EE 640 - HW 1

Note Title 04-08-2008

1) Direct Solution of State-Space Eqns

a) Solve the state equations

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x(t) \qquad x(0) = x_0$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \qquad t \geq 0$$

These are the equations for the simple harmonic motion of a particle of unit mass.

b) Consider the system described by:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \end{bmatrix} u(t)$$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

* Calculate $x_1(t)$, $x_2(t)$ and $y(t)$ for $t > 0$.

2) Realizations of a Non-strictly proper $H(s)$:

Find realizations using all the (possible) methods covered in class of the transfer function:

$$H(s) = \frac{4s^3 + 25s^2 + 45s + 34}{s^3 + 6s^2 + 10s + 8}$$

Give both state space equations and block diagrams.

3) Draw a block diagram corresponding to the realization

$$A = \begin{bmatrix} \lambda_1 & c_2 & c_3 & \cdots & c_{n-1} & 1 \\ & \lambda_2 & c_3 & \cdots & c_{n-1} & 1 \\ & & \ddots & \cdots & c_{n-1} & 1 \\ & & & \ddots & \vdots & 1 \\ & 0 & & & \vdots & \vdots \\ & & & & \lambda_{n-1} & 1 \\ & & & & & \lambda_n \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_{n-1} \end{bmatrix} \qquad C = \begin{bmatrix} c_1 & c_2 & \cdots & c_{n-1} & 1 \end{bmatrix}$$

This is called the cascade form.

4) Write state equations for two realizations $\{A_i, b_i, c_i\}$ connected in (a) series (b) parallel (c) feedback, with $\{A_1, b_1, c_1\}$ in the forward loop and $\{A_2, b_2, c_2\}$ in the feedback loop.

5) Time dependent Similarity Transformation

We discussed the use of similarity transformations

$$x(t) = T\bar{x}(t)$$

where T is a constant non-singular matrix. We can also allow time-variant transformations,

$$x(t) = T(t)\,\bar{x}(t)$$

provided $T(t)$ is non-singular and differentiable for all t. Find the state equations for $\bar{x}(t)$ in this case, and show that it is possible to choose $T(t)$ so that the new "$\bar{A}$" matrix for $\bar{x}(t)$ is identically zero.

6) Consider a body of unit mass moving along a line under the influence of a force u. Let $y(t)$ = its displacement at time t, and let

$\{y_0, \dot{y}_0\}$ be its initial position and velocity.

a) Let $u = -y$; solve the equations and plot trajectories in the phase plane (i.e. the plane where the respective states form the x-axis & the y-axis) for

$$\dot{y}_0 = y_0 = 1 \ , \ \dot{y}_0 = y_0 = -1$$

(Extension: Let $u = -ky$ and discuss the effect of k on the shape and period of the trajectories.) Trajectories behaving as in this part are said to have a center at $\dot{y} = y = 0$. Why? Discuss the suitability of $u = -y$ as a control. Find the eigenvalues of the "A" matrix. To what eigenvalue locations does a center correspond?

(b) Since part (a) was not satisfactory, we add velocity feedback and let $u = -3\dot{y} - 2y$. Plot the phase-plane portrait and discuss the suitability of this control. The portrait here is called node. Where are the eigenvalues of A? (You <u>can</u> use MATLAB for the phase portrait in this part).

6) a) If $\{A, b, c\}$ and $\{\bar{A}, \bar{b}, \bar{c}\}$ are related by a constant similarity transformation, show that they have the same transfer function.

b) Realizations can have different numbers of states. Show that the constant realizations

$$\begin{bmatrix} A & A_1 \\ 0 & A_2 \end{bmatrix}, \begin{bmatrix} b \\ 0 \end{bmatrix}, [c \;\; q]$$

and
$$\begin{bmatrix} A & 0 \\ A_1 & A_2 \end{bmatrix}, \begin{bmatrix} b \\ q \end{bmatrix}, [c \;\; 0]$$

and $\{A, b, c\}$ all have the same transfer function for all values and (compatible) dimensions of A_1, A_2, q.

7) The Relative Order of a System

a) Show that the numerator polynomial of the transfer function of a system realization has degree m if and only if

$$cA^i b = 0 \quad \text{for } i = 0, 1, 2, \ldots, n-m-2.$$

and
$$cA^{n-m-1} b \neq 0$$

b) Conversely, if the difference between the degrees of the numerator and denominator polynomials of a given transfer function is p and if (A, b, c) is a realization of this transfer function, then how many and which of the $\{cA^i b\}_{i=0}^{n-1}$ must be zero?

The integer p is known as the relative order of the system.

8) Consider the system given in Problem (1 b) above:

a) What is the transfer function of the system described by these state equations?

b) Does the transfer function give an adequate description of this system? Give reasons for your answer.