Note Title 11-06-2008

EE - 302 (Need for State Variables)

Q We have learnt about transfer fn and their properties. Then,
* Why do we need state variables?

Problem 1: Consider the following block diagram:

$$G_{c}(s) = \frac{Y(s)}{V(s)} = \frac{s-1}{s+1} \cdot \frac{1}{s-1} = \frac{1}{s+1}$$

Lets calculate the step response

$$v(t) = u(t), \quad V(s) = \frac{1}{s}; \; Y(s) = \frac{1}{(s+1)s}$$

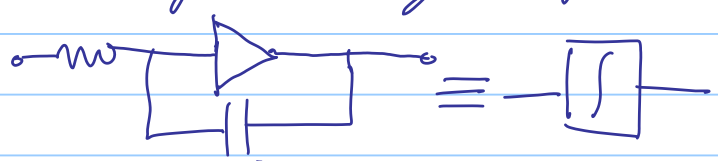
$$y(t) = \mathcal{L}^{-1}\left[\frac{1}{s} - \frac{1}{s+1}\right] = \left[1 - e^{-t}\right]u(t)$$

Q) Is this $y(t)$ calculation a true representation of reality?

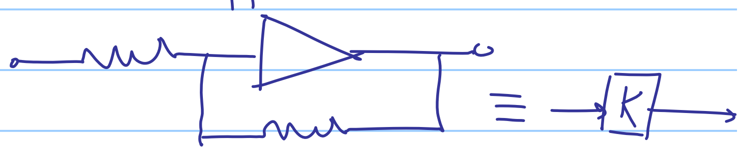
To verify let us try and simulate this system on an analog computer

Recall that an analog computer usually have the following components

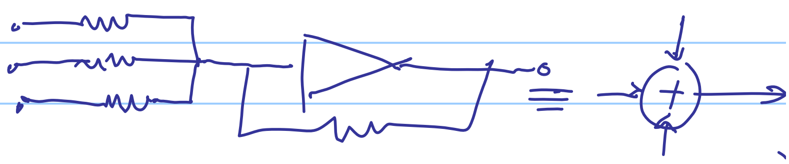
1) Integrator

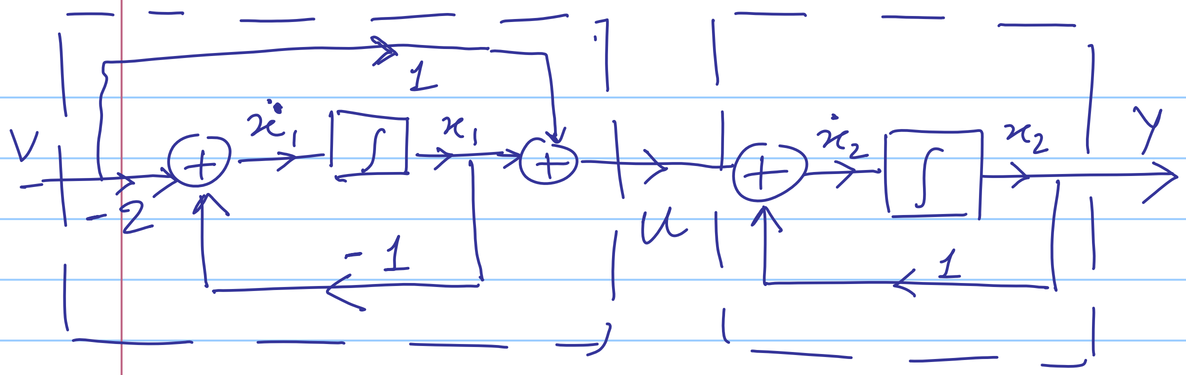


2) Amplifier



3) Adder





If you actually implement this figure on a analog computer in the lab, you will find that the output y (visible on a CRO e.g.) saturates or the system burns out.

➡ The T.F. $G_c(s)$ DID NOT say anything about this!

To find out why this happens look at the differential equation involved in the diagram above.

$$\text{(1)} \begin{cases} \dot{x}_1 = -x_1 - 2v \\ \dot{x}_2 = x_1 + x_2 + v \end{cases} \quad \begin{array}{|l} x_1(0) = x_{10} \\ x_2(0) = x_{20} \end{array}$$

For solving these equations we need <u>initial conditions</u> for $x_1(0)$ & $x_2(0)$. In the analog computer simulation these are voltages at the output of the integrators when the simulation started.

Among various other methods, these equations can be solved using Laplace transforms:

Taking L.T. of (1),

$$sX_1(s) - X_1(0^-) = -X_1(s) - 2V(s)$$

$$sX_2(s) - X_2(0^-) = X_1(s) + X_2(s) + V(s)$$

$$\therefore Y(s) = X_2(s) = \frac{X_2(0)}{s-1} + \frac{X_1(0)}{(s-1)(s+1)} + \frac{V(s)}{s+1}$$

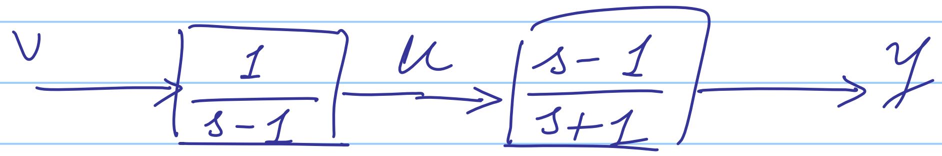
$$\Rightarrow y(t) = x_2(t) = e^{t} x_{20} + \frac{1}{2}\left(e^{t} - e^{-t}\right) x_{10} + e^{-t} * v(t)$$

Hence the "actual" transfer function matches the "original" $G_c(s)$, only when $x_{10} = x_{20} = 0$.

For most other initial conditions, $y(t)$ saturates in a short while. and THAT IS WHAT IS VISIBLE ON THE ANALOG COMPUTER.

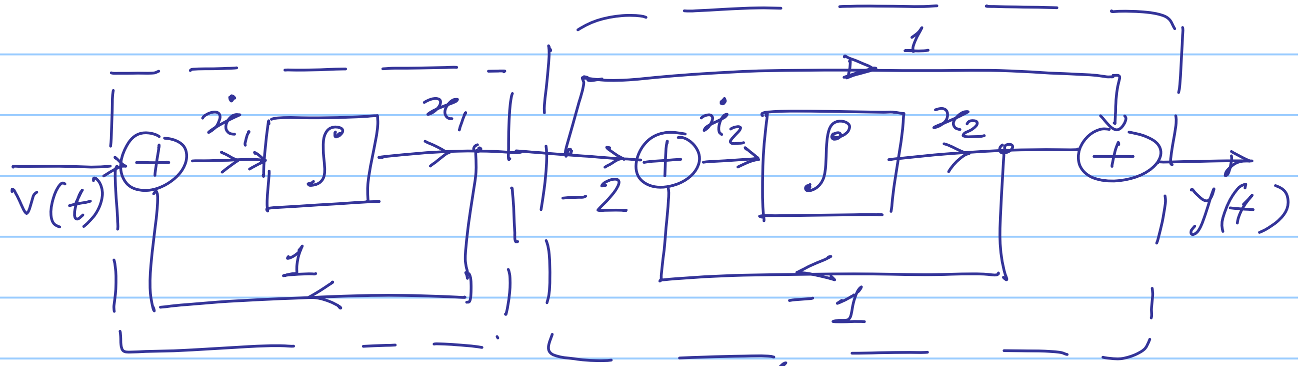
Exercise: For what other initial conditions in this problem the output does not saturate?

Problem 2: Now consider the blocks in reverse order:



$$G_c'(s) = \frac{Y(s)}{V(s)} = \frac{1}{s-1} \cdot \frac{s-1}{s+1} = \frac{1}{s+1}$$

Hence, from a transfer function approach, $G_c(s)$ and $G_c'(s)$ are identical.



The corresponding differential eqns:

$$(2)\quad \begin{cases} \dot{x}_1 = x_1 + v(t) \\ \dot{x}_2 = -2x_1 - x_2 \end{cases} \quad \Bigg| \quad \begin{matrix} x_1(0) = x_{10} \\ x_2(0) = x_{20} \end{matrix}$$

Solving these equations:

$$y(t) = (x_{10} + x_{20})e^{-t} + e^{-t} * v(t)$$

In this case $y(t)$ don't saturate even for non-zero x_{10}, x_{20}.

But consider, $x_1(t)$

$$x_1(t) = e^{t}x_{10} + e^{t} * v(t)$$

Hence $u(t) = x_1(t)$ saturates ..

SO, The analog computer simulation won't work here either.

SUMMARY: INTERNAL BEHAVIOR OF SYSTEMS ARE COMPLICATED.
T.F. cannot describe internal behavior. We require STATE VARIABLES.