10 Feb:

$$p(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0$$

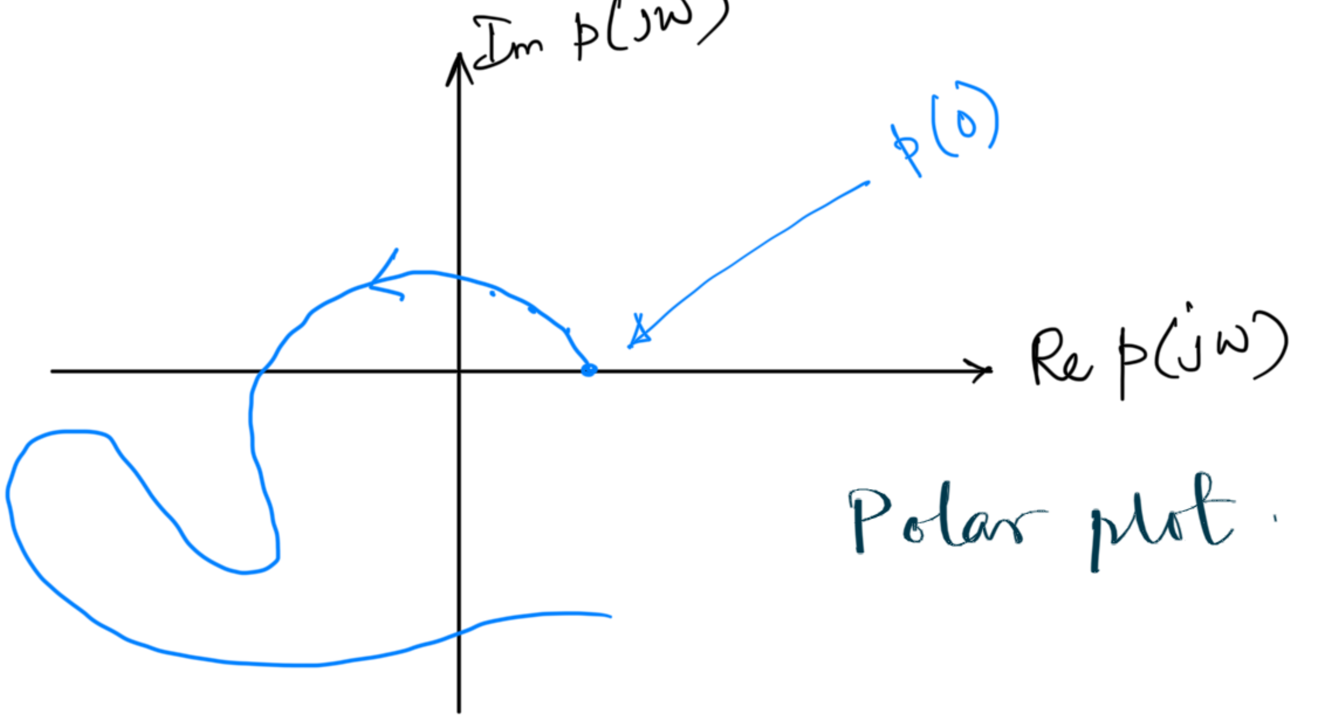
$$a_i \in \mathbb{R}$$

p is said to be "Hurwitz" iff roots $p(s) \subseteq \mathbb{C}^-$

$$p(s) = a_n (s+\lambda_1)(s+\lambda_2) \cdots (s+\lambda_n)$$

theorem: p is Hurwitz $\Rightarrow$ a_i are all of the same sign.

$p(s) \longrightarrow p(j\omega)$ vary $\omega = 0$ to $\omega = +\infty$

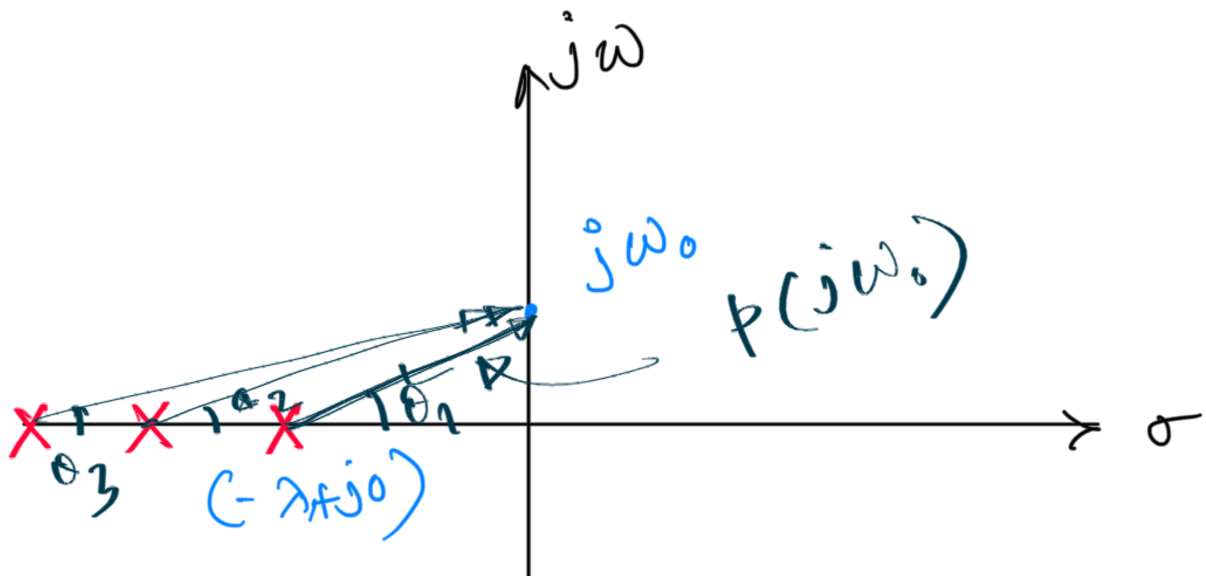


Polar plot.

$$\theta(p(j\omega)) = \text{angle of } p(j\omega)$$

$$\int_{\omega=0}^{\omega=\infty} d\theta = \text{total angle accumulated by the polar plot.}$$

theorem: p is Hurwitz $\Leftrightarrow$ total angle accumulated by $p(j\omega)$ is $n\frac{\pi}{2}$.

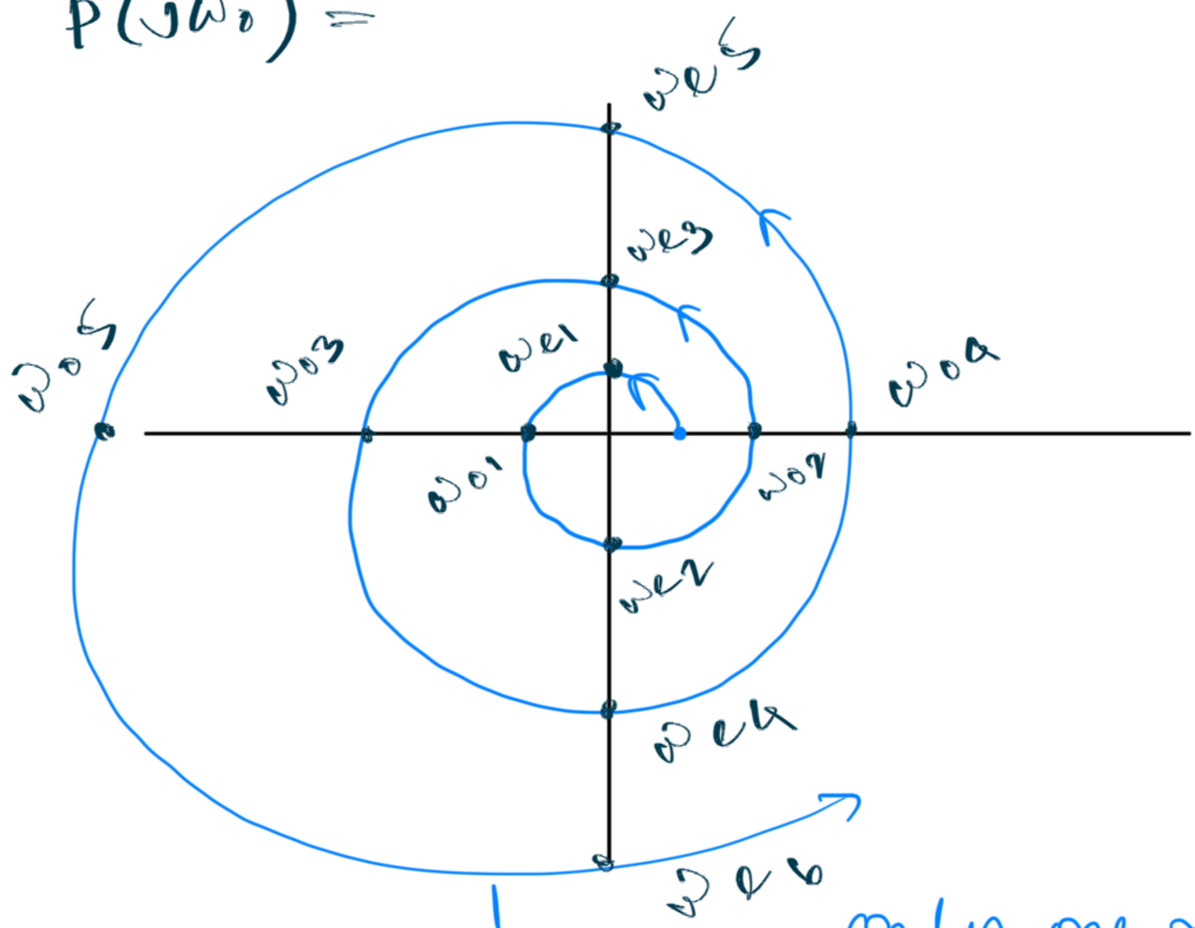


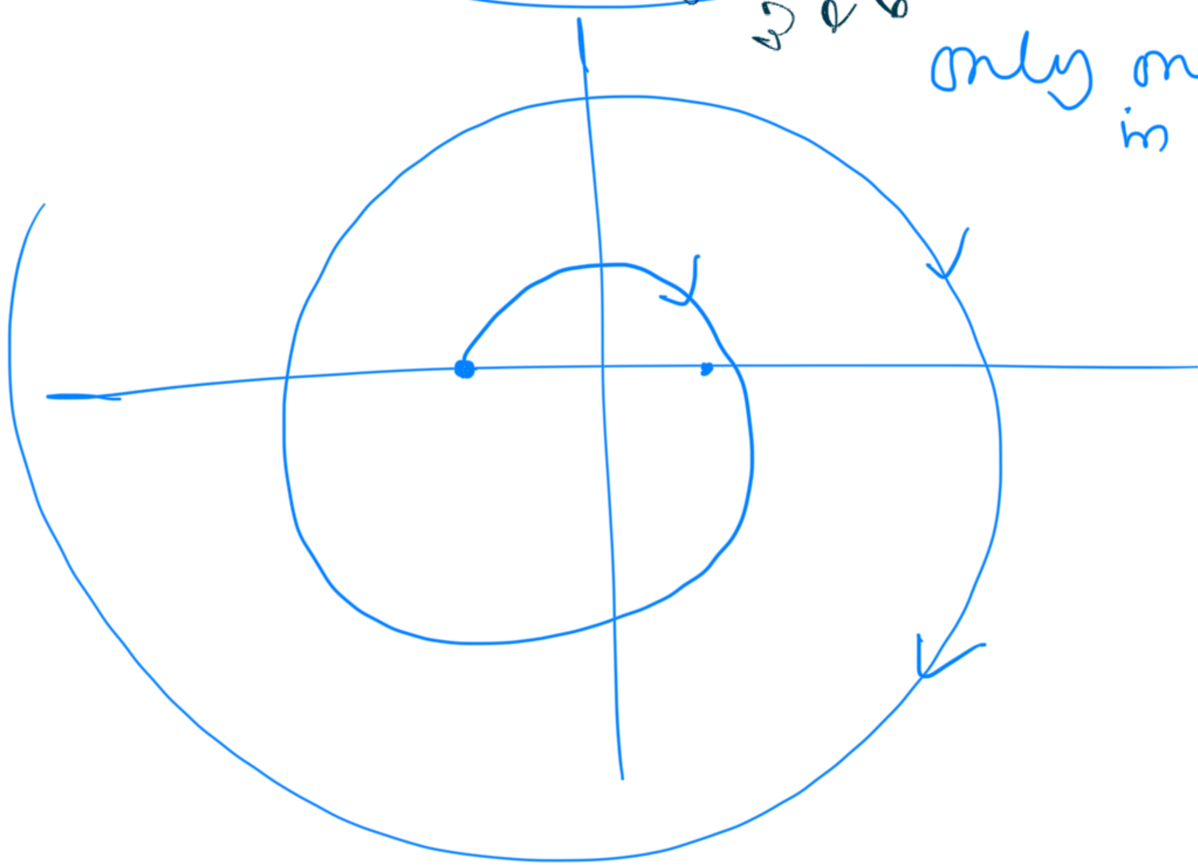
$$p(s) = (s+\lambda) \qquad \lambda \in \mathbb{R}$$

$$p(j\omega_0) = (j\omega_0 + \lambda)$$

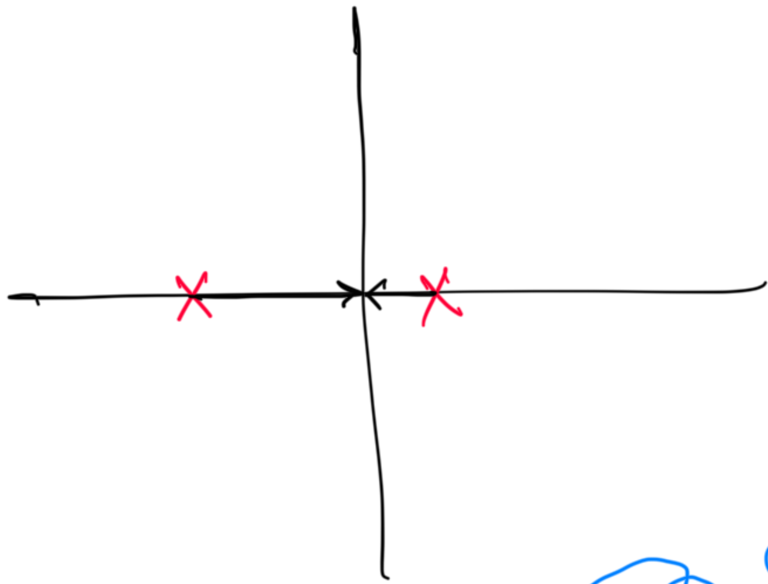
$$p(s) = (s+\lambda_1)(s+\lambda_2)(s+\lambda_3)$$

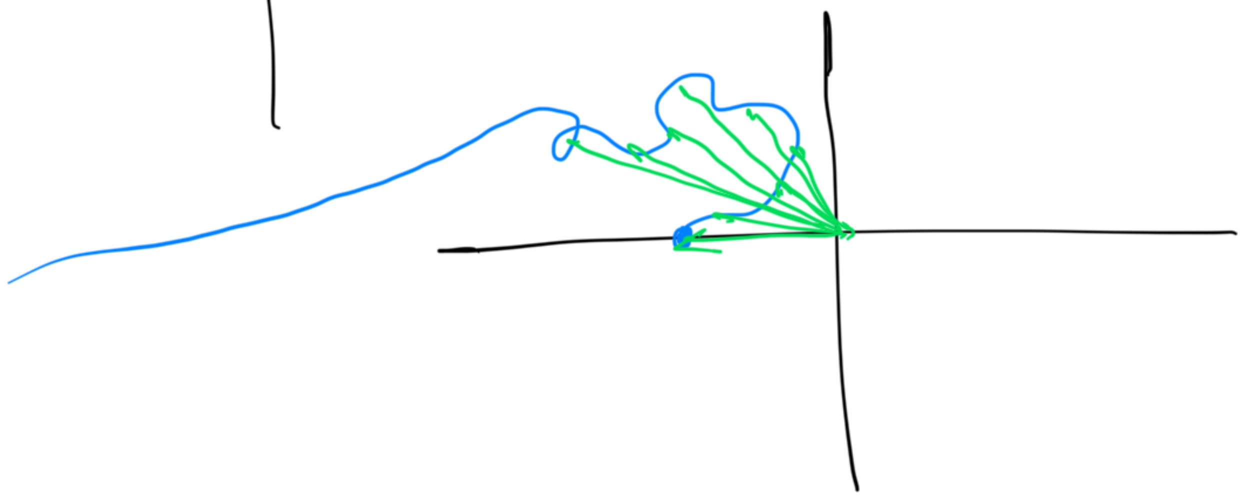
$$p(j\omega_0) =$$





only one root in $\mathbb{C}^+$





$$p(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0$$

$n =$ even

s^n: $a_n \quad a_{n-2} \quad a_{n-4} \quad \cdots \quad a_0$

s^{n-1}: $a_{n-1} \quad a_{n-3} \quad a_{n-5} \quad \cdots \quad a_1$

$$p^{even}(s) = a_n s^n + a_{n-2} s^{n-2} + a_{n-4} s^{n-4} + \cdots + a_0$$

$$p^{odd}(s) = a_{n-1} s^{n-1} + \cdots + a_1 s$$

$$\Rightarrow p^{even}(j\omega) = a_n (j\omega)^n + a_{n-2}(j\omega)^{n-2} + \cdots + a_0 \in \mathbb{R}$$

$$=: P^e(\omega)$$

$$p^{odd}(j\omega) = j\omega \underbrace{\left[a_{n-1}(j\omega)^{n-2} + a_{n-3}(j\omega)^{n-4} + \cdots + a_1\right]}_{P^o(\omega)} \in j\mathbb{R}$$

$$\Rightarrow p(j\omega) = P^e(\omega) + j\omega P^o(\omega)$$

$$\boxed{p(j\omega) = p^e(\omega) + j\omega\, p^o(\omega)}$$

$$p^e(\omega) \xrightarrow{\text{roots}} 0 < \omega_{e,1} < \omega_{e,2} < \omega_{e,3} < \cdots$$

$$p^o(\omega) \xrightarrow{\text{roots}} 0 < \omega_{o,1} < \omega_{o,2} < \omega_{o,3} < \cdots$$

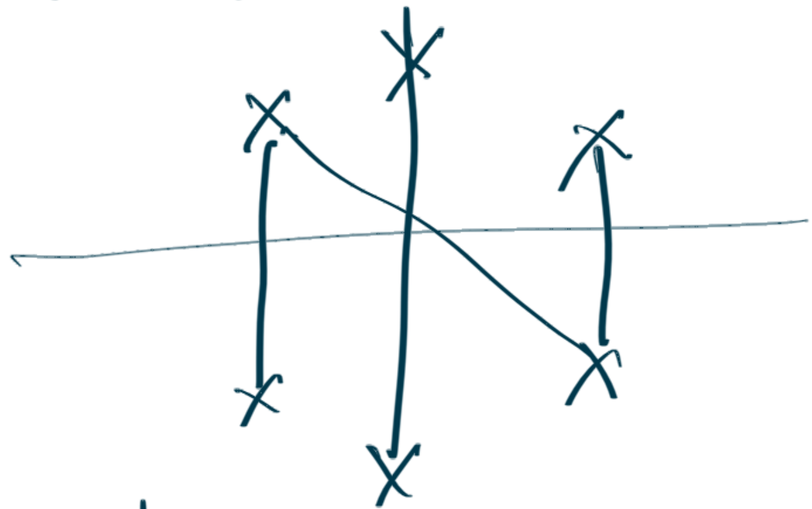
$$0 < \omega_{e,1} < \omega_{o,1} < \omega_{e,2} < \omega_{o,2} < \omega_{e,3} < \omega_{o,3} < \cdots < \omega_{o,\frac{n-2}{2}} < \omega_{e,\frac{n}{2}} < \infty$$

Interlacing property.

Hermite-Bieler Theorem:

p is Hurwitz $\Leftrightarrow$ Interlacing property is satisfied.

$$s^4 + s^2 + 1 + 4s^3 + 2s$$



$p(j\omega)$