3. What would happen to a physical system that becomes unstable?
4. Why are marginally stable systems considered unstable under the BIBO definition of stability?
5. Where do system poles have to be to ensure that a system is not unstable?
6. What does the Routh-Hurwitz criterion tell us?
7. Under what conditions would the Routh-Hurwitz criterion easily tell us the actual location of the system's closed-loop poles?
8. What causes a zero to show up only in the first column of the Routh table?
9. What causes an entire row of zeros to show up in the Routh table?
10. Why do we sometimes multiply a row of a Routh table by a positive constant?
11. Why do we not multiply a row of a Routh table by a negative constant?
12. If a Routh table has two sign changes above the even polynomial and five sign changes below the even polynomial, how many right-half-plane poles does the system have?
13. Does the presence of an entire row of zeros always mean that the system has right-half-plane poles?
14. If a seventh-order system has a row of zeros at the $s^3$ row and two sign changes below the $s^4$ row, how many right-half-plane poles does the system have?
15. Is it true that the eigenvalues of the system matrix are the same as the closed-loop poles?
16. How do we find the eigenvalues?

### PROBLEMS

1. Tell how many roots of the following polynomial are in the right half-plane, in the left half-plane, and on the $j\omega$-axis: [Section: 6.2]
   \[ P(s) = s^3 + 3s^2 + 5s^2 + 4s^2 + 3 + 3 \]

2. Tell how many roots of the following polynomial are in the right half-plane, in the left half-plane, and on the $j\omega$-axis: [Section: 6.3]
   \[ P(s) = s^5 + 6s^2 + 5s^2 + 8s + 20 \]

3. Using the Routh table, tell how many poles of the following function are in the right half-plane, in the left half-plane, and on the $j\omega$-axis: [Section: 6.3]
   \[ T(s) = \frac{s + 8}{s^2 - 4s^2 - 4s + 3s - 2} \]

4. The closed-loop transfer function of a system is [Section: 6.3]
   \[ T(s) = \frac{s^2 + 2s^2 + 7s + 21}{s^3 - 2s^3 + 6s^2 + 6s + 2s - 4} \]

Determine how many closed-loop poles lie in the right half-plane, in the left half-plane, and on the $j\omega$-axis.

5. How many poles are in the right half-plane, the left half-plane, and on the $j\omega$-axis for the open-loop system of Figure P6.17?

6. How many poles are in the right half-plane, the left half-plane, and on the $j\omega$-axis for the open-loop system of Figure P6.27 [Section: 6.3]

7. Use MATLAB and the Symbolic Math Toolbox to generate a Routh table to solve Problem 3.

8. Determine whether the system of Figure 1 [Section: 6.2]
   \[ G(s) = \frac{1}{s+1} \]
   \[ R(s) = \frac{s^2 + 2s^2 - 4}{s^2 + 4s^2 - 6s^2 + 6s + 6} \]

9. Consider the unity with
   \[ G(s) \]
   Using the Routh-Hurwitz the $s$-plane where $s$ are located. [Section: 6.4]

10. In the system of $F$
    \[ G(s) = \]
    Find the range
    [Section: 6.4]

11. Given the unity feedback system:
    \[ G(s) = \frac{1}{s^2 + 5s^2 + 1} \]
    Tell how many poles lie in the right half-plane and on the $j\omega$-axis.

12. Using the Routh-Hurwitz criterion for the system:
    \[ G(s) = \]
    Tell whether or not
    [Section: 6.2]
Determine whether the unity feedback system of Figure P6.3 is stable if 
[Section: 6.2]

\[ G(s) = \frac{240}{(s + 1)(s + 2)(s + 3)(s + 4)} \]

9. Consider the unity feedback system of Figure P6.3 with

\[ G(s) = \frac{1}{4s^2(s^2 + 1)} \]

Using the Routh-Hurwitz criterion, find the region of the s-plane where the poles of the closed-loop system are located. [Section: 6.3]

10. In the system of Figure P6.3, let

\[ G(s) = \frac{K(s + 2)}{s(s - 1)(s + 3)} \]

Find the range of K for closed-loop stability. [Section: 6.4]

11. Given the unity feedback system of Figure P6.3 with 
[Section: 6.3]

\[ G(s) = \frac{84}{s^3 + 5s^2 + 12s^2 + 25s^2 + 43s + 50s^2 + 82s + 60} \]

tell how many poles of the closed-loop transfer function lie in the right half-plane, in the left half-plane, and on the jω-axis. [Section: 6.3]

12. Using the Routh-Hurwitz criterion and the unity feedback system of Figure P6.3 with

\[ G(s) = \frac{1}{2s^2 + 5s + s^2 + 2s} \]

tell whether or not the closed-loop system is stable. [Section: 6.2]

13. Given the unity feedback system of Figure P6.3 with

\[ G(s) = \frac{8}{s^6 - 2s^5 - s^4 + 2s^3 + 4s^2 - 8s - 4} \]

tell how many closed-loop poles are located in the right half-plane, in the left half-plane, and on the jω-axis. [Section: 6.3]

14. Repeat Problem 13 using MATLAB.

15. Consider the following Routh table. Notice that the s^6 row was originally all zeros. Tell how many roots of the original polynomial were in the right half-plane, in the left half-plane, and on the jω-axis. [Section: 6.3]

<table>
<thead>
<tr>
<th>s^6</th>
<th>s^5</th>
<th>s^4</th>
<th>s^3</th>
<th>s^2</th>
<th>s^1</th>
<th>s^0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>-1</td>
<td>-2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>-1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>-3</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>21</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

16. For the system of Figure P6.4, tell where the closed-loop poles are located (i.e., right half-plane, left half-plane, jω-axis). Notice that there is positive feedback.

17. Using the Routh-Hurwitz criterion, tell how many closed-loop poles of the system shown in Figure P6.5 lie in the left half-plane, in the right half-plane, and on the jω-axis. [Section: 6.3]
18. Determine if the unity feedback system of Figure P6.3 with
\[ G(s) = \frac{K(s^2 + 1)}{(s + 1)(s + 2)} \]
can be unstable. [Section: 6.4]

19. For the unity feedback system of Figure P6.3 with
\[ G(s) = \frac{K(s + 6)}{s(s + 1)(s + 3)} \]
determine the range of \( K \) to ensure stability. [Section: 6.4]

20. In the system of Figure P6.3, let
\[ G(s) = \frac{K(s - a)}{s(s - b)} \]
Find the range of \( K \) for closed-loop stability when:
[Section: 6.4]
\[ \begin{align*}
  a &< 0, \ b < 0 \\
  a &< 0, \ b > 0 \\
  a > 0, \ b < 0 \\
  a > 0, \ b > 0
\end{align*} \]

21. For the unity feedback system of Figure P6.3 with
\[ G(s) = \frac{K(s + 1)}{s(s + 2)(s + 3)(s + 4)} \]
determine the range of \( K \) for stability.

22. Repeat Problem 21 using MATLAB.

23. Use MATLAB and the Symbolic Math Toolbox to generate a Routh table in terms of \( K \) to solve Problem 21.

24. Find the range of \( K \) for stability for the unity feedback system of Figure P6.3 with [Section: 6.4]
\[ G(s) = \frac{K(s + 2)(s - 2)}{(s^2 + 3)^2} \]

25. For the unity feedback system of Figure P6.3 with
\[ G(s) = \frac{K(s + 1)}{s^2(s + 2)} \]
find the range of \( K \) for stability. [Section: 6.4]

26. Find the range of gain, \( K \), to ensure stability for a unity feedback system of Figure P6.3 with [Section 6.4]
\[ G(s) = \frac{K(s - 2)(s + 4)(s + 5)}{(s^2 + 3)} \]

27. Find the range of gain, \( K \), to ensure stability in a unity feedback system of Figure P6.3 with [Section 6.4]
\[ G(s) = \frac{K(s + 2)}{(s^2 + 1)(s + 4)(s - 1)} \]

28. Using the Routh-Hurwitz criterion, find the value of \( K \) that will yield oscillations for the unity feedback system of Figure P6.3 with [Section: 6.4]
\[ G(s) = \frac{K(s + 2)}{(s + 15)(s + 27)(s + 38)} \]

29. Use the Routh-Hurwitz criterion to find the range of \( K \) for which the system of Figure P6.6 is stable [Section: 6.4]
\[ G(s) = \frac{K}{s^2 + 2s + 1} \]

30. Repeat Problem 29 for the system of Figure P6.7. [Section: 6.4]
\[ G(s) = \frac{K(s^2 + 2)}{(s^2 + 3)^2} \]

31. Given the unity feedback system of Figure P6.3 with [Section 6.4]
\[ G(s) = \frac{K(s + 4)}{s(s + 1)(s + 2)} \]
find the following: [Section: 6.4]
\[ a. \text{ The range of } K \text{ that keeps the system stable} \\
b. \text{ The value of } K \text{ that makes the system oscillate} \\
c. \text{ The frequency of oscillation when } K \text{ is set to the value that makes the system oscillate} \]

32. Repeat Problem 31 for [Section 6.4]
\[ G(s) = \frac{K(s - 2)}{(s + 2)} \]

33. For the system shown in Figure P6.8, find the frequency of oscillation of gain, \( K \), that will make the system marginally stable.

34. Given the unity feedback system of Figure P6.3 with [Section 6.4]
\[ G(s) = \frac{K}{s^2 - 4s} \]
\[ a. \text{ Find the range of } K \text{ for which the system is stable} \\
b. \text{ Find the frequency of oscillation when the system is marginally stable} \]

35. Repeat Problem 34 using MATLAB.

36. For the unity feedback system of Figure P6.9
\[ G(s) = \frac{K}{(s^2 + 4)(s^2 + 3)} \]
find the range of \( K \) for which the system has oscillations.

37. For the unity feedback system [Section: 6.4]
\[ G(s) = \frac{K}{s + 1} \]
\[ a. \text{ Find the range of } K \text{ for which the system is stable} \\
b. \text{ Find the frequency of oscillation when the system is marginally stable} \]
42. The closed-loop transfer function of a system is

\[ T(s) = \frac{s^2 + K_1 s + K_2}{s^4 + K_1 s^3 + K_2 s^2 + 5s + 1} \]

Determine the range of \( K_4 \) in order for the system to be stable. What is the relationship between \( K_1 \) and \( K_2 \) for stability? [Section: 6.4]

43. For the transfer function below, find the constraints on \( K_1 \) and \( K_2 \) such that the function will have only two \( j\omega \)-poles. [Section: 6.4]

\[ T(s) = \frac{K_1 s + K_2}{s^2 + K_1 s + s + K_2 s + 1} \]

44. The transfer function relating the output engine fan speed (rpm) to the input main burner fuel flow rate (lb/hr) in a short takeoff and landing (STOL) fighter aircraft, ignoring the coupling between engine fan speed and the pitch control command, is (Schierman, 1992):

\[ G(s) = \frac{6.5s^2 + 90.5s + 1076s + 15.996s^2 + 3126s - 11060 - 3900s - 1640}{s^2 + 110s + 1186s + 4640s + 2156s + 890s - 1000s - 1350s - 415} \]

a. Find how many poles are in the right half-plane, in the left half-plane, and on the \( j\omega \)-axis.
b. Is this open-loop system stable?

45. An interval polynomial is of the form

\[ P(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + \cdots \]

with its coefficients belonging to intervals \( x_i \leq a_i \leq y_i \), where \( x_i, y_i \) are prescribed constants. Khartitonov’s theorem says that an interval polynomial has all its roots in the left half-plane if each one of the following four polynomials have its roots in the left half-plane (Mincelli et al., 2000):

\[ K_1(s) = a_0 + x_1 s + y_2 s^2 + x_3 s^3 + y_4 s^4 + y_5 s^5 + \cdots \]
\[ K_2(s) = a_0 + y_1 s + y_2 s^2 + x_3 s^3 + y_4 s^4 + y_5 s^5 + \cdots \]
\[ K_3(s) = a_0 + y_1 s + y_2 s^2 + y_3 s^3 + y_4 s^4 + y_5 s^5 + \cdots \]
\[ K_4(s) = a_0 + y_1 s + y_2 s^2 + x_3 s^3 + y_4 s^4 + y_5 s^5 + \cdots \]

Use Khartitonov’s theorem and the Routh-Hurwitz criterion to find if the following polynomial has any zeros in the right half-plane.

\[ P(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 \]

\[ 2 \leq a_0 \leq 4; \quad 1 \leq a_1 \leq 2; \quad 4 \leq a_2 \leq 6; \quad a_3 = 1 \]

46. A linearized model of a torque-controlled aircraft having a load with a fixed rope length is

\[ P(s) = \frac{X_r(s)}{F_y(s)} = \frac{1}{my^2(s^2 + aw_0^2)} \]

where \( w_0 = \sqrt{\frac{f}{L}} \) is the rope length, \( my = \) the mass of the car, \( a = \) the combined rope and car mass, \( f \) is the force input applied to the car, and \( x = \) the resulting rope displacement (Martinen, 1996).

If the system is controlled in a feedback configuration by placing it in a loop as shown in Figure P6.11, with \( K > 0 \), where will the closed-loop poles be located?

![Figure P6.11](image)

47. The read/write head assembly arm of a computer hard disk drive (HDD) can be modeled as a rigid rotating body with inertia \( I \). Its dynamics can be described by the transfer function

\[ P(s) = \frac{X(s)}{F(s)} = \frac{1}{I s^2} \]

where \( X(s) \) is the displacement of the read/write head and \( F(s) \) is the applied force (Kro, 2007). Show that if the HDD is controlled in the configuration shown in Figure P6.11, the arm will oscillate and cannot be positioned with any precision over a HDD track. Find the oscillation frequency.

48. A system is represented in state space as

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} 0 & 1 & 2 \\ 3 & 1 & -4 \\ 1 & 1 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x
\end{align*}
\]

Determine how many eigenvalues are in the right half-plane, in the left half-plane, and on the \( j\omega \)-axis.
The MATLAB to find the eigenvalues of the following system:

\[
\begin{pmatrix}
0 & 1 & 0 \\
-1 & 1 & 3 \\
1 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
x \\
x \\
u
\end{pmatrix}
\]

The system in state space represents the forward path of a unity feedback system. Use the Routh-Hurwitz criterion to determine if the closed-loop system is stable. (Section 6.5)

A Butterworth polynomial is of the form

\[B_T(s) = 1 + (-1)^n \left( \frac{s}{\omega_n} \right)^{2n}, n > 0.\]

Use the Routh-Hurwitz criterion to find the zeros of a Butterworth polynomial for:

a. \( n = 1 \);

b. \( n = 2 \)

Design Problems

A model for an airplane’s pitch loop is shown in Figure P6.12. Find the range of gain, \( K \), that will keep the system stable. Can the system ever be unstable for positive values of \( K \)?

A common application of control systems is in controlling the temperature of a chemical process (Figure P6.13). The flow of chemical reactants to a process is controlled by an actuator and valve. The reactant causes the temperature in the vat to change. This system is designed to control the temperature of a desired set-point temperature in a closed loop, where the flow of reactant is adjusted to yield the desired temperature. In Chapter 9 we will learn how a PID controller is used to improve the performance of such process control systems. Figure P6.11 shows the control system prior to the addition of the PID controller. The PID controller is replaced by the shaded box with a gain of unity. For this system, prior to the design of the PID controller, find the range of the PID controller, find the range of amplifier gain, \( K \), to keep the system stable.

A robot arm called ISAC (Intelligent Soft Arm Control) can be used as part of a system to feed people with disabilities (see Figure P6.14(a)). The control system guides the spoon to the food and then to a position near the person’s mouth. The arm uses a special pneumatic controlled actuator called a rubber actuators. The structure consists of rubber tubes covered with fiber cord. The actuator contracts in length when pneumatic pressure is decreased. This expansion and contraction in length can drive a pulley or other device. A video camera provides the sight for the robot and the tracking loop (Kara, 1992). Assume the simplified block diagram shown in Figure P6.14(b) for regulating the speed of the motor. Find the range of \( K \) for stability. (Use of a program with symbolic capability is recommended.)
56. Often an aircraft is required to tow another vehicle, such as a practice target or glider. To stabilize the towed vehicle and prevent it from rolling, pitching, and yawing, an autopilot is built into the towed vehicle. Assume the block diagram shown in Figure P6.15 represents the autopilot roll control system (Cochran, 1992). Find the range of $K$ to keep the roll angle stable.

Figure P6.15  Towed vehicle roll control

57. Cutting forces should be kept constant during machining operations to prevent changes in spindle speeds or work position. Such changes would deteriorate the accuracy of the work's dimensions. A control system is proposed to control the cutting force. The plant is difficult to model, since the factors that affect cutting force are time varying and not easily predicted.

Figure P6.16  Cutting force control system

However, assuming the simplified force model shown in Figure P6.16, use the Routh-Hurwitz criterion to find the range of $K$ to keep the system stable (Roberts, 1997).

58. Transportation systems that use magnetic levitation can reach very high speeds, since contact friction on the rails is eliminated (see Figure P6.17(a)). Electric magnets can produce the force to elevate the vehicle. Figure P6.17(b) is a simulation model of a complete system that can be used to regulate the magnetic field. In the figure, $Z_{act}(s)$ represents a voltage proportional to the desired amount of levitation, or gap. $Z_{mag}$ represents a voltage proportional to the actual amount of levitation. The plant models the dynamic response of the vehicle to signals from the controller (Hancock, 1998). Use the Routh-Hurwitz criterion to find the range of $K$, to keep the closed-loop system stable.

$F(s) = \frac{1 + \frac{1}{s}}{s^2}$

where $k = 2.1 \times 10^5$ N/kg, $R_C = 1.5 \times 10^3$ A, a parameter that is used in the range of $K$, and $m$ is the mass of the levitated object (Hancock, 1999).

Figure P6.18 depicts the shift oscillator.
The transfer function from indoor radiator power, \( Q(t) \), room temperature, \( T(t) \), in an 11 m\(^3\) room is

\[
Q(t) = 1 \times 10^{-5}T^2 + 1.314 \times 10^{-5}T + 2.66 \times 10^{-11}
\]

where \( Q \) is in watts and \( T \) is in °C. (Thomas, 2005).

The room’s temperature will be controlled by embedding it in a closed loop, such as that of Figure P6.11, and the range of \( K \) for closed-loop stability.

During vertical spindle surface grinding, adjustments are made on a multi-axis CNC machine by measuring the applied force with a dynamometer and applying appropriate corrections. This feedback force control results in higher homogeneity and better tolerances in the resulting finished product. In a specific experiment with an extremely high feed rate, the transfer function from the desired depth of cut (DOC) to applied force was

\[
F(x) = \frac{K_C}{DOC(x)} = \frac{1}{1 + \frac{K_C}{m} \frac{x}{s^2 + b} + \frac{K_C}{K_I} \frac{1}{Ts + 1}}
\]

where \( k = 2.1 \times 10^3 \text{ N/m}, b = 0.78 \text{ Ns/m}, m = 1.2 \times 10^{-4} \text{ Kg}, K_C = 1.5 \times 10^3 \text{ N/mm} \), and \( T = 0.044 \text{ s}. \) \( K_I \) is a parameter that is varied to adjust the system. Find the range of \( K_I \) under which the system is stable (Holman, 1999).

Figure P6.18 depicts the schematic diagram of a phase shift oscillator.

![Figure P6.18 Phase shift oscillator.](image)

The circuit will oscillate if it is designed to have pole on the \( j\omega \)-axis.

### Problems

a. Show that the transfer function for the passive network in the circuit is given by

\[
V_2(s) = \frac{-1}{(1 + \frac{1}{2RC}) (1 + \frac{1}{2SRC})^2 - \frac{2}{2SRC}}
\]

b. Show that the oscillator’s characteristic equation is given by

\[
1 - K \left(1 + \frac{1}{2SRC} \right) \left(2 + \frac{1}{2SRC} \right)^2 - \frac{2}{2SRC} = 0
\]

where \( K = \frac{R_2}{R_1} \).

c. Use the Routh-Hurwitz criterion to obtain the oscillation condition and the oscillation frequency.

(62.) Look-ahead information can be used to automatically steer a bicycle in a closed-loop configuration. A line is drawn in the middle of the lane to be followed, and an arbitrary point is chosen in the vehicle’s longitudinal axis. A look-ahead offset is calculated by measuring the distance between the look-ahead point and the reference line and is used by the system to correct the vehicle’s trajectory. A linearized model of a particular bicycle traveling on a straight-line path at a fixed longitudinal speed is

\[
\begin{bmatrix}
V \\
\dot{V}
\end{bmatrix} = \begin{bmatrix}
-11.7 & 6.8 & 61.6K & 7.7K \\
-3.5 & -24 & -66.9K & 8.4K
\end{bmatrix} \begin{bmatrix}
V \\
\dot{V}
\end{bmatrix} + \begin{bmatrix}
0 & 1 \end{bmatrix} \begin{bmatrix}
\dot{f}
\end{bmatrix}
\]

In this model \( V \) = bicycle’s lateral velocity, \( \dot{r} \) = bicycle’s yaw velocity, \( \psi \) = bicycle’s yaw acceleration, and \( Y_e \) = bicycle’s center of gravity coordinate on the y-axis. \( K \) is a controller parameter to be chosen by the designer (Özgüner, 1995). Use the Routh-Hurwitz criterion to find the range of \( K \) for which the system is closed-loop stable.

### Progressive Analysis and Design Problems

63. High-speed rail pantograph. Problem 19 in Chapter 1 discusses active control of a pantograph mechanism for high-speed rail systems. In Problem 72(a), Chapter 5, you found the block diagram for the active
64. Control of HIV/AIDS. The HIV infection linearized model developed in Problem 77, Chapter 4 can be shown to have the transfer function

\[
P(s) = \frac{Y(s)}{U(s)} = \frac{-5200 - 10.3844}{s^2 + 2.6817s^2 + 0.11s + 0.0125}
\]

It is desired to develop a policy for drug delivery to maintain the virus count at prescribed levels. For the purpose of obtaining an appropriate \( u_i(t) \), feedback will be used as shown in Figure P6.19

*Figure P6.19*

As a first approach, consider \( G(s) = K \), a constant, be selected. Use the Routh-Hurwitz criteria to find the range of \( K \) for which the system is closed-loop stable.

**CYBER EXPLORATION LABORATORY**

Experiment 6.1

Objectives To verify the effect of pole location upon stability. To verify the effect upon stability of loop gain in a negative feedback system.

Minimum Required Software Packages MATLAB, Simulink, and the Control System Toolbox

Prelab

1. Find the equivalent transfer function of the negative feedback system of Figure P6.20 if

\[
G(s) = \frac{-K}{s(s + 2)^2} \quad \text{and} \quad H(s) = 1
\]

2. For the system of Prelab 1, find two values of gain that will yield closed-loop overdamped, second-order poles. Repeat for underdamped poles.

3. For the system of Prelab 1, find the value of gain, \( K \), that will make the system critically damped.

4. For the system of Prelab 1, find the value of gain, \( K \), that will make the system marginally stable. Also, find the frequency of oscillation at that value of \( K \) that makes the system marginally stable.

5. For each of Prelab 2 through 4, plot on one graph the pole locations for each case and write the corresponding value of gain, \( K \), at each pole.

Lab

1. Using Simulink, set up the negative feedback system of Prelab 1. Plot the step response of the system at each value of gain calculated to yield overdamped, underdamped, critically damped, and marginally stable responses.

**BIBLIOGRAPHY**