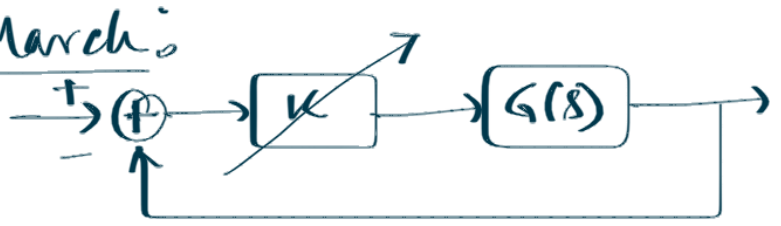
[5] 12 March:



$$\text{closed loop tf} = \frac{kG(s)}{1+kG(s)}$$

$$\text{CL poles} = \text{zeros}\,(1+kG(s))$$

$$\text{Suppose } G(s) = \frac{p(s)}{q(s)}$$

$$\text{CLTF denominator} = \text{CL ch. poly} = \chi_k(s)$$
$$= q(s) + k\,p(s)$$

@ $k=0$ $\quad \chi_k(s) = q(s)$

@ $k=\infty$ $\quad \chi_k(s) = k\,p(s)$

Rule #1: R-L plot starts from OL poles and ends at OL zeros.

Def: $n :=$ no. of OL poles
$m :=$ no. of OL zeros

Typically $n \geq m$, when $n > m$,

$n-m$ branches of the RL go off to infinity.

— Angle criterion:

CL poles = zeros of $1 + kG(s)$

$\lambda \in \mathbb{C}$ is a zero of $1+kG(s)$ for some $k \geq 0$
[i.e., λ is a pt. on the R-L plot]

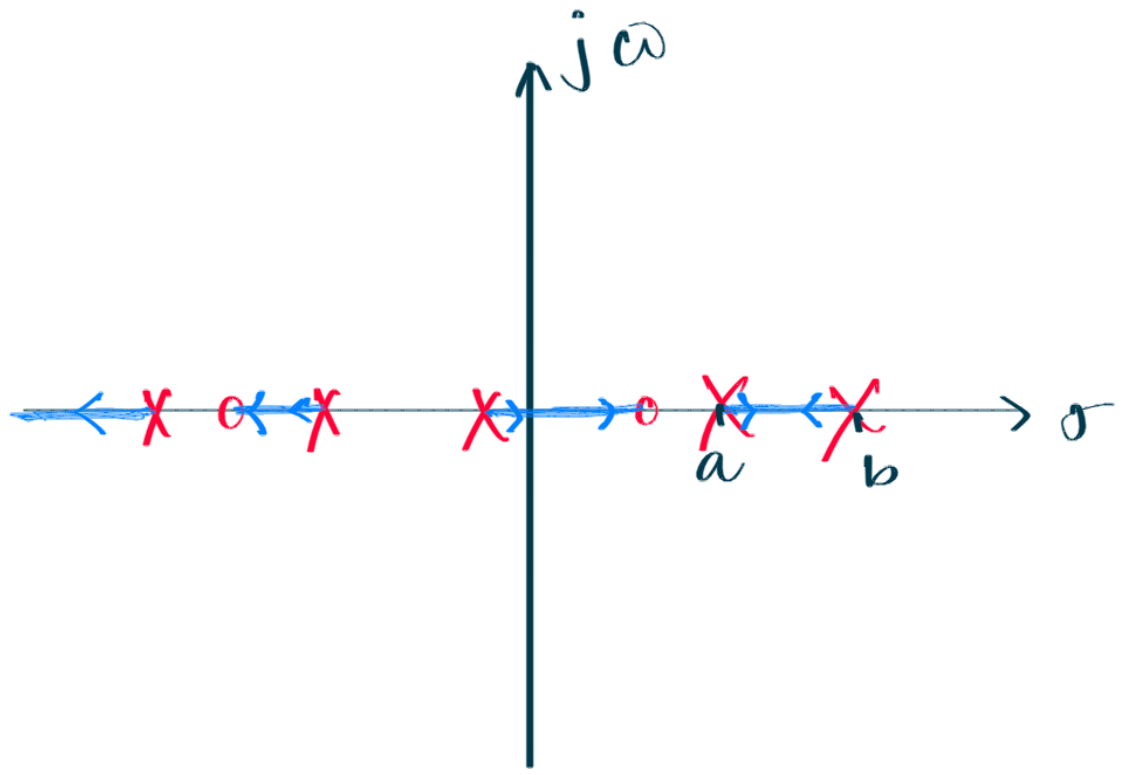
$$\iff \quad 1 + kG(\lambda) = 0$$

$$\iff \quad G(\lambda) = -\frac{1}{k} \leq 0$$

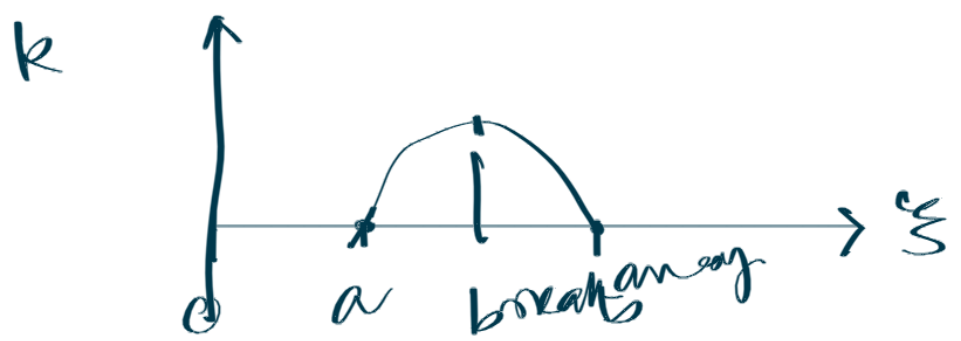
$$\Leftrightarrow \quad \angle G(\lambda) = (2\ell+1)\,180^\circ \cdot \ell = 0, \pm 1, \pm 2, \pm 3, \ldots$$

$$\text{and} \quad k = \frac{1}{|G(\lambda)|}$$

Rule #2: (Real axis segments)



Rule #3: (Break-in / break-away)

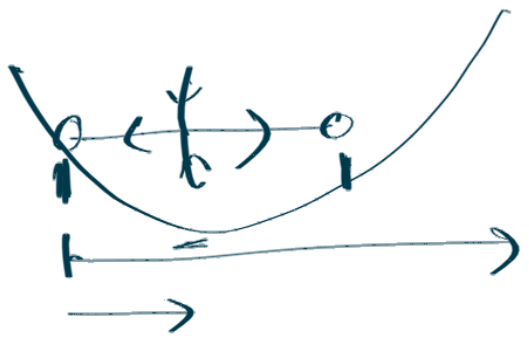


$$\underline{\underline{k}} = -\frac{1}{G(\xi)}$$

$$\frac{dk}{d\xi} = 0 = + \frac{\frac{d}{d\xi} G(\xi)}{G(\xi)^2}$$

$$\Leftrightarrow \quad \frac{d}{d\xi} G(\xi) = 0$$

These are candidates for breakaway/ break-in.



Rule #4: (Asymptotes)

$$\text{Centroid} = \frac{\sum p_i - \sum z_i}{n - m}$$

$$\text{Angles} \quad \varphi = \frac{1}{(n-m)} (2l+1)\, 180^\circ \qquad l = 0, \pm 1, \pm 2, \ldots$$