Freq. Resp.

Damping Ratio from P.M.



Put:

$$|G(j\omega)| = \frac{\omega_n^2}{|-\omega^2 + j\,2\zeta\omega_n\omega|} = 1 \quad \text{—— } (*)$$

Solving (*) for ω: $\omega_1 = \omega_n\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}$

$$\angle G(j\omega_1) = -90^\circ - \tan^{-1}\left[\frac{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}{2\zeta}\right]$$

Phase margin: $\phi_M = 180^\circ + \angle G(j\omega_1)$

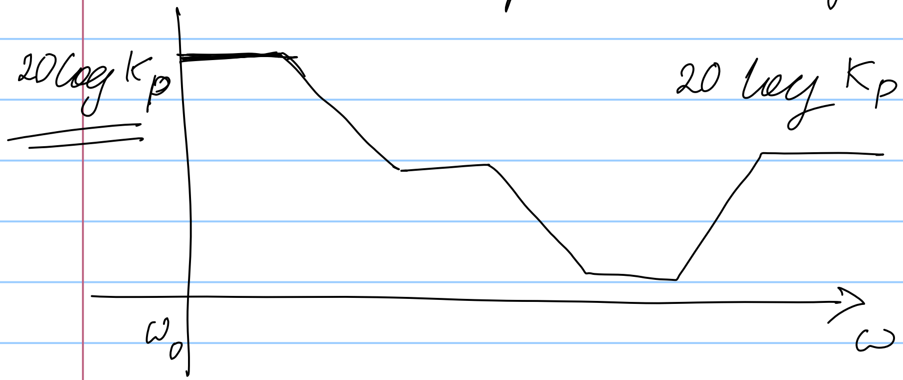
$$= 90^\circ - \tan^{-1}\frac{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}{2\zeta}$$

$$= \tan^{-1}\left[\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}\right]$$

SSE (K_p, K_v and K_a) from (O.L.) Bode Plots

$$G(s) = K\,\frac{\prod(s+z_i)}{\prod(s+p_i)} \qquad \text{"Type zero"}$$

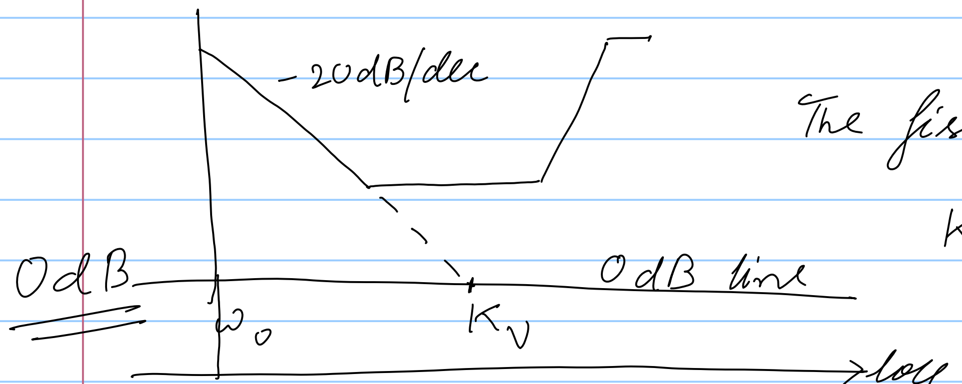
In Bode magnitude plot



$$20\log K_p = 20\log K \frac{\prod z_i}{\prod p_i}$$

"Type 1"

$$G(s) = K\frac{\prod (s+z_i)}{s\prod(s+p_i)}$$



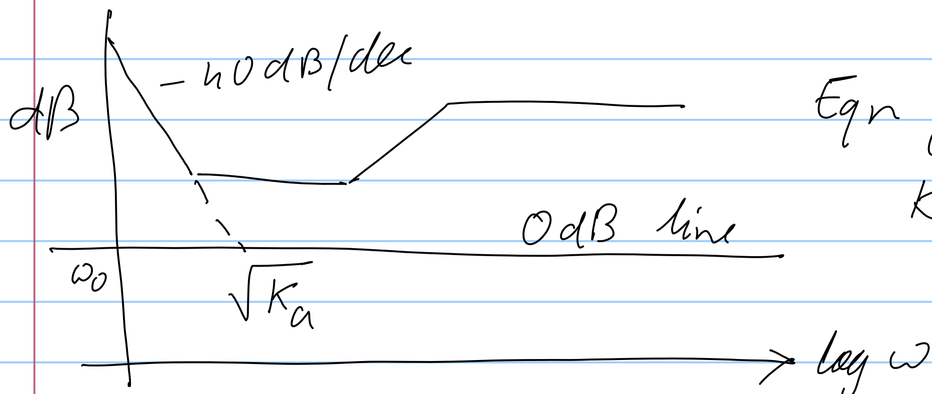
The first part eqn:

$$K\frac{\prod z_i}{\omega \prod p_i}$$

Find where it intersects 0dB line:

$$K\frac{\prod z_i}{\omega_1\prod p_i} = 1 \Rightarrow \omega_1 = K\frac{\prod z_i}{\prod p_i} = K_v$$

"Type 2" : $$G(s) = K\frac{\prod (s+z_i)}{s^2\prod(s+p_i)}$$



Eqn for 1st part

$$K\frac{\prod z_i}{\omega_1^2\prod p_i} = 1$$

$$\omega_1 = \sqrt{\frac{K\prod z_i}{\prod p_i}} = \sqrt{K_a}$$

Note Title 12-04-2012

Transient Response via Gain Adjustment

[%OS req]

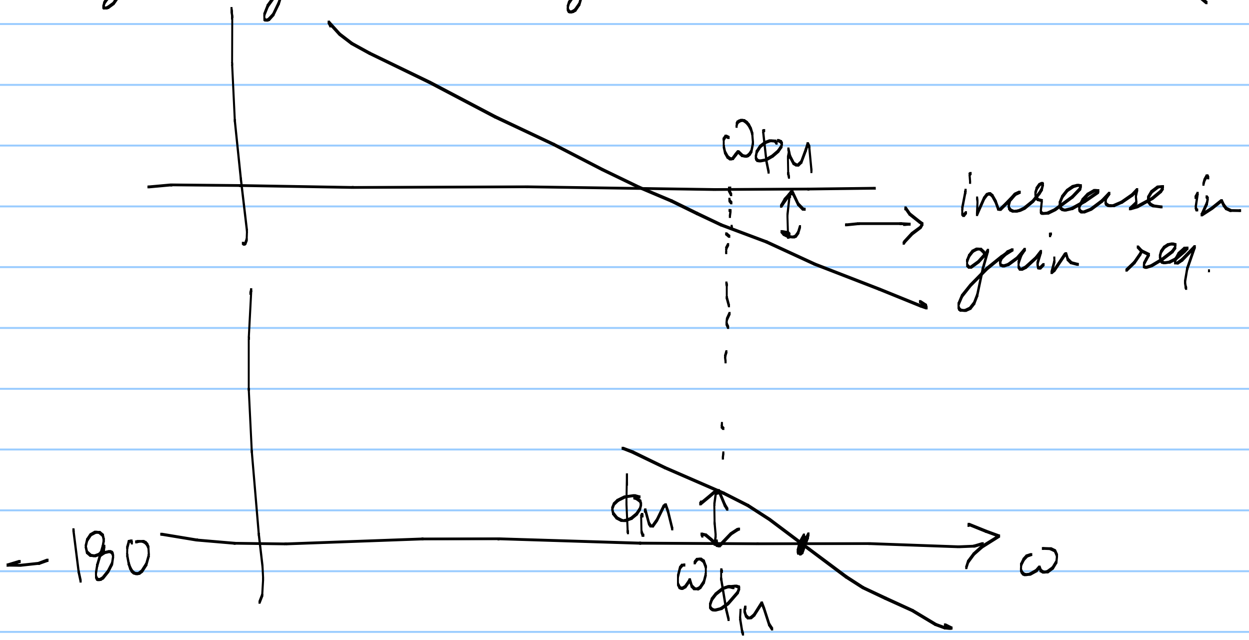
1) Draw Bode plots

2) Calculate ϕ_M from %OS (or ζ)

$$\phi_M = \tan^{-1}\left[\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}\right]$$

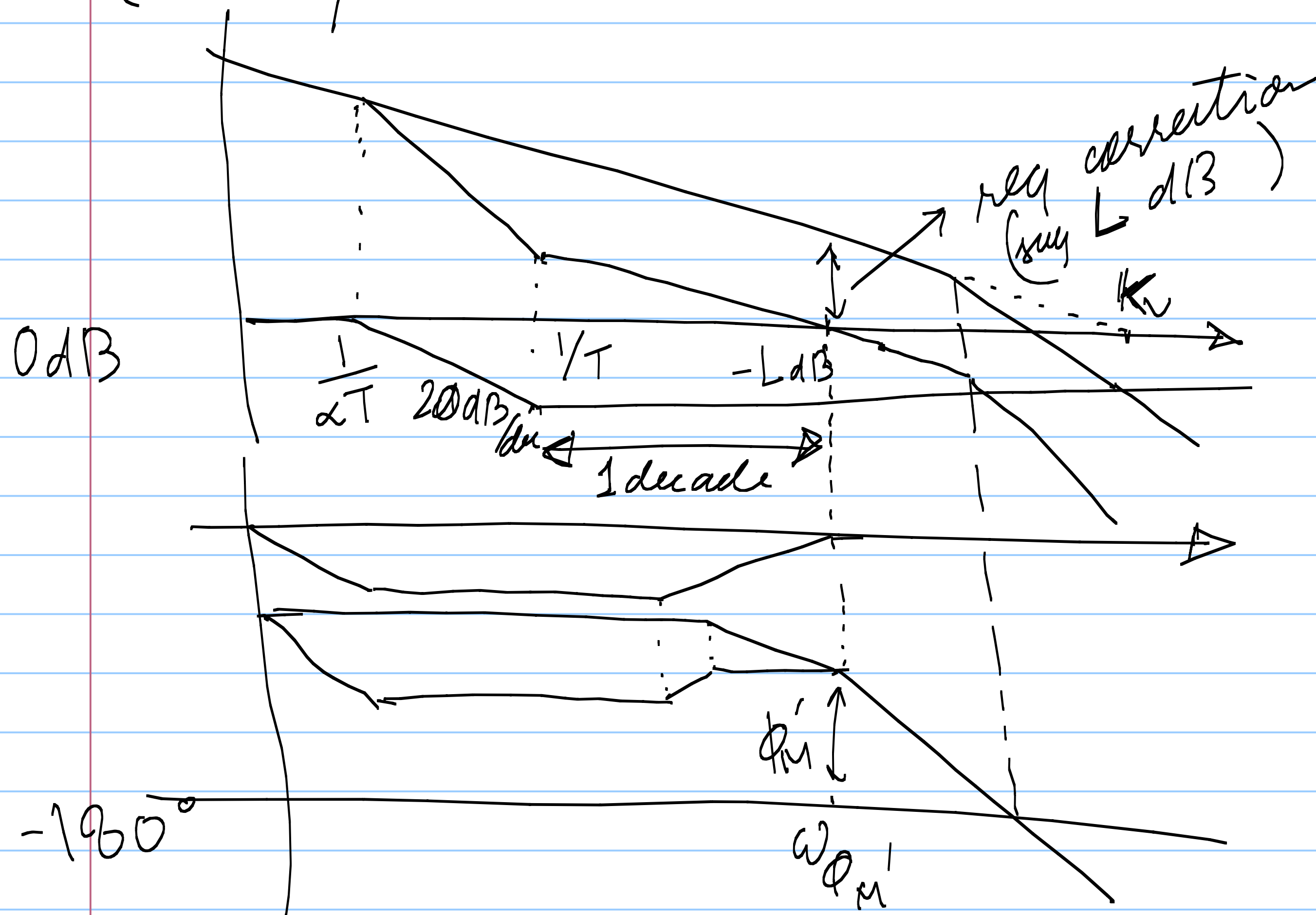
3) Calculate ω_{ϕ_M} corr. to ϕ_M from Bode plot

4) Change gain to get 0dB at ω_{ϕ_M}



Lag Compensation : Improve S.S.E.

(i.e. K_p, K_v, K_a) while preserving ($\%OS$ / Φ_M)



1) Adjust gain to get req SSE (in the fig above K_v)
2) Calculate Φ_M req from $\%OS$ req.
3) Calculate $\Phi_M' = \Phi_M + (5 \text{ to } 12°)$
4) Choose Lag comp a) upper corner freq ($1/T$) 1 decade below $\omega_{\Phi_M'}$

b) high freq asymptote $= -L$ dB

c) We get $\frac{1}{\alpha T}$ by a 20 dB/dec line from $-L$ dB up to 0 dB