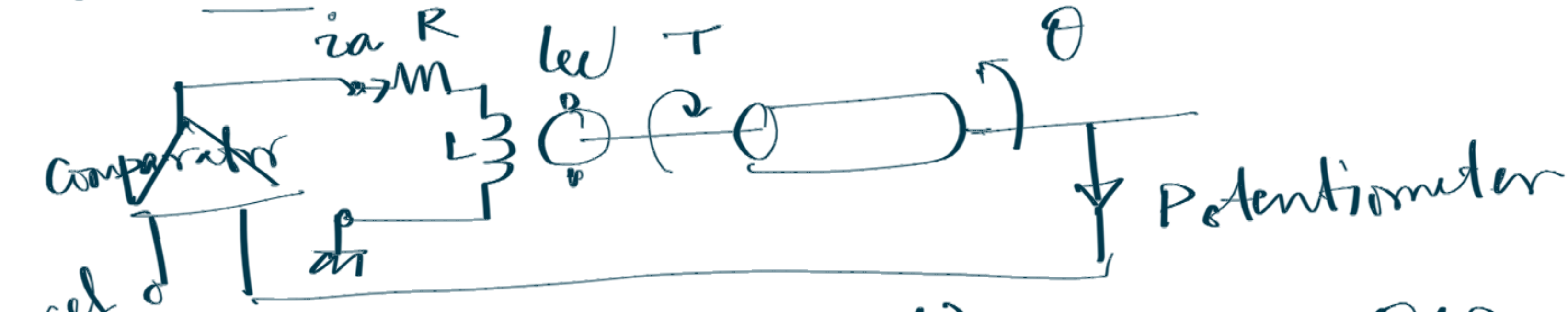
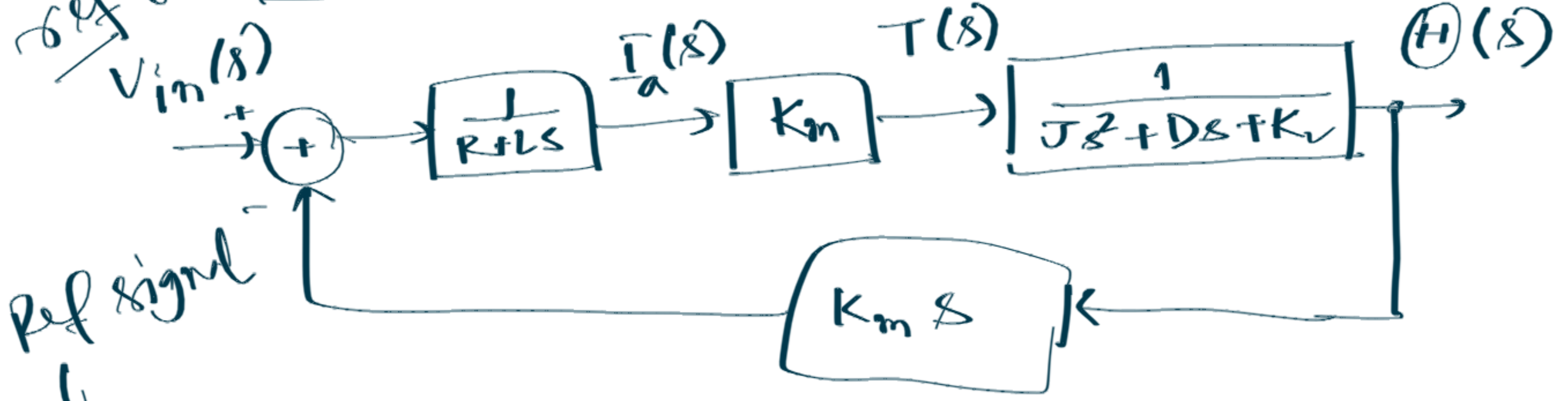
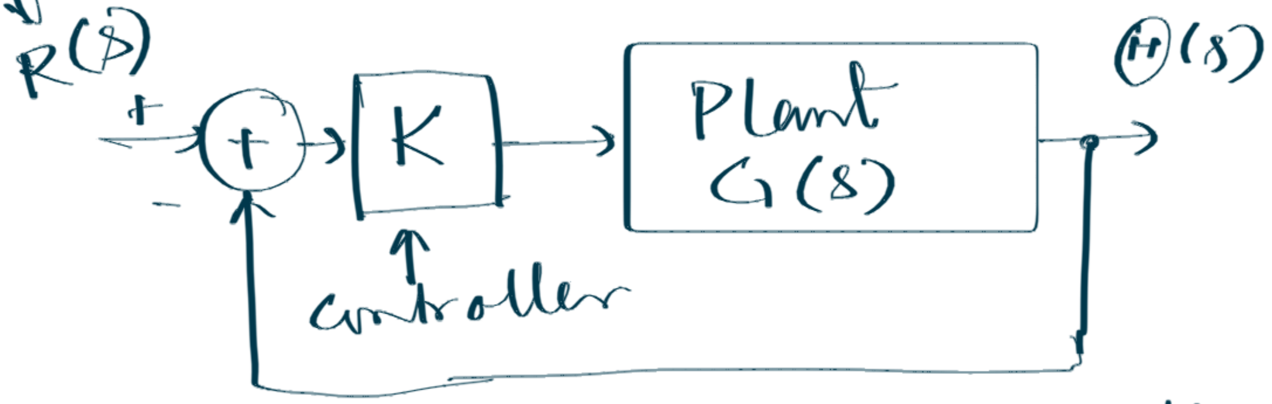
19 Feb:



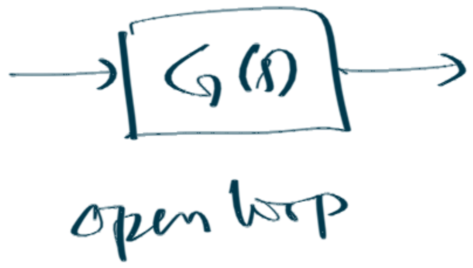


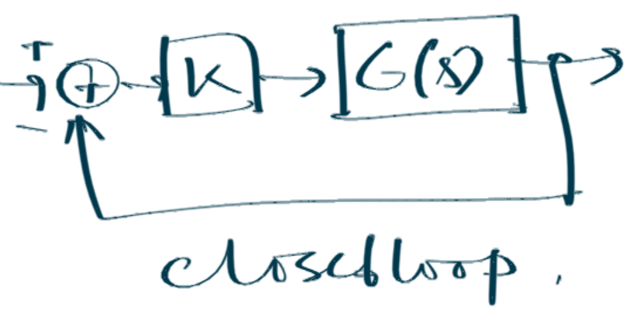


Unity negative feedback.

→ Quantative analysis of tracking:

→ Aside: Benefit of feedback in terms of robustness.





Sensitivity:

$$S := \frac{(\Delta OLTF/OLTF)}{\Delta G/G}$$

$$= \frac{d(OLTF)}{dG} \cdot \frac{G}{OLTF}$$

$$= 1$$

$$S := \frac{(\Delta CLTF/CLTF)}{\Delta G/G}$$

$$= \frac{d(CLTF)}{dG} \cdot \frac{G}{CLTF}$$

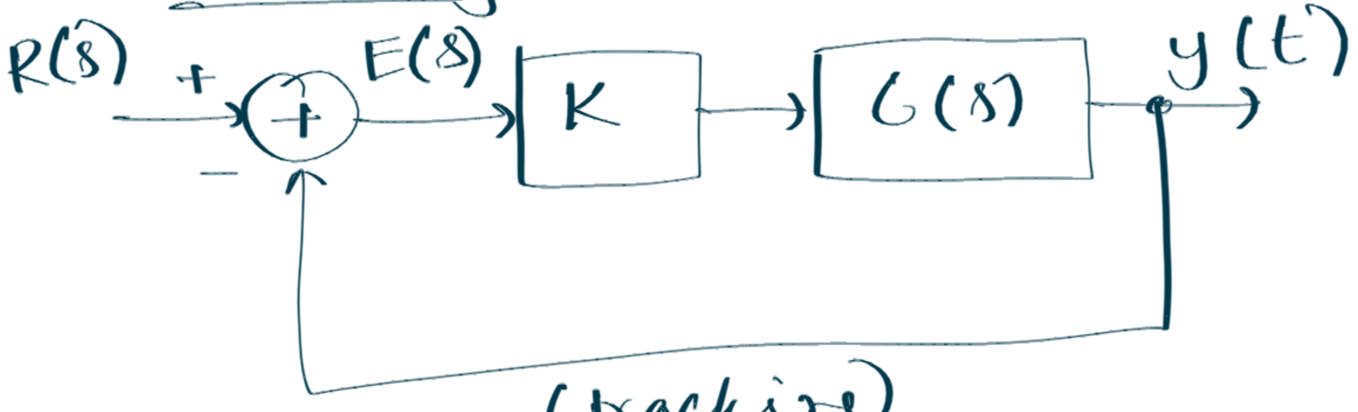
$$= \frac{d}{dG}\left[\frac{KG}{1+KG}\right] \cdot \frac{G(1+KG)}{KG}$$

$$= \frac{K(1+KG) - K KG}{(1+KG)^2} \cdot \frac{G\cancel{(1+KG)}}{KG}$$

$$= \frac{\cancel{K} + (\cancel{K^2G - K^2G}) \cdot \cancel{G}}{(1+KG)\,\cancel{KG}}$$

$$= \frac{1}{1+KG}$$

Tracking:



E(s) is the (tracking) error Laplace transform

$$e(t) := r(t) - y(t)$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+KG}$$

Steady state error:

$$e(\infty) := \lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot E(s)$$

Provided limit exists

FVT:

$$\int_{0^-}^{\infty} f(t)\, e^{-st}\, dt = F(s)$$

$$\Rightarrow sF(s) = \int_{0}^{\infty} \dot{f}(t)\, e^{-st}\, dt$$

$$\Rightarrow \lim_{s \to 0} sF(s) = \int_{0^-}^{\infty} \dot{f}(t)\,dt$$

$$= f(\infty) - f(0^-)$$

$$f(0^-) = 0$$

$$\Rightarrow f(\infty) = \lim_{s \to 0} sF(s).$$

<u>IVT:</u> $f(t) \xrightarrow{\mathcal{L}} F(s)$

$$\lim_{s \to \infty} sF(s) = f(0^+)$$

$$sF(s) = \int_{0^-}^{\infty} f(t)\, \underline{s}e^{-st}\, \underline{dt}$$

$$st =: \tau$$

$$s\,dt = d\tau$$

$$\lim_{s \to \infty} sF(s) = \lim_{s \to \infty} \int_{0^-}^{\infty} f\left(\frac{\tau}{s}\right) e^{-\tau}\, d\tau$$

$$= f(0^+) \int_{0^-}^{\infty} e^{-\tau}\, d\tau$$

$$= f(0^+) \left[-e^{-\tau}\right]_{0^-}^{\infty}$$

$$= \underline{f(0^+)}$$

$$E(s) = \frac{1}{1 + KG(s)} \cdot R(s)$$

[let's assume that the error transfer f^n is stable]

$$R(s) = \frac{1}{s}, \quad \frac{1}{s^2}, \quad \frac{1}{s^3}$$

step ramp parabola

Step:

$$e(\infty) = \lim_{s \to 0} s \cdot E(s) = \lim_{s \to 0} \cancel{s} \frac{1}{1 + KG(s)} \frac{1}{\cancel{s}}$$

$$= \frac{1}{1 + K \lim_{s \to 0} G(s)}$$

Ramp:

$$e(\infty) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \cancel{s} \frac{1}{1 + KG(s)} \cdot \frac{1}{s^{\cancel{2}}}$$

$$= \frac{1}{K \lim_{s \to 0} s\,G(s)}$$

Parabola:

$$e(\infty) = \lim_{s \to 0} sE(s) = \frac{1}{K \lim_{s \to 0} s^2 G(s)}$$

$$K_p := \lim_{s \to 0} G(s)$$ Position error constant

$$K_v := \lim_{s \to 0} s\,G(s)$$ Velocity error constant.

$$K_a := \lim_{s \to 0} s^2 G(s)$$ Acceleration error constant.

	step	ramp	parabola
$e(\infty)$	$\frac{1}{1+KK_p}$	$\frac{1}{KK_v}$	$\frac{1}{KK_a}$