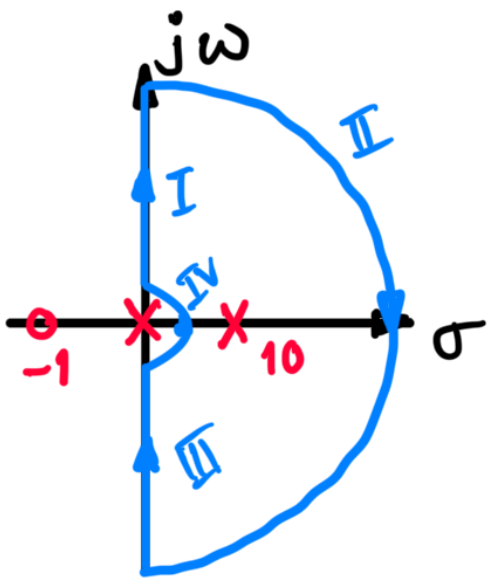
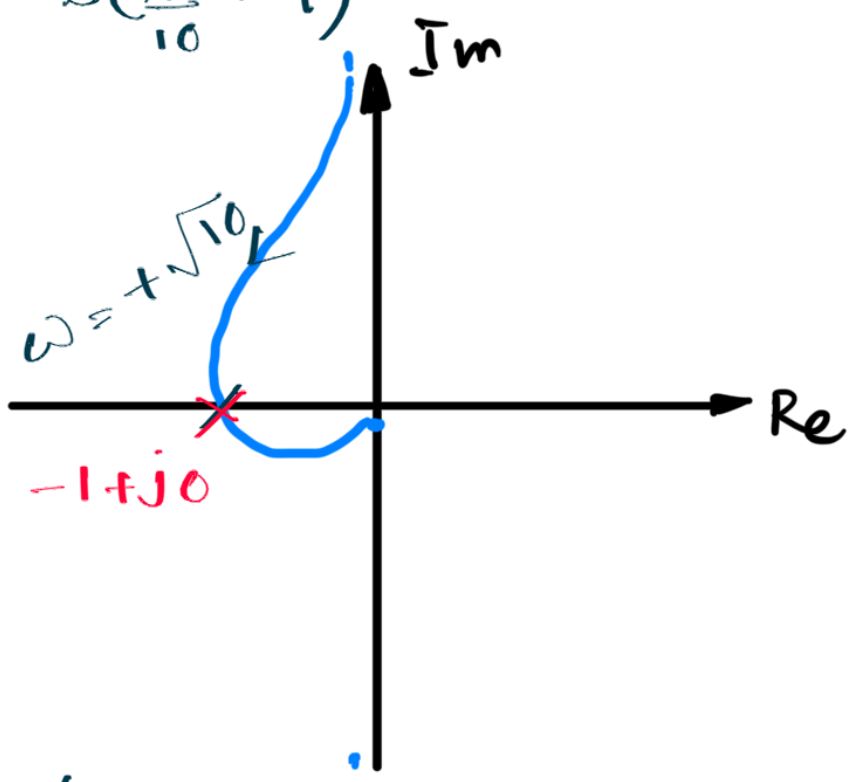
10 March:

Nyquist plot of $G(s) = \dfrac{K(s+1)}{s\left(\frac{s}{10} - 1\right)}$





"$N = P - Z$"

nyquist(G)

$$G(j\omega) = \frac{j\omega + 1}{j\omega\left(\frac{j\omega}{10} - 1\right)}$$

$$= \frac{1 + j\omega}{\left(\frac{-\omega^2}{10} - j\omega\right)}$$

$$= \frac{(1+j\omega)\left(\frac{-\omega^2}{10} + j\omega\right)}{\left(\frac{-\omega^2}{10} - j\omega\right)\left(\frac{-\omega^2}{10} + j\omega\right)}$$

$$= \frac{-\frac{\omega^2}{10} + j\omega - \frac{j\omega^3}{10} - \omega^2}{\left[\frac{\omega^4}{100} + \omega^2\right]}$$

$$\omega - \frac{\omega^3}{10} = 0$$

$\Rightarrow \omega \neq 0$, or $1 - \frac{\omega^2}{10} = 0$ → Not possible

$\Rightarrow \omega^2 = 10$

$\Rightarrow \omega = \pm\sqrt{10}$

$$G(j\sqrt{10}) = \frac{1 + j\sqrt{10}}{-1 - j\sqrt{10}} = -1$$

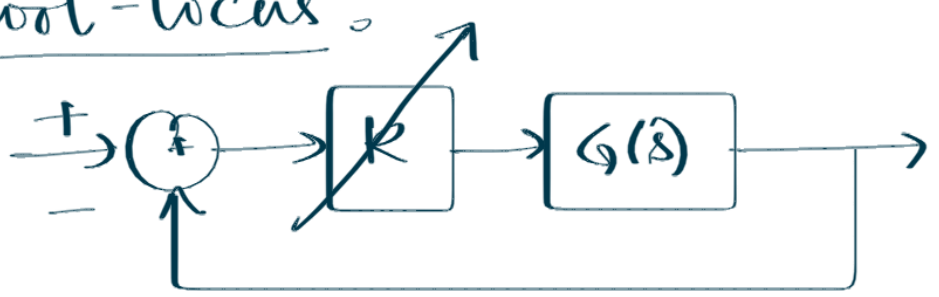
CLTF:

$$\frac{\frac{(s+1)}{s\left(\frac{s}{10} - 1\right)}}{1 + \frac{s+1}{s\left(\frac{s}{10} - 1\right)}}$$

$$= \frac{s+1}{\frac{s^2}{10} - s + s + 1}$$

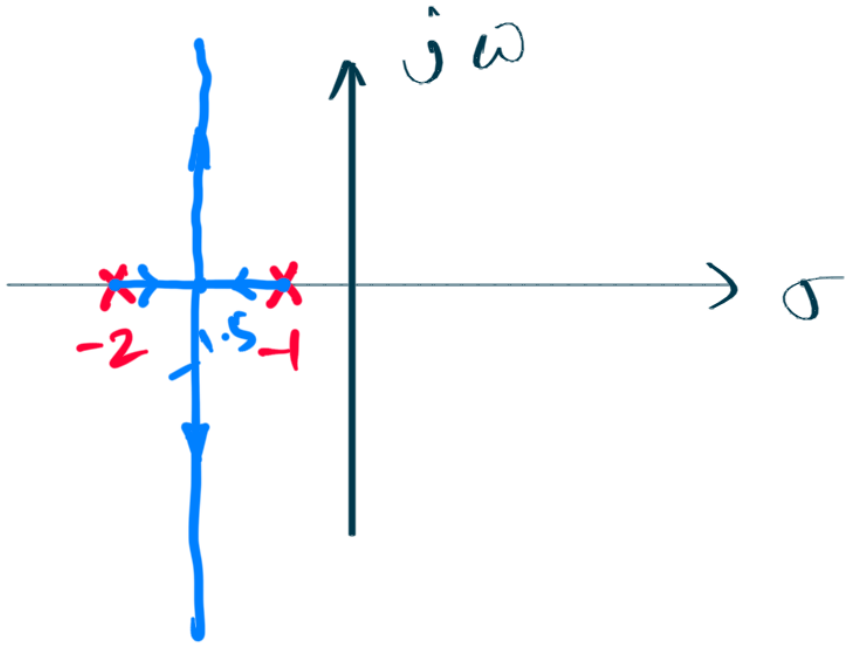
$$= \frac{s+1}{\frac{s^2}{10} + 1}$$

Root-locus:



closed loop pole = zeros of $(1 + kG(s))$

$$G(s) = \frac{1}{(s+1)(s+2)}$$

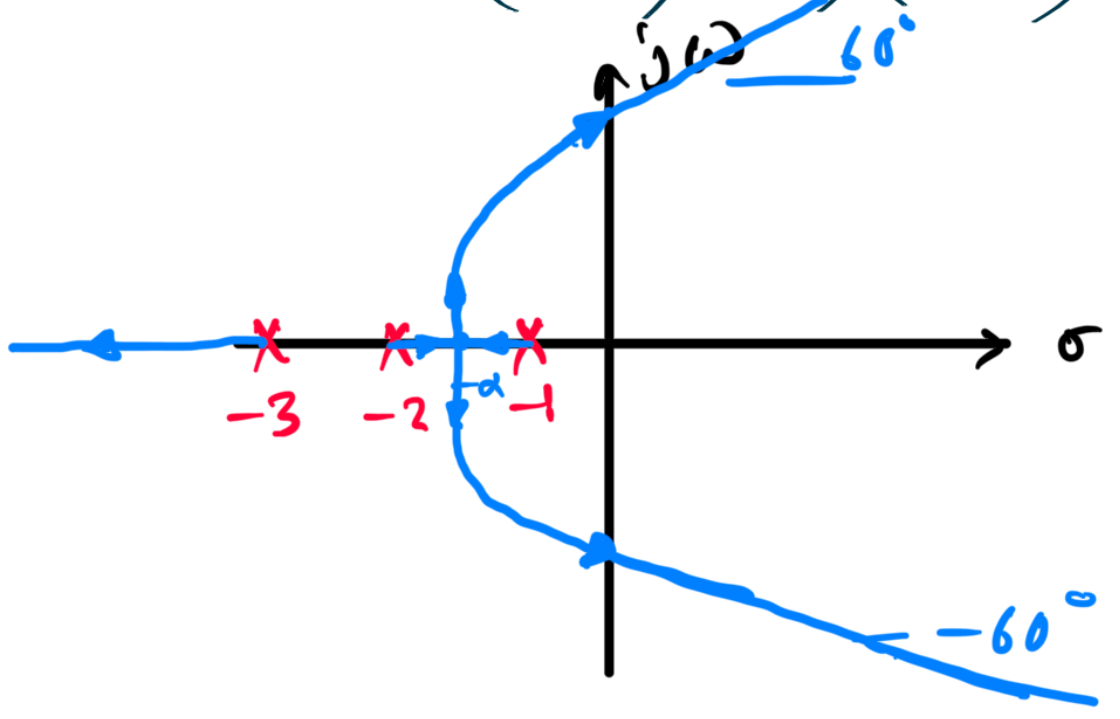


Cl ch-poly

$= s^2 + 3s + 2 + k$

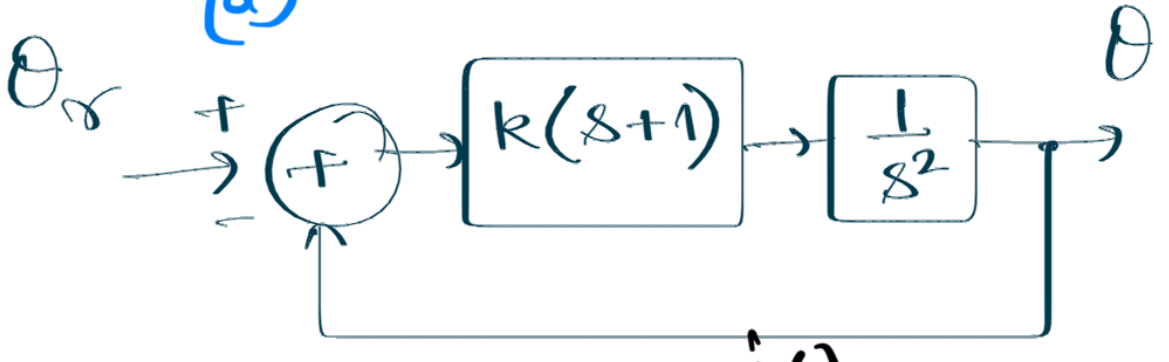
— Ex 2:

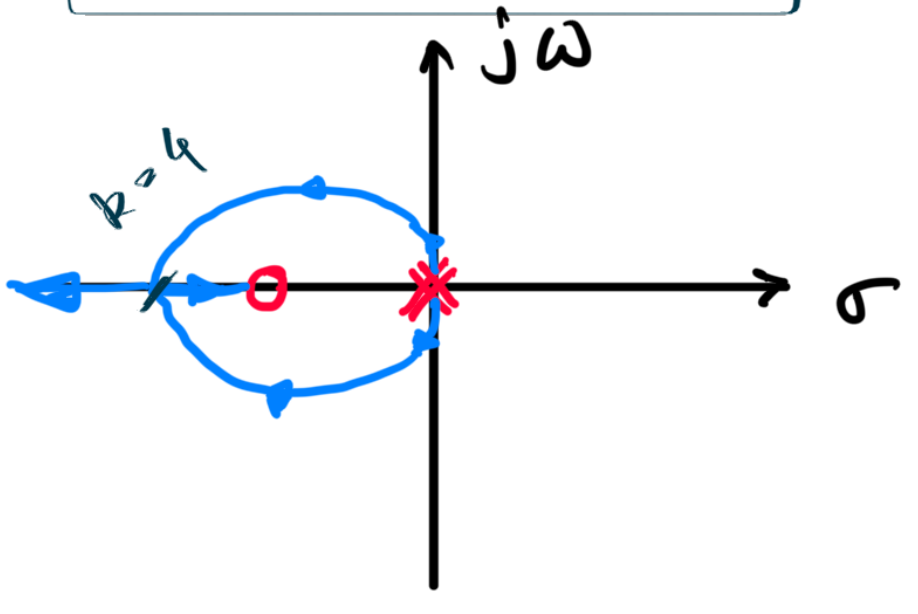
$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}$$



Ex 3:

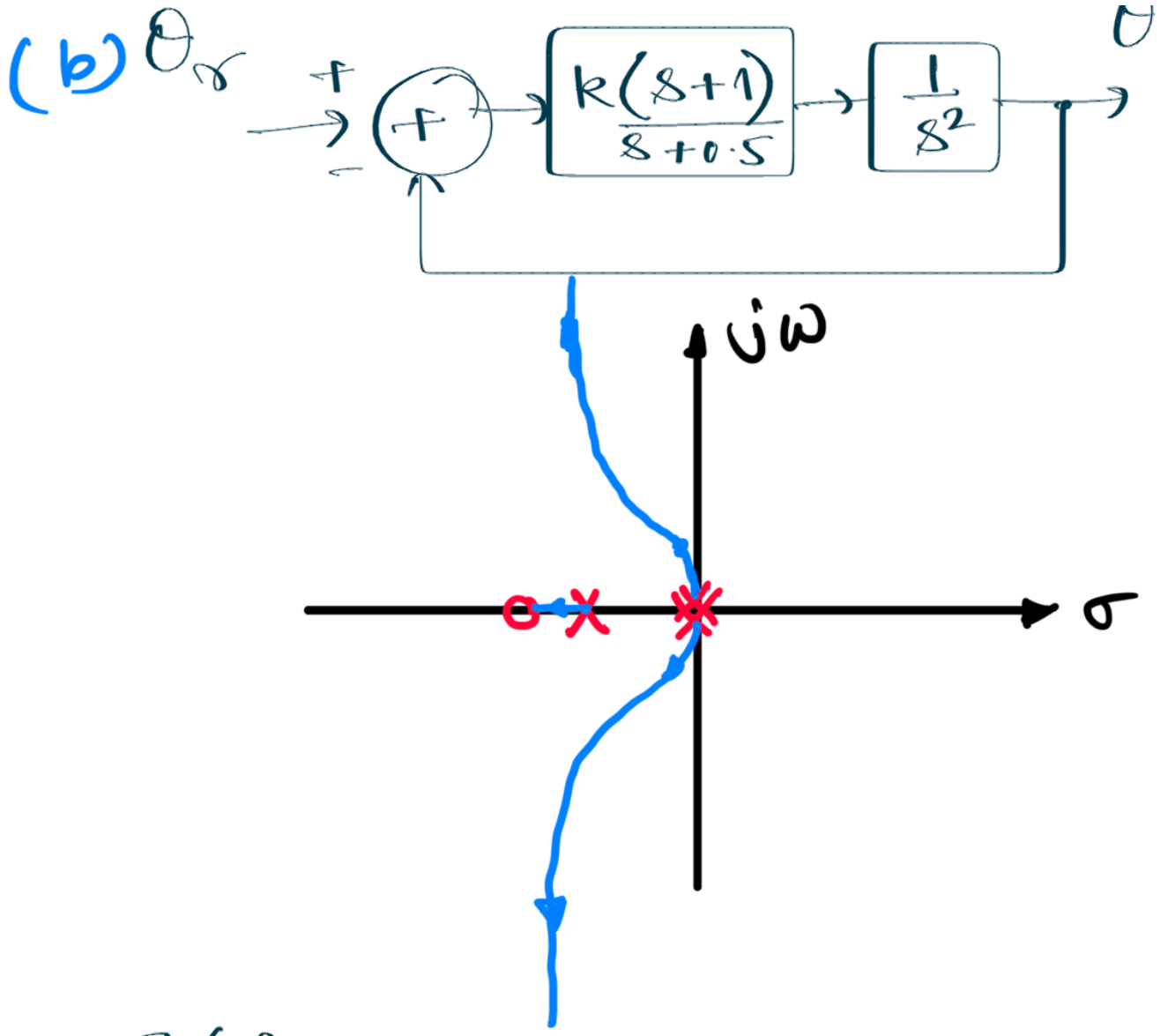
(a)





(b)

(b)

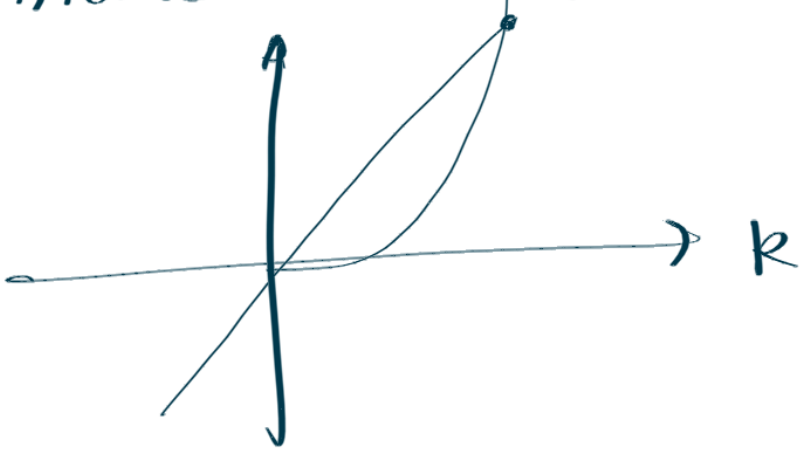


3 (a)

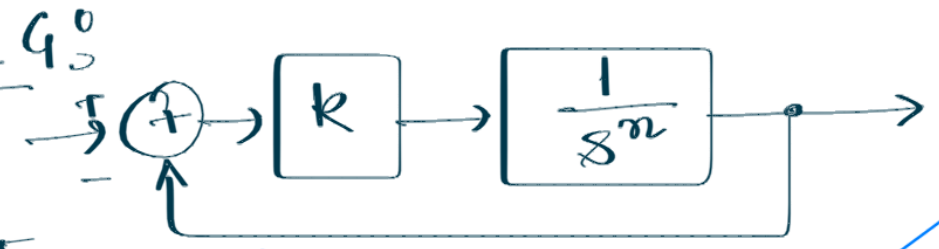
$$CLTF = \frac{\frac{k(s+1)}{s^2}}{1 + \frac{k(s+1)}{s^2}}$$

$$= \frac{k(s+1)}{(s^2 + ks + k)}$$

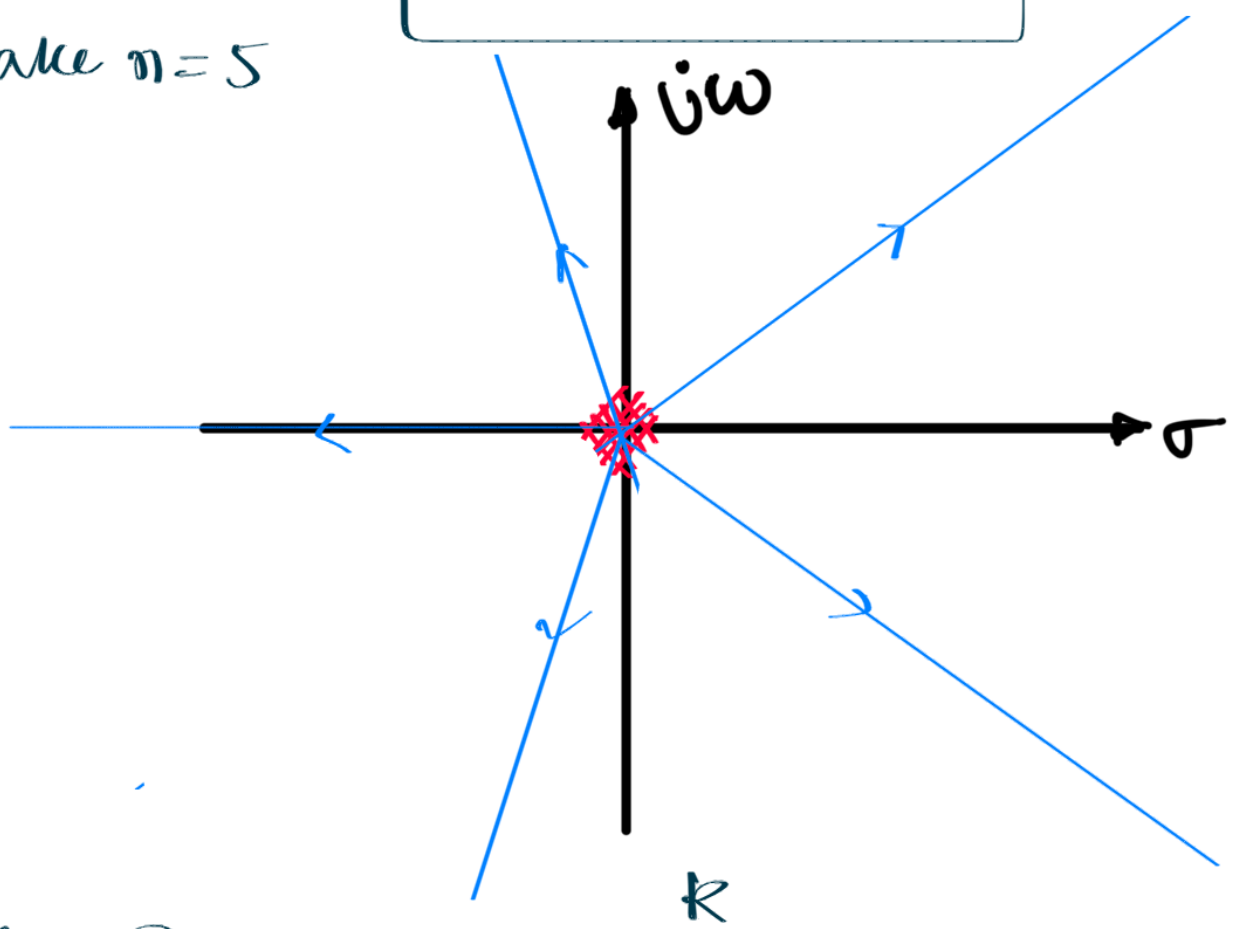
discriminant $k^2 - 4k$



Example 6:



Take $n = 5$



$$CLTF = \frac{\frac{K}{s^5}}{1 + \frac{K}{s^5}}$$

$$= \frac{K}{(s^5 + K)}$$

To find roots of $s^5 + k$

$$s = \lambda e^{j\phi}$$

$$\lambda^5 e^{j5\phi} = -K$$

$$\lambda^5 = K \Rightarrow \lambda = \sqrt[5]{K}.$$

$$5\phi = (2m+1)180^\circ \quad m = 0, \pm 1, \pm 2,$$

$$\Rightarrow 5\phi = 180^\circ$$

$$\Rightarrow \phi_1 = 36^\circ$$

$$\phi_2 = \frac{1}{5}(3 \times 180^\circ)$$

$$= 108^\circ$$

$$\phi_3 = \frac{1}{\cancel{5}}(\cancel{5} \times 180^\circ)$$

$$= 180^\circ$$

$$\phi_4 = \frac{1}{5}(7 \times 180^\circ)$$

$$= 252^\circ$$

$$\phi_5 = \frac{1}{5}(9 \times 180^\circ)$$

$$= 324^\circ$$

Example 2