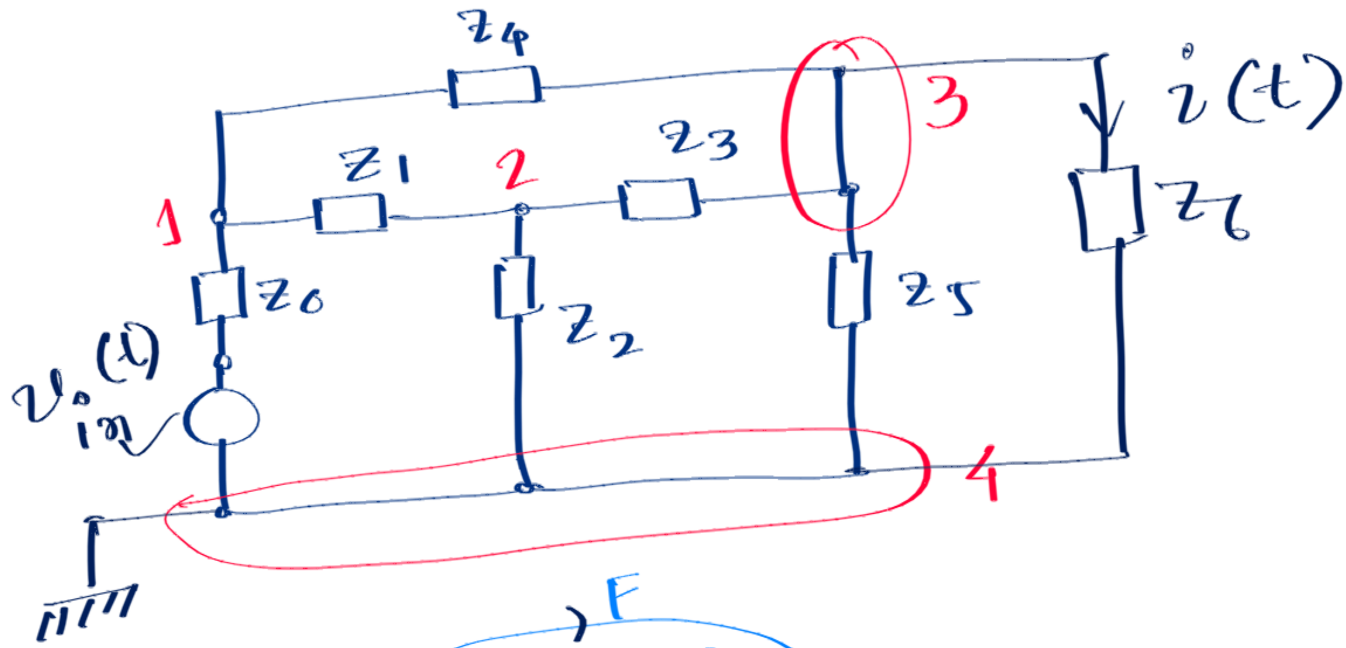
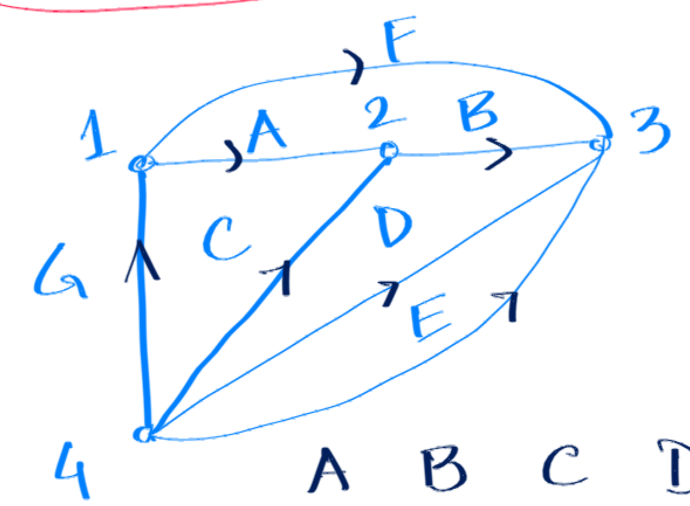




$$\mathcal{E} = \begin{array}{c|ccccccc} & A & B & C & D & E & F & G \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 2 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 3 & 0 & -1 & 0 & -1 & -1 & -1 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array} \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \\ i_E \\ i_F \\ i_G \end{bmatrix}$$

$i_b \rightarrow$ Branch current vector

$$i_b = \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \\ i_E \\ i_F \\ i_G \end{bmatrix}$$

$v_b \rightarrow$ Branch voltage vector

$$v_b = \begin{bmatrix} v_A \\ v_B \\ v_C \\ v_D \\ v_E \\ v_F \\ v_G \end{bmatrix}$$

$$\boxed{\mathcal{E}\, i_b = 0} \quad \text{KCL}$$

$\bar{\lambda}_n \rightarrow$ Node potential vector.

$$\bar{\lambda}_n = \begin{bmatrix} \bar{\lambda}_1 \\ \bar{\lambda}_2 \\ \bar{\lambda}_3 \\ \bar{\lambda}_4 \end{bmatrix}$$

$$v_b = \mathcal{E}^T \bar{\lambda}_n$$

$$\mathcal{E}^T \bar{\lambda}_n = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{\lambda}_1 \\ \bar{\lambda}_2 \\ \bar{\lambda}_3 \\ \bar{\lambda}_4 \end{bmatrix} = \begin{bmatrix} v_A \\ v_B \\ v_C \\ v_D \\ v_E \\ v_F \\ v_G \end{bmatrix}$$

$$\boxed{\mathcal{E}^T \bar{\lambda}_n = v_b} \quad \underline{\checkmark} \quad \text{KVL}$$

$$v_G + v_A - v_C = (-\cancel{\bar{\lambda}_1} + \cancel{\bar{\lambda}_4}) + (\cancel{\bar{\lambda}_1} - \cancel{\bar{\lambda}_2}) - (-\cancel{\bar{\lambda}_2} + \cancel{\bar{\lambda}_4}) = 0$$

In Laplace domain

$$V_A(s) = \underline{Z_1}(s)\, I_A(s)$$

$$V_B(s) = \underline{Z_3}(s)\, I_B(s)$$

$$V_C(s) = Z_2(s)\, I_C(s)$$

$$V_D(s) = Z_5(s)\, I_D(s)$$

$$V_E(s) = Z_6(s)\, I_E(s)$$

$$V_F(s) = Z_4(s)\, I_F(s)$$

$$V_G(s) = Z_0(s)\, I_G(s) - V_{in}(s)$$

$$\boxed{\underline{V}_b(s) = Z(s)\, I_b(s) - V^{\text{source}}(s)}$$

$\uparrow$ device impedance matrix

— Device properties

$$I_b(s) = Z^{-1}(s)\left[V_b(s) + V^{\text{source}}(s)\right]$$

$$= Z^{-1}(s)\, \mathcal{E}^T \Pi_n(s) + Z^{-1}(s) V^{\text{source}}(s)$$

$$\Rightarrow \mathcal{E} I_b(s) = \mathcal{E} Z^{-1}(s) \mathcal{E}^T \underline{\Pi_n}(s) + \mathcal{E} Z^{-1}(s) V^{\text{source}}(s)$$

$= 0$

$$\left[\mathcal{E} Z^{-1}(s)\, \mathcal{E}^T\right] \Pi_n(s) = -\mathcal{E} Z^{-1}(s) V^{\text{source}}(s)$$

Circuit eq$^{\underline{n}}$

π_n is independent but not uniquely associated with v_b.

Indeed, if π_n^* corresponds to v_b^* then $\underline{\pi_n^* + \mathbb{1}\alpha}$ $\quad \mathbb{1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$

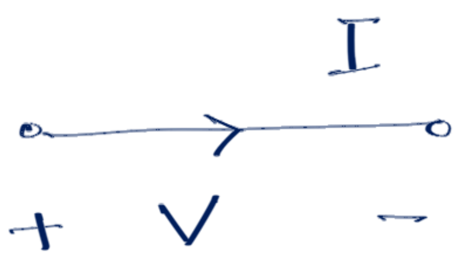
also corresponds to v_b^* $\quad \alpha \in \mathbb{R}$

To get rid of this non uniqueness, we take any $\pi_i = 0$. In this case, let's take $\bar{n}_4 = 0$,

$\equiv 4^{\text{th}}$ node grounded.

Here,

$$V^{\text{Source}}(s) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_{in}(s) \end{bmatrix}$$



$$\tilde{\mathcal{E}} \in \mathbb{R}^{3 \times 7}$$

$\parallel$

$\underline{\mathcal{E}} \setminus 4^{\text{th}}$ row.

$$\tilde{\mathcal{E}}\, Z^{-1}(s)\, \tilde{\mathcal{E}}^{T} \begin{bmatrix} \pi_1(s) \\ \pi_2(s) \\ \pi_3(s) \end{bmatrix} = -\tilde{\mathcal{E}}\, Z^{-1}(s)\, V^{\text{Source}}(s)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} Y_1(s) & & & & & & \\ & Y_3(s) & & & & & \\ & & Y_2(s) & & & & \\ & & & Y_5(s) & & & \\ & & & & Y_6(s) & & \\ & & & & & Y_4(s) & \\ & & & & & & Y_8(s) \end{bmatrix}$$

$z_i^{-1}(s) =: Y_i(s)$ (define)

$$\tilde{\mathcal{E}}^T \begin{bmatrix} \Pi_1(s) \\ \Pi_2(s) \\ \Pi_3(s) \end{bmatrix} = -\tilde{\mathcal{E}}\, \underline{Z^{-1}}(s) \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ V_{in}(s) \end{bmatrix}$$

$$= -\tilde{\mathcal{E}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \underline{Y_0(s) V_{in}(s)} \end{bmatrix} = \begin{bmatrix} Y_0(s) V_{in}(s) \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} Y_1 & & & & & & \\ & Y_3 & & & & 0 & \\ & & Y_2 & & & & \\ & & & Y_5 & & & \\ & & & & Y_6 & & \\ & 0 & & & & Y_4 & \\ & & & & & & Y_0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \\ 1 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} Y_1 & -Y_1 & 0 \\ 0 & Y_3 & -Y_3 \\ 0 & -Y_2 & 0 \\ 0 & 0 & -Y_5 \\ 0 & 0 & -Y_6 \\ Y_4 & 0 & -Y_4 \\ -Y_0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} Y_1 + Y_4 + Y_0 & -Y_1 & -Y_4 \\ -Y_1 & Y_1 + Y_3 + Y_2 & -Y_3 \\ -Y_4 & -Y_3 & Y_3 + Y_5 + Y_6 + Y_4 \end{bmatrix}$$

$$\tilde{Y}(s)\,\tilde{\Pi}_n(s) = \begin{bmatrix} Y_0 V_{in}(s) \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \boxed{\tilde{\Pi}_n(s) = \tilde{Y}(s)^{-1} \begin{bmatrix} Y_0 V_{in}(s) \\ 0 \\ 0 \end{bmatrix}}$$