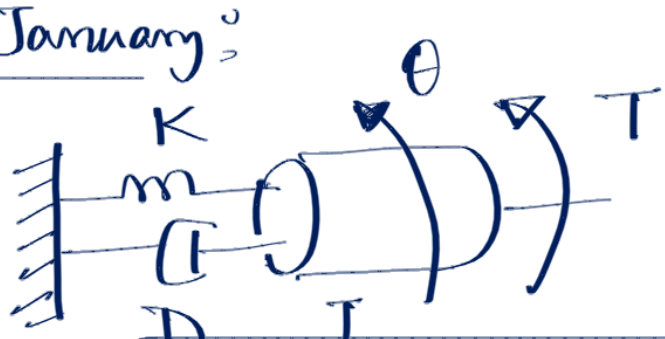
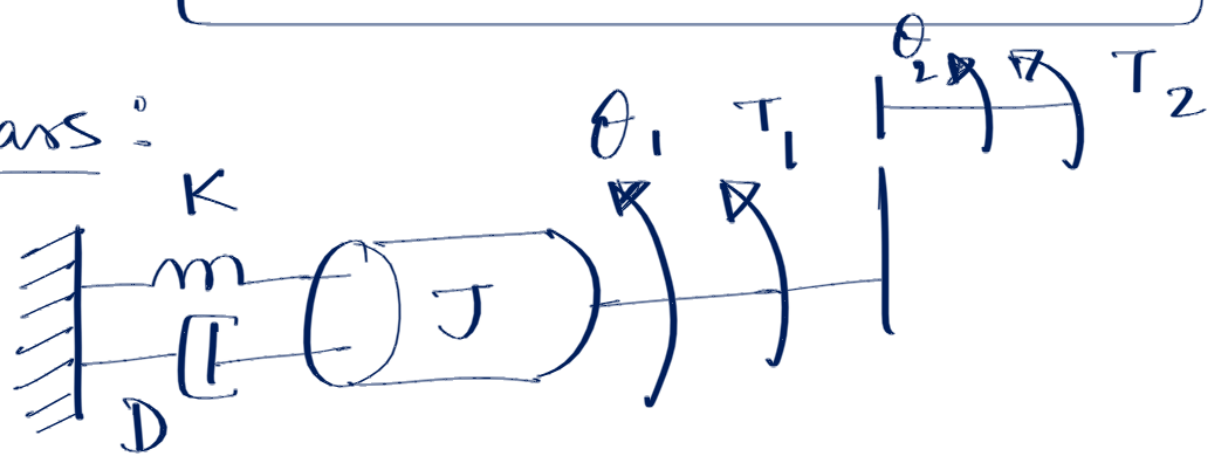
20th January:

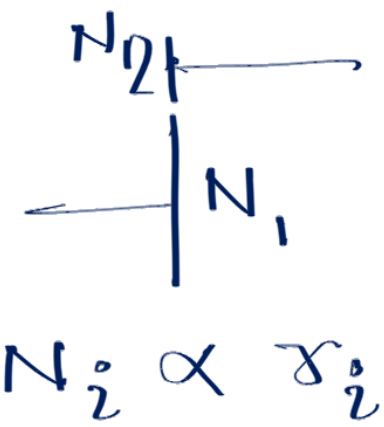


$$T = J\ddot{\theta} + D\dot{\theta} + K\theta$$

Gears:



$$J\ddot{\theta}_1 + D\dot{\theta}_1 + K\theta_1 = T_1$$



$$N_i \propto r_i$$

$$r_1\theta_1 = r_2\theta_2$$

$$\Rightarrow N_1\theta_1 = N_2\theta_2 \Rightarrow \boxed{\frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}} \quad \text{—(1)}$$

$$T_1\theta_1 = T_2\theta_2$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{N_1}{N_2} \Rightarrow \boxed{\frac{T_1}{T_2} = \frac{N_1}{N_2}} \quad \text{—(2)}$$

$$J\ddot{\theta}_1 + D\dot{\theta}_1 + K\theta_1 = T_1 = \left(\frac{N_1}{N_2}\right) T_2$$

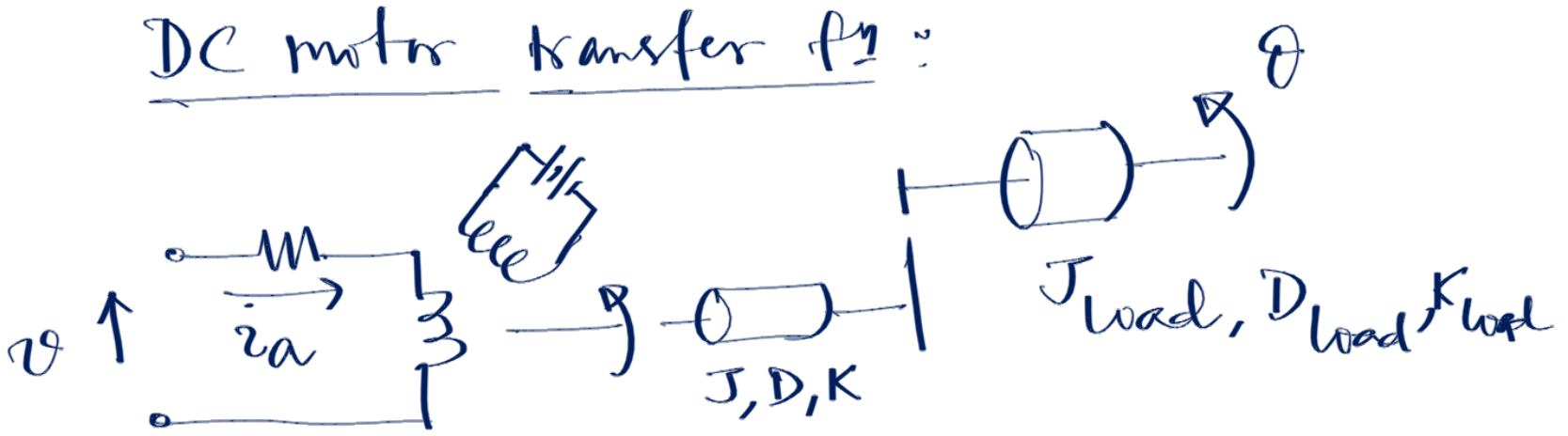
$$\Rightarrow J\left(\frac{N_2}{N_1}\right)\ddot{\theta}_2 + D\left(\frac{N_2}{N_1}\right)\dot{\theta}_2 + K\left(\frac{N_2}{N_1}\right)\theta_2 = \left(\frac{N_1}{N_2}\right) T_2$$

$$\Rightarrow \boxed{J_2 \ddot{\theta}_2 + D_2 \dot{\theta}_2 + K_2 \theta_2 = T_2}$$

$$J_2 := J\left(\frac{N_2}{N_1}\right)^2, \quad D_2 := D\left(\frac{N_2}{N_1}\right)^2,$$

$$K_2 := K\left(\frac{N_2}{N_1}\right)^2.$$

DC motor transfer fn:

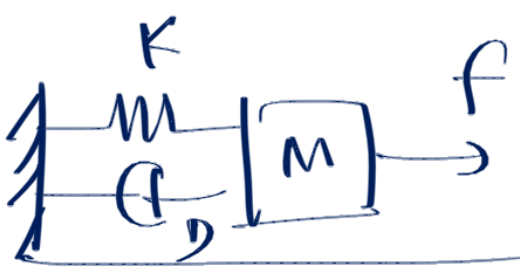


Find out TF $\boxed{\frac{\Theta(s)}{V(s)}}$ (Homework).

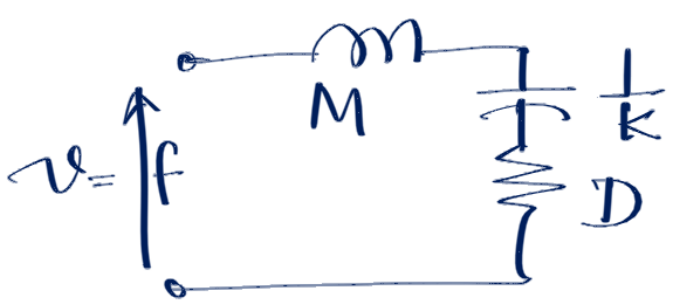
Electrical - Mechanical analogy:

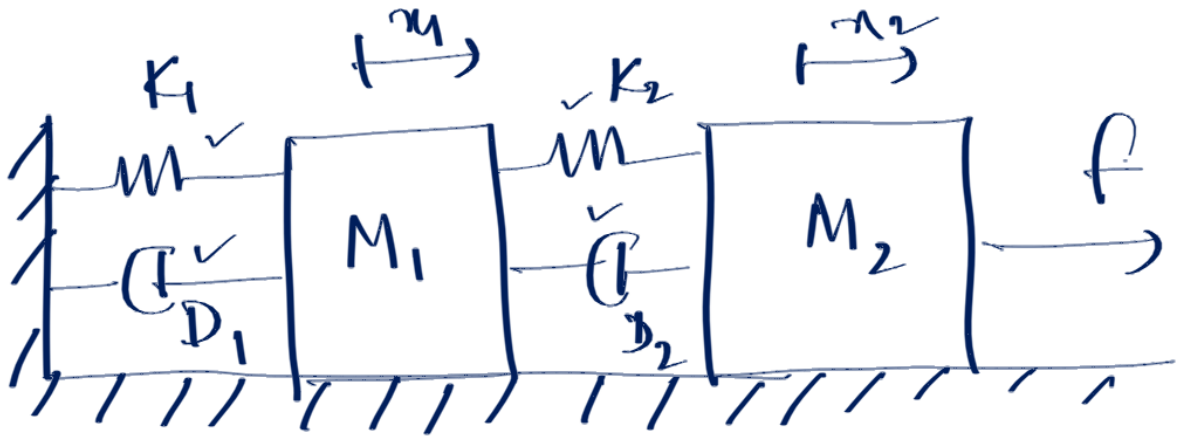
1. Force - voltage analogy:
   velocity - current

	Mechanical		Electrical	
(a)	M	$f = M\ddot{x}$	m	$v = L\ddot{q}$
(b)	K	$f = Kx$	C	$v = \frac{1}{C}q$
(c)	D	$f = D\dot{x}$	R	$v = R\dot{q}$



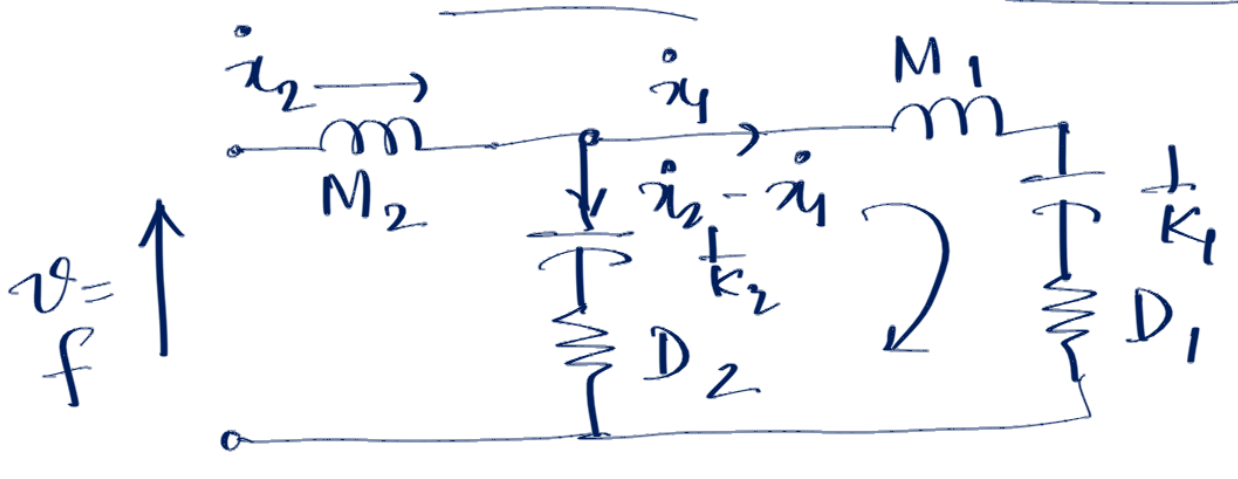
$$f = M\ddot{x} + D\dot{x} + Kx$$





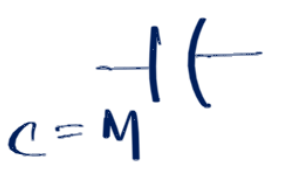
$$M_1\ddot{x}_1 + K_1 x_1 + D_1\dot{x}_1 + K_2 x_1 - K_2 x_2 + D_2\dot{x}_1 - D_2\dot{x}_2 = 0$$

$$M_2\ddot{x}_2 + K_2(x_2 - x_1) + D_2(\dot{x}_2 - \dot{x}_1) = f$$



2. Force - current analogy : velocity - voltage

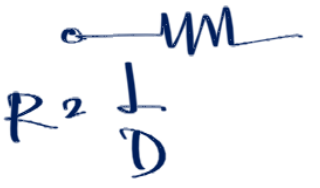
(a) 

$f = M\ddot{x}$ 

$i = C\dot{v}$

(b) 
$f = Kx$ 

$i = \frac{1}{L}\int v(t)dt$

(c) 
$f = D\dot{x}$ 

$i = \frac{v}{R}$