17 March:

$$1 + k\,G(s) = 1 + k \cdot \frac{p(s)}{q(s)}$$

$$\deg p = m < \deg q = n$$

$k = K \in \mathbb{R}_+$ fixed

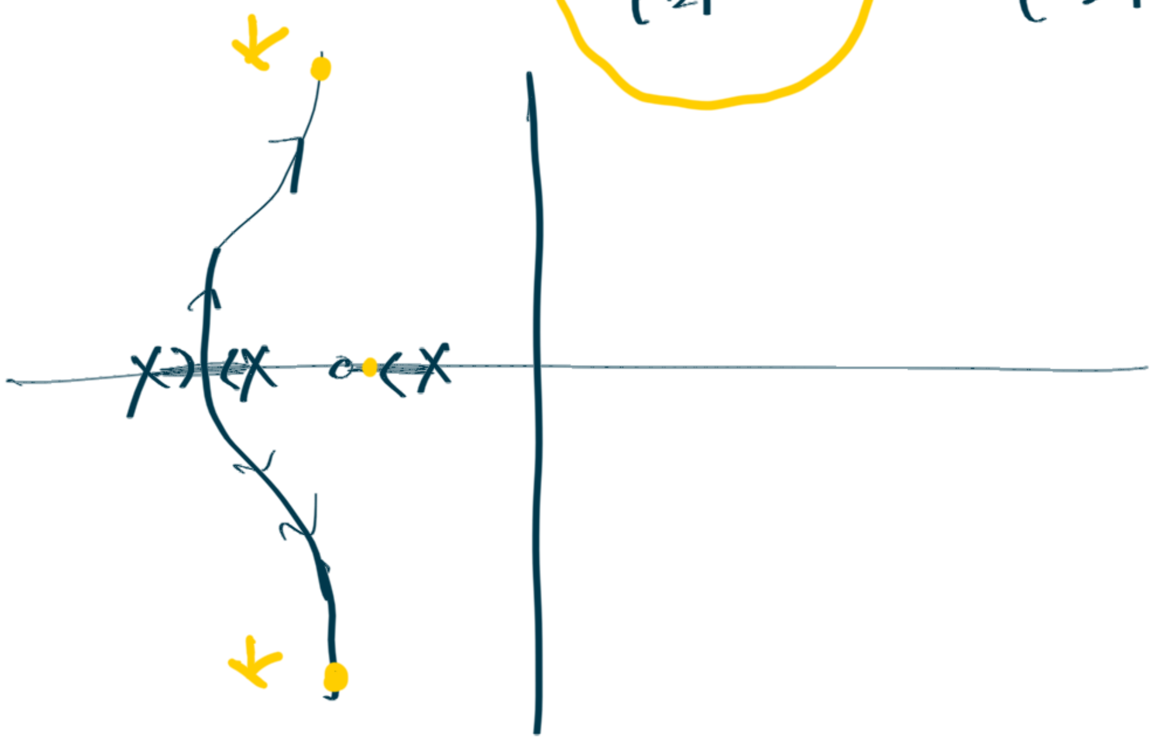
$$1 + (K + \Delta k)\frac{p(s)}{q(s)} = \frac{q(s) + K\,p(s) + \Delta k\,p(s)}{q(s)}$$

$$\text{zeros} = \text{zeros}\left(1 + \boxed{\frac{\Delta k\; p(s)}{q(s) + K\,p(s)}}\right)$$

$$q(s) + K\,p(s) = (s - \hat{p}_1)(s - \hat{p}_2)\cdots(s - \hat{p}_n)$$

→ Centroid location

$$(n-m)\alpha = \sum_{i=1}^{n} \hat{p}_i - \sum_{i=1}^{m} z_i$$



$$1 + k\frac{p(s)}{q(s)} = \left[q(s) + k\,p(s)\right]/q(s)$$

$$= \frac{\left[(s^n + a_{n-1}s^{n-1} + \cdots + a_0) + k(s^m + b_{m-1}s^{m-1} + \cdots + b_0)\right]}{q(s)}$$

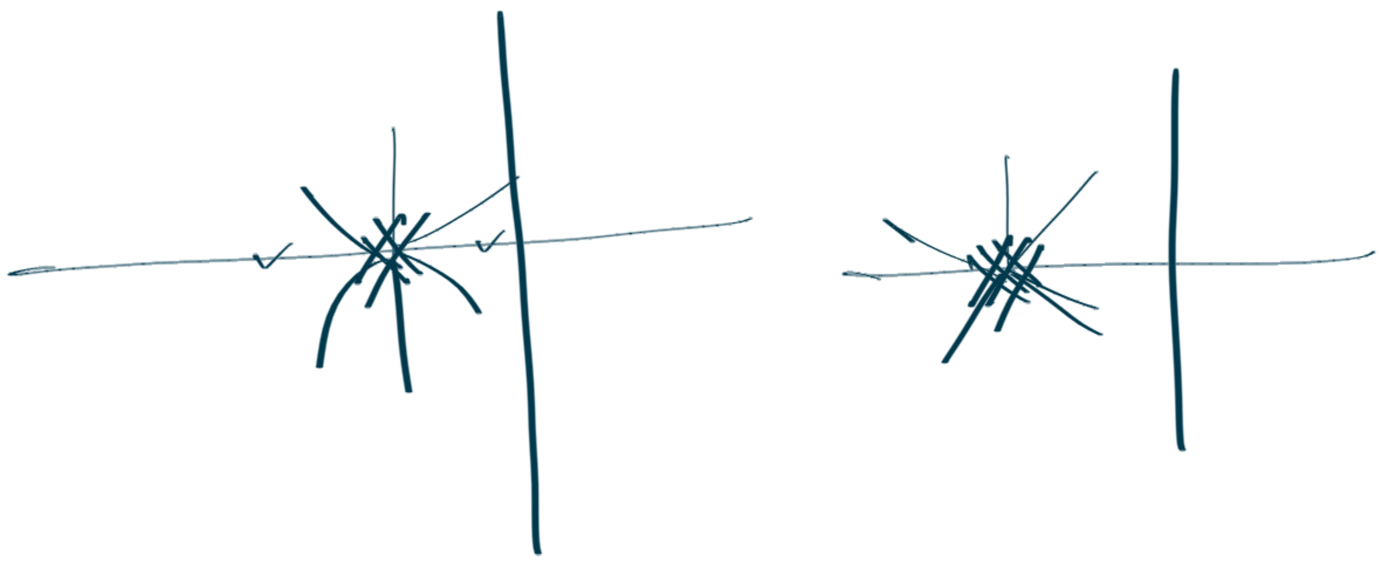
If $(n-m) \geq 2$ then

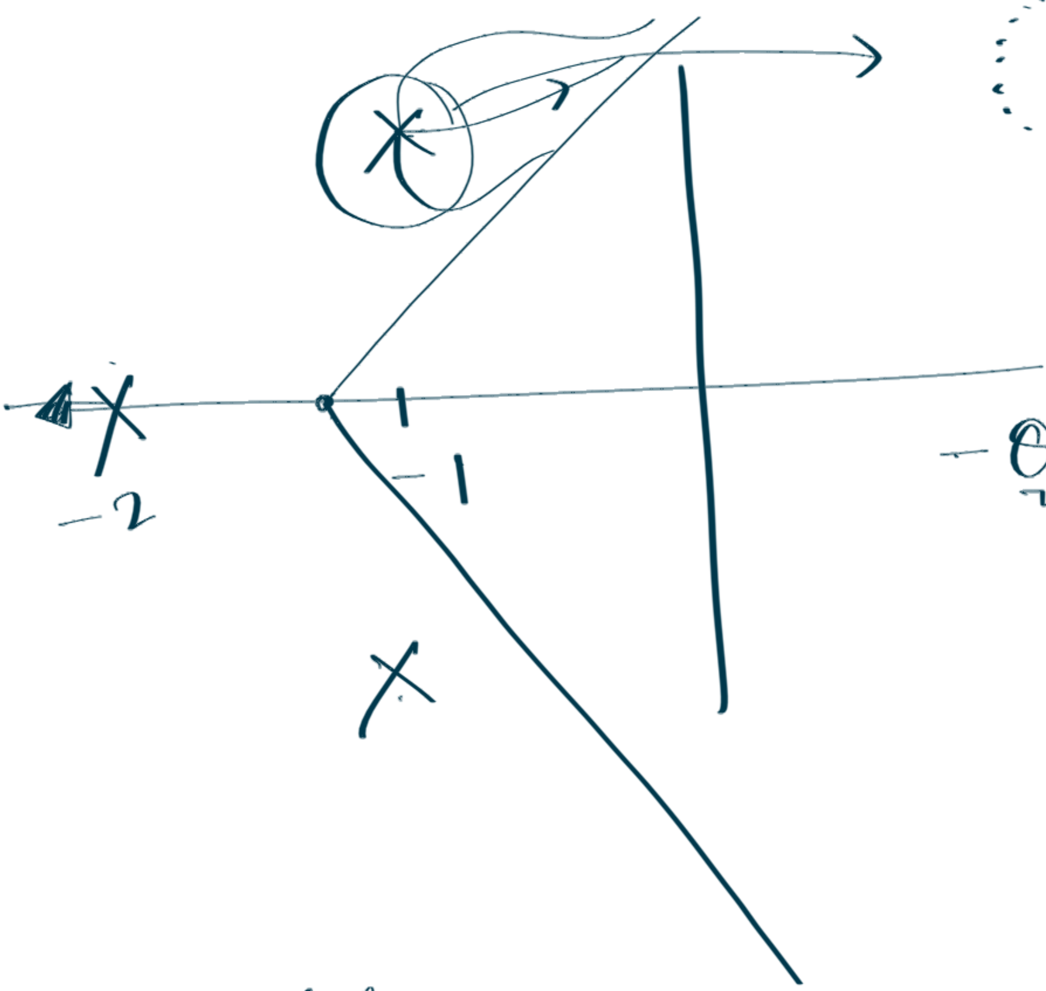
coeff. of s^{n-1} in the CL ch. poly is unchanged by k.

$\Rightarrow \sum_{i=1}^{n} \hat{p}_i$ is invariant under the variation of k.

$\Rightarrow \sum_{i=1}^{n} \hat{p}_i = \sum_{i=1}^{n} p_i$.

Rule #5: (Angles of arrival & departure)





$$-\theta_{\nu,dep} - \sum_{i \neq \nu} \angle \text{poles}_i + \sum \angle \text{zeros}_i = 180^\circ(2l+1)$$

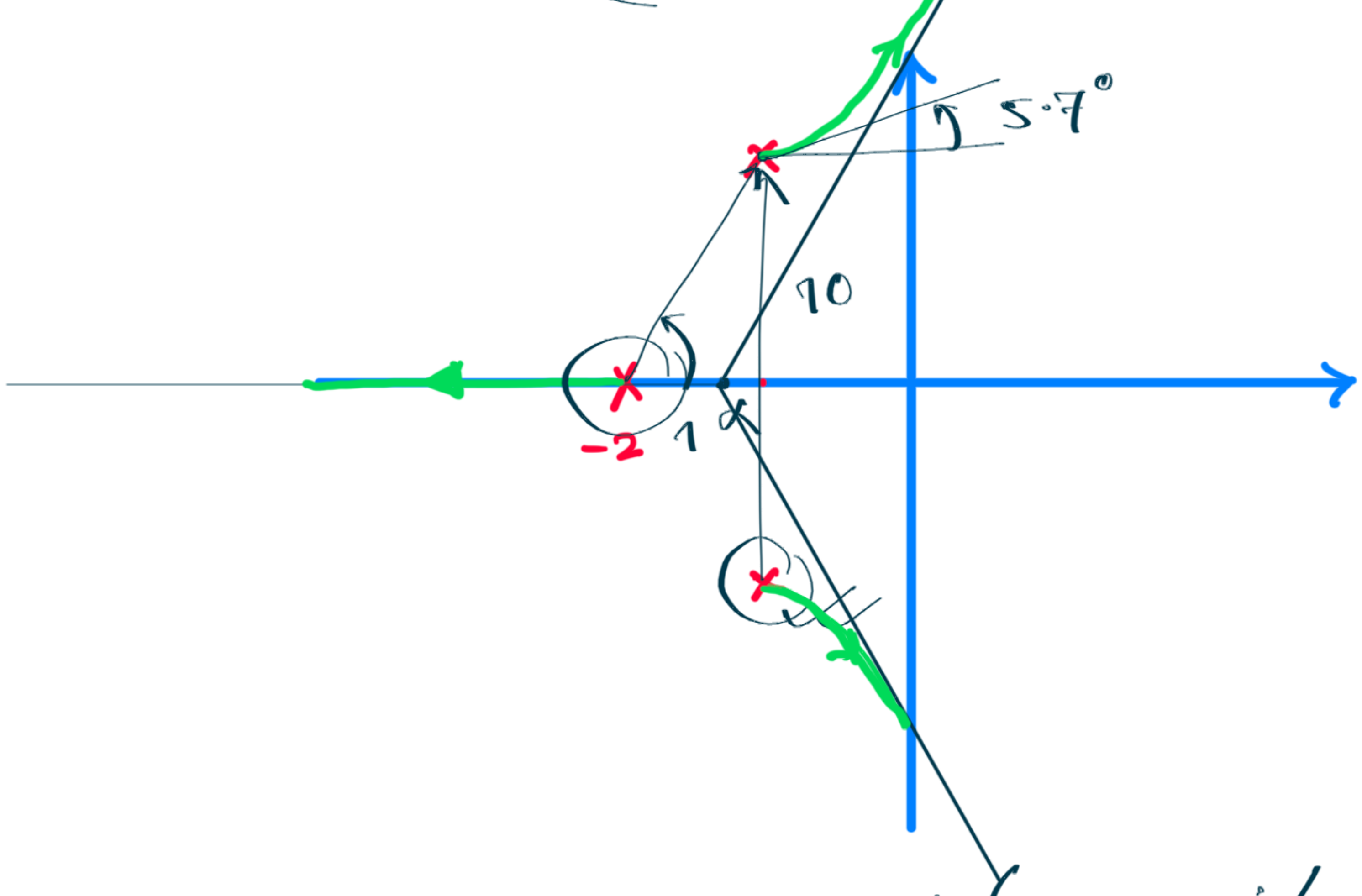
$l = 0, \pm 1, \pm 2, \dots$

Example:

$$G(s) = \frac{1}{(s+2)(s+1+j10)(s+1-j10)}$$

$$= \frac{1}{(s+2)\left((s+1)^2+100\right)}$$

$$= \frac{1}{(s+2)(s^2+2s+101)}$$



<u>Centroid loc^n</u> : $\alpha = \frac{1}{3}\left(-2 - 1 - \cancel{j10} - 1 + \cancel{j10}\right)$

$$= -\frac{4}{3}$$

<u>Angle of departure</u> :

$$\left(-\theta - 90^\circ - \tan^{-1} 10\right) = 180^\circ(2l+1)$$

$$= 180^\circ$$

$$\Rightarrow \theta = -90^\circ - \tan^{-1} 10 + 180^\circ$$

$$= 5.7^\circ$$