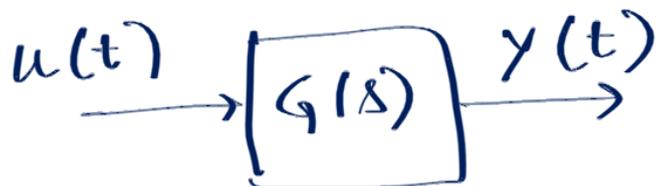


□



$$\frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_0 y = b_m \frac{d^m u}{dt^m} + b_{m-1} \frac{d^{m-1} u}{dt^{m-1}} + \dots + b_0 u$$

Assume all initial condⁿs to be zero and take \mathcal{L} -transform.

$$(s^n + a_{n-1} s^{n-1} + \dots + a_0) Y(s) = (b_m s^m + \dots + b_0) U(s)$$

$$\Rightarrow Y(s) = \frac{b_m s^m + \dots + b_0}{s^n + a_{n-1} s^{n-1} + \dots + a_0} \cdot \underline{U(s)}$$

Laplace transform:

$$f: [0, \infty) \rightarrow \mathbb{R}$$

$$F(s) := \mathcal{L}(f) := \int_{0_-}^{\infty} e^{-st} f(t) dt$$

$$\begin{aligned} \mathcal{L}(f_1 + f_2) &= \int_{0_-}^{\infty} e^{-st} (f_1(t) + f_2(t)) dt \\ &= \int_{0_-}^{\infty} e^{-st} f_1(t) dt + \int_{0_-}^{\infty} e^{-st} f_2(t) dt \\ &= \mathcal{L}(f_1) + \mathcal{L}(f_2) \end{aligned}$$

linearity \Rightarrow { one-to-one \Leftrightarrow nullspace = {0} }

* Piecewise cont. fⁿs with finitely many

impulses and their finitely many derivatives \Leftrightarrow Laplace transformable fⁿs.

In this class

\mathcal{L} is one-to-one.

\Rightarrow Laplace inverse exists.

$$f(t) \xrightarrow{\mathcal{L}} F(s) \Rightarrow$$

$$f(t) = \frac{1}{2\pi j} \int_{\sigma_c - j\infty}^{\sigma_c + j\infty} F(s) e^{st} ds$$

Not needed

\square

$$\boxed{1(t)} \quad \cancel{u(t)}$$

$$1(t) := \begin{cases} 1 & \forall t > 0 \\ 0 & t = 0 \end{cases} \text{ unit step}$$

$$\mathcal{L}(1(t)) := \int_0^{\infty} e^{-st} 1(t) dt$$

$$= \int_0^{\infty} e^{-st} dt$$
$$= -\frac{1}{s} e^{-st} \Big|_0^{\infty}$$

$$= \boxed{+\frac{1}{s}}$$

$$- \boxed{f(t) = e^{-at} 1(t)}$$

$$\begin{aligned} \mathcal{L}(e^{-at}) &= \int_0^{\infty} e^{-st} e^{-at} dt \\ &= \int_0^{\infty} e^{-(s+a)t} dt \\ &= \boxed{\frac{1}{s+a}} \end{aligned}$$

$$\boxed{\sin \omega t}$$

$$\boxed{\cos \omega t}$$

$$\sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$\cos \omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\mathcal{L}(\sin \omega t) = ? \quad \mathcal{L}(\cos \omega t) = ?$$

(HW)

PFE:

$$\frac{p(s)}{q(s)} = r(s) + \frac{A_1}{s+\lambda_1} + \frac{A_2}{s+\lambda_2} + \dots + \frac{A_k}{s+\lambda_k} + \dots$$

$$\frac{bs+c}{(s+\lambda_1)^2}$$

$$\frac{bs+c}{(s^2+as+a_0)}$$

