$p(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0$

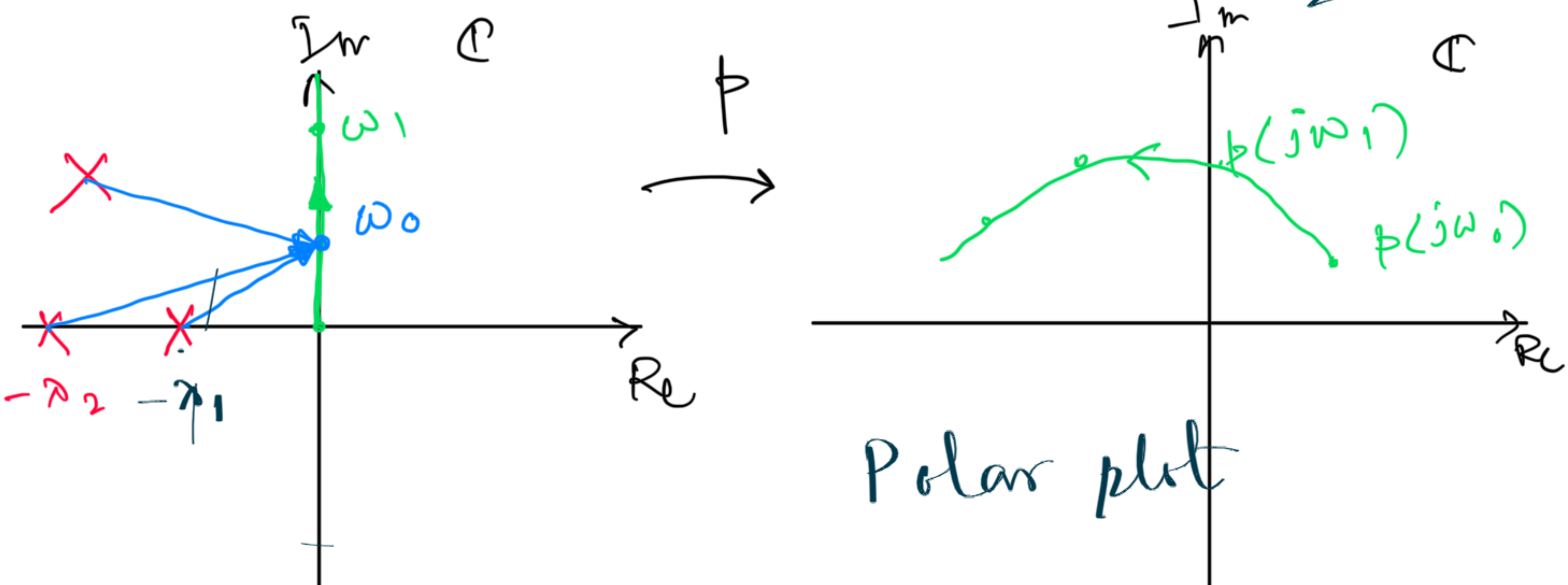
p is Hurwitz (i.e. $\text{roots}(p) \subseteq \mathbb{C}^-$)

$\Rightarrow a_0, a_1, \ldots, a_n$ are of the same sign.

WLOG $a_0 > 0, a_1 > 0, \ldots, a_n > 0$.

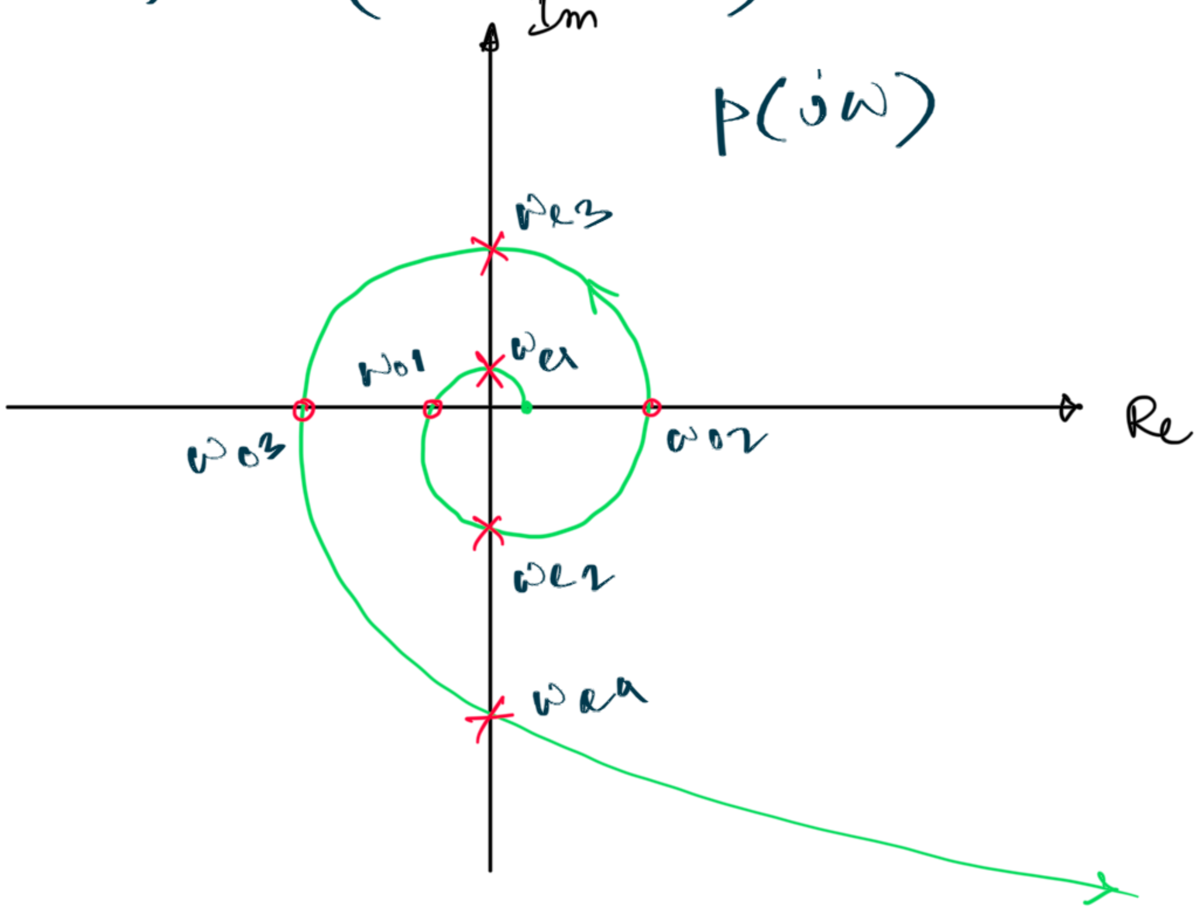
$p(s)$ Hurwitz $\iff$ $p(j\omega)$ vary ω from 0 to $+\infty$.

then $\angle p(j\omega)$ is monotonically increasing from 0 to $\frac{n\pi}{2}$.



$p(s) = (s+\lambda_1)(s+\lambda_2)$

$p(j\omega_0) = (j\omega_0+\lambda_1)(j\omega_0+\lambda_2)$



$$p(j\omega) = a_8(j\omega)^8 + a_7(j\omega)^7 + \cdots + a_1(j\omega) + a_0$$

$$= a_8\omega^8 - a_6\omega^6 + a_4\omega^4 - a_2\omega^2 + a_0$$
$$- j a_7\omega^7 + j a_5\omega^5 - j a_3\omega^3 + j a_1\omega$$

$$= \underline{p^e(\omega)} + \underline{j\omega}\, \underline{p^o(\omega)}.$$

$$p^e(\omega) = a_8\omega^8 - a_6\omega^6 + a_4\omega^4 - a_2\omega^2 + a_0$$

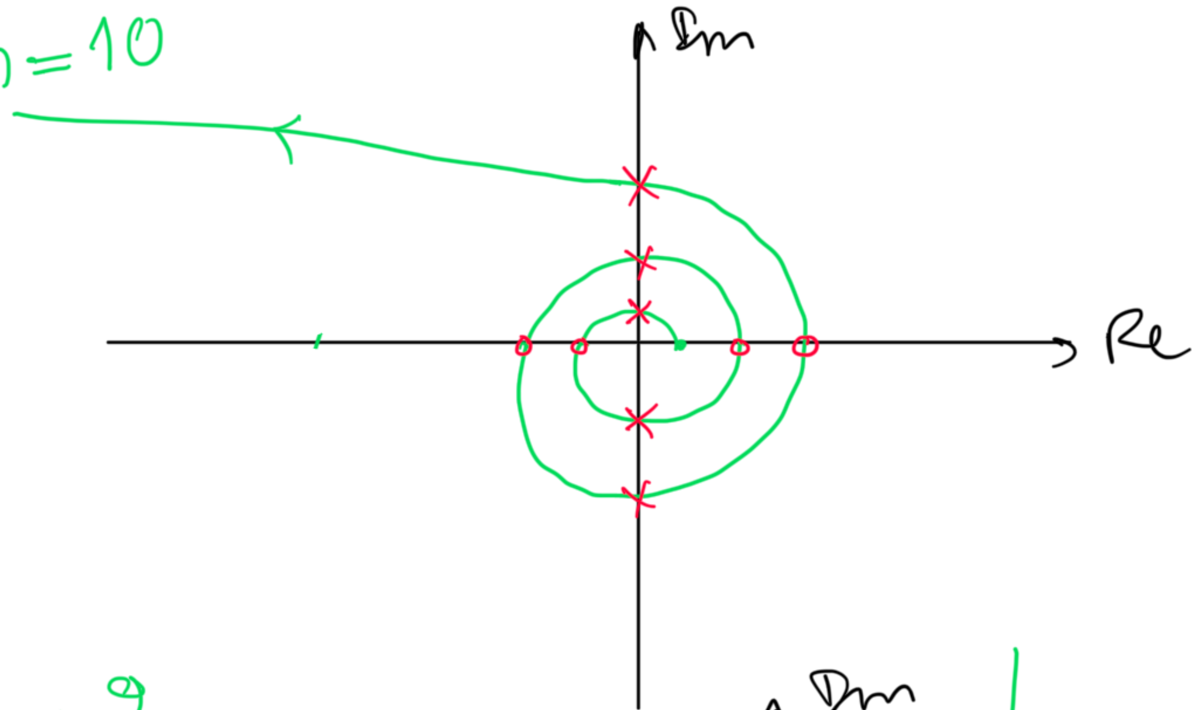
$$p^o(\omega) = -a_7\omega^6 + a_5\omega^4 - a_3\omega^2 + a_1$$

$$0 < \omega_{e1} < \omega_{o1} < \omega_{e2} < \omega_{o2} < \omega_{e3} < \omega_{o3} < \omega_{e4} < \infty$$

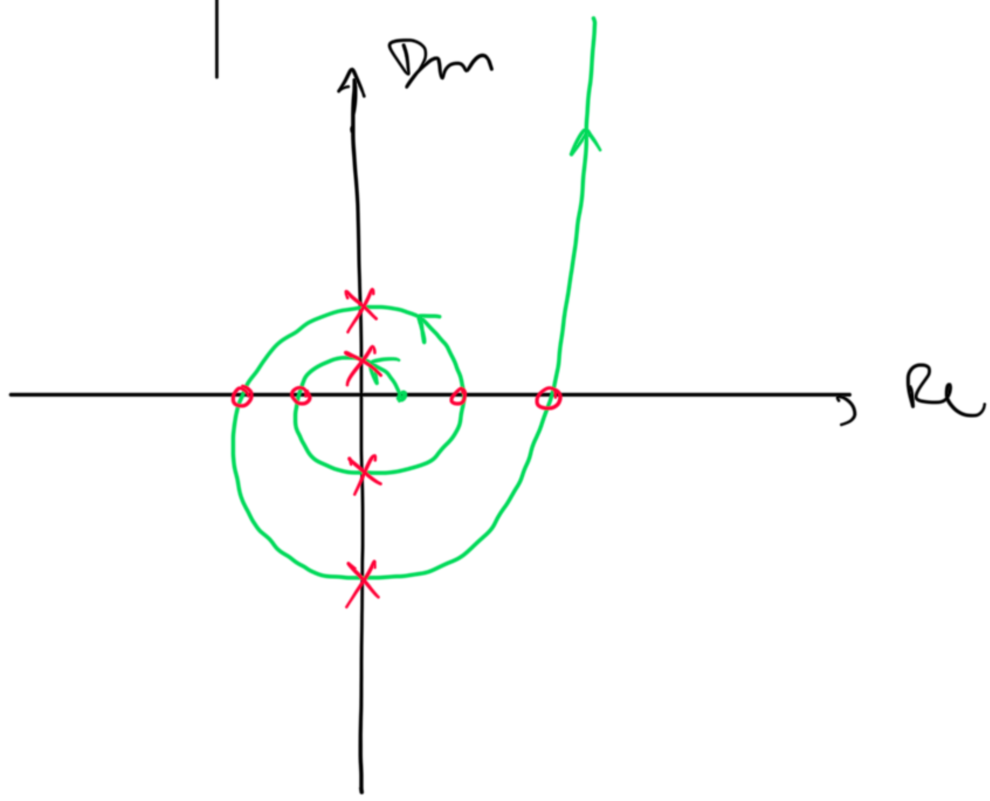
root of $p^e(\omega)$

root of $p^o(\omega)$

$n = 10$



$n = 9$



$$0 < \omega_{e1} < \omega_{o1} < \omega_{e2} < \omega_{o2} < \omega_{e3} < \omega_{o3}$$
$$< \omega_{e4} < \omega_{o4} < \infty$$

<u>Interlacing property:</u>

$p(s)$ is said to have the interlacing property if

1. $a_n, a_{n-1} > 0$.
2. $0 < \omega_{e1} < \omega_{o1} < \dots < \omega_{e\frac{n}{2}} < \infty$

For n even

or, $0 < \omega_{e1} < \omega_{o1} < \dots < \omega_{e\frac{n-1}{2}} < \omega_{o,\frac{n-1}{2}} < \infty$

For n odd.

<u>Theorem:</u> $a_n, \dots, a_0 > 0$.

$p(s)$ is Hurwitz $\Leftrightarrow$ p has interlacing property.

$\boxed{4}$ $p(s) = s^4 + s^2 + 1 \checkmark \rightarrow \underline{p(j\omega)}$

$\frac{dp}{ds} = 4s^3 + 2s$

$$\left.\begin{array}{llll} s^4: & 1 & 1 & 1 \\ s^3: & 4 & 2 & \\ s^2: & \frac{1}{2} & 1 & \\ s^1: & -6 & & \\ s^0: & 1 & & \end{array}\right\}$$

$s^4 + 4s^3 + s^2 + 2s + 1$

$\tilde{p}(s) =:$

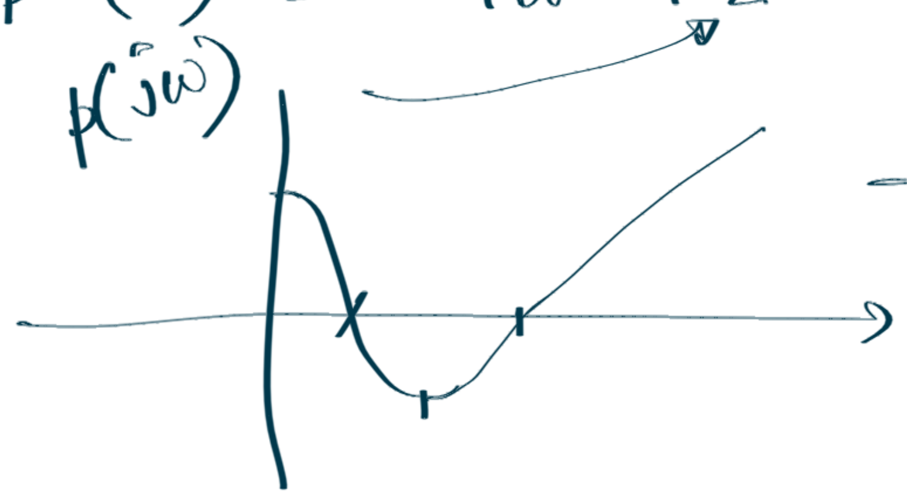
$$\tilde{p}(j\omega) = (j\omega)^4 + 4(j\omega)^3 + (j\omega)^2 + 2j\omega + 1$$

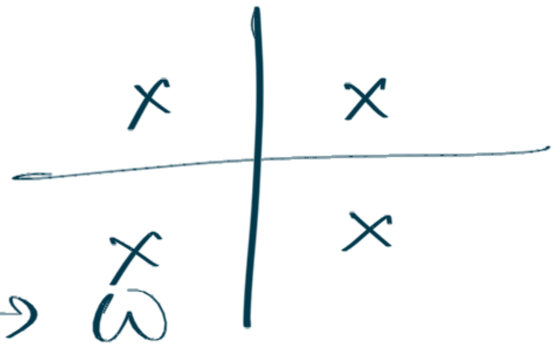
$$= \omega^4 - \omega^2 + 1 - j4\omega^3 + 2j\omega$$

$$= \omega^4 - \omega^2 + 1 - j\omega(4\omega^2 - 2)$$

$$\tilde{p}^e(\omega) = \omega^4 - \omega^2 + 1 = p(j\omega)$$

$$\tilde{p}^o(\omega) = -4\omega^2 + 2$$





$p(s)$ even polynomial

$p(s)$ has all roots on $j\mathbb{R}$

$$\iff \quad p(s) + \frac{dp(s)}{ds} \text{ is Hurwitz.}$$