$$u(t) \longrightarrow \boxed{G(s)} \longrightarrow y(t)$$

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_0 y = b_m \frac{d^m u}{dt^m} + b_{m-1}\frac{d^{m-1} u}{dt^{m-1}} + \cdots + b_0 u$$

Assume all initial cond^{n}s to be zero and take $\mathcal{L}$-transform.

$$(s^n + a_{n-1} s^{n-1} + \cdots + a_0) Y(s) = (b_m s^m + \cdots + b_0) \underline{U(s)}$$

$$\Rightarrow Y(s) = \frac{b_m s^m + \cdots + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_0} \cdot \underline{U(s)}$$

<u>Laplace transform</u>:

$$f : [0, \infty) \to \mathbb{R}$$

$$F(s) := \mathcal{L}(f) := \int_{0_-}^{\infty} e^{-st} f(t)\, dt$$

$$\mathcal{L}(f_1 + f_2) = \int_{0_-}^{\infty} e^{-st} (f_1(t) + f_2(t))\, dt$$

$$= \int_{0_-}^{\infty} e^{-st} f_1(t)\, dt + \int_{0_-}^{\infty} e^{-st} f_2(t)\, dt$$

$$= \mathcal{L}(f_1) + \mathcal{L}(f_2)$$

Linearity $\Rightarrow$ {one-to-one $\Leftrightarrow$ nullspace $= \{0\}$}

" Piecewise cont. f^{n}s with finitely many impulses and their finitely many derivatives $\Leftrightarrow$ Laplace transformable f^{n}s.

In this class

$\mathcal{L}$ is one-to-one.

$\Rightarrow$ Laplace inverse exists.

$f(t) \overset{\mathcal{L}}{\mapsto} F(s) \Rightarrow$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma_c - j\infty}^{\sigma_c + j\infty} F(s)\, e^{st}\, ds$$

Not needed

$\boxed{1(t)}$ ~~u(t)~~

$$1(t) := 1 \quad \forall\, t > 0 \qquad = 0 \quad t = 0 \Big\} \text{ unit step}$$

$$\mathcal{L}(1(t)) := \int_{0^-}^{\infty} e^{-st}\, 1(t)\, dt$$

$$= \int_0^{\infty} e^{-st}\, dt$$

$$= -\frac{1}{s} e^{-st} \Big|_0^{\infty}$$

$$= \boxed{+\frac{1}{s}}$$

- $\boxed{f(t) = e^{-at}\, 1(t)}$

$$\mathcal{L}(e^{-at}) = \int_{0^-}^{\infty} e^{-st} e^{-at} \, dt$$

$$= \int_{0^-}^{\infty} e^{-(s+a)t} \, dt$$

$$= \boxed{\frac{1}{s+a}}$$

$\boxed{\sin \omega t}$ $\boxed{\cos \omega t}$

$$\sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$\cos \omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$\mathcal{L}(\sin \omega t) = ?$ $\mathcal{L}(\cos \omega t) = ?$

(HW)

PFE:

$$\frac{p(s)}{q(s)} = r(s) + \frac{A_1}{s+\lambda_1} + \frac{A_2}{s+\lambda_2} + \dots + \frac{A_k}{s+\lambda_k} +$$

$$\frac{bs+c}{(s+\lambda_1)^2}$$

$$\frac{bs+c}{(s^2+as+a_0)}$$