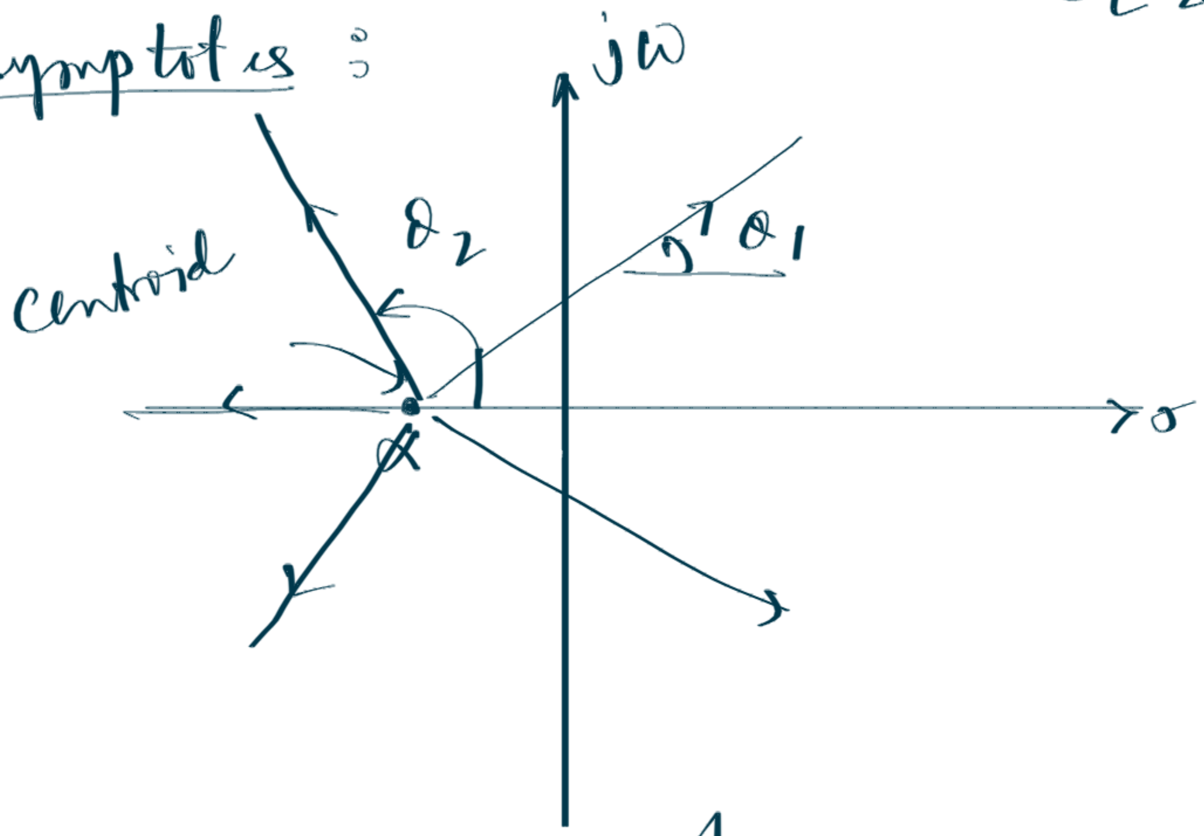
16 March: $n :=$ no. of OL poles, $m :=$ no. of OL zeros.

Asymptotes:



$$G_{asymptote}(s) = \frac{1}{(s-\alpha)^{n-m}}$$

Angle of the asymptotes,

$$(n-m)\theta = (2l+1)180^\circ. \quad l = 0, \pm 1, \pm 2, \pm 3.$$

$$\theta_1 = \frac{180^\circ}{5} = 36^\circ, \quad \theta_2 = \frac{3}{5} \times 180^\circ = 108^\circ, \ldots$$

Centroid:

$$\alpha = \frac{1}{n-m}\left[\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i\right]$$

$$1 + K\frac{p(s)}{q(s)}$$

$$= 1 + K\frac{s^m + b_{m-1}s^{m-1} + \cdots + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_0}$$

$$b_{m-1} = -\sum_{i=1}^{m} z_i$$

$$a_{n-1} = -\sum_{i=1}^{n} p_i$$

$$p(s) = (s-z_1)(s-z_2)\cdots(s-z_m)$$

$$q(s) = (s-p_1)(s-p_2)\cdots(s-p_n)$$

$1 + k\,G(s)$

$$= 1 + \frac{k\cdot(s^{m} + b_{m-1}s^{m-1} + \cdots + b_0)}{[(s^{n} + a_{n-1}s^{n-1} + \cdots + a_0)]}$$

$$= 1 + \frac{k}{(s^{n-m} + \alpha_{n-m-1}s^{n-m-1} + \cdots + \alpha_0) + \frac{r(s)}{p(s)}}$$

$\deg r < \deg p$.

as $s \to \infty$ $\quad \frac{r(s)}{p(s)} \to 0$.

$$1 + kG(s) \simeq 1 + \frac{k}{s^{n-m} + \alpha_{n-m-1}s^{n-m-1} + \cdots + \alpha_0}$$

We need to compare this with

$$1 + \frac{k}{(s-\alpha)^{n-m}}.$$

$$1 + \frac{k}{s^{n-m} - \alpha(n-m)\,s^{n-m-1} + \cdots}$$

Comparing coefficients of the largest degrees up to s^{n-m-1}, we get

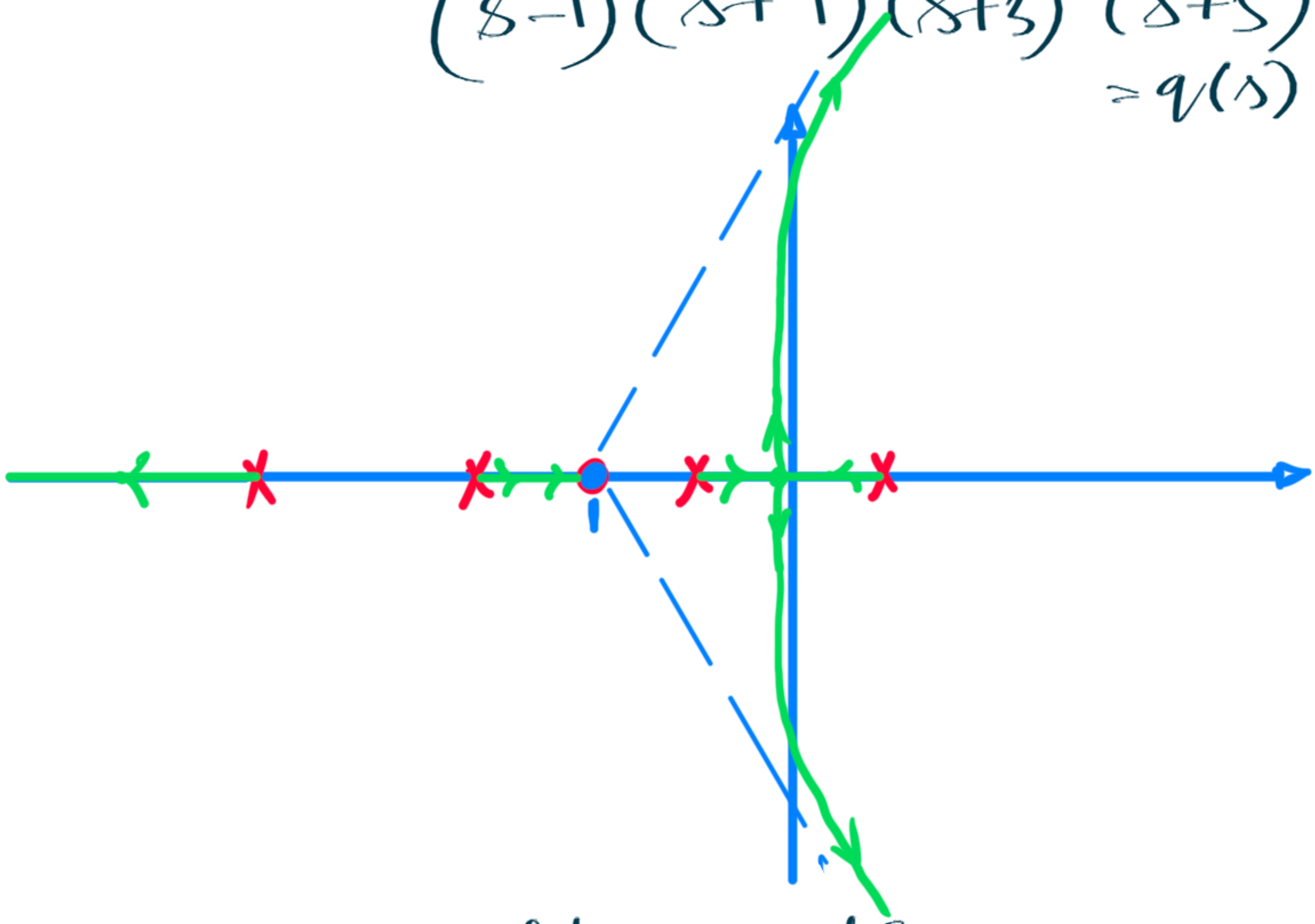
$$-\alpha(n-m) = \alpha_{n-m-1}$$
$$= a_{n-1} - b_{m-1}$$

$$= -\sum_{i=1}^{n} p_i + \sum_{i=1}^{m} z_i$$

$$\Rightarrow \alpha = \frac{1}{(n-m)} \left(\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} z_i \right)$$

Example:

$$G(s) = \frac{(s+2)}{(s-1)(s+1)(s+3)(s+5)} \quad \begin{matrix} = p(s) \\ = q(s) \end{matrix}$$



$$\frac{dG(s)}{ds} = \frac{q(s)\frac{dp}{ds} - p\frac{dq}{ds}}{q(s)^2}$$

Angles of asymptotes

$$\theta_1 = 60^\circ, \ \theta_2 = 180^\circ, \ \theta_3 = 300^\circ$$

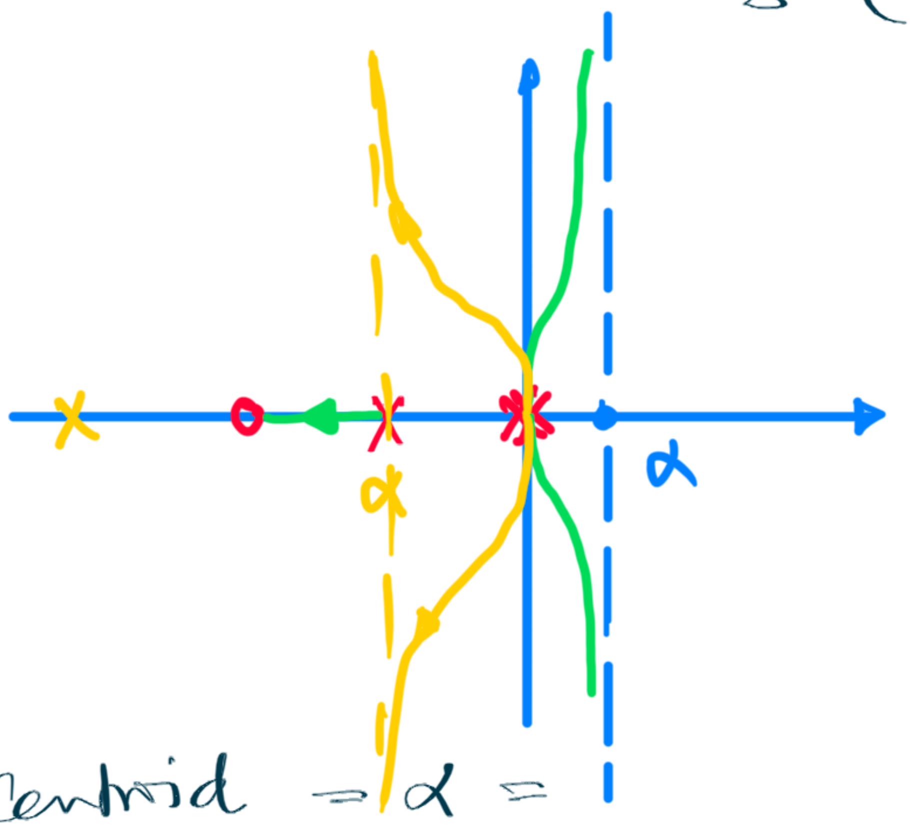
Location of the centroid

$$\alpha = \frac{1}{3}\left[-8 + 2\right]$$

$$= -2$$

Example :-

$$G(s) = \frac{(s+2)}{s^2(s+1)}$$



Centroid $= \alpha =$