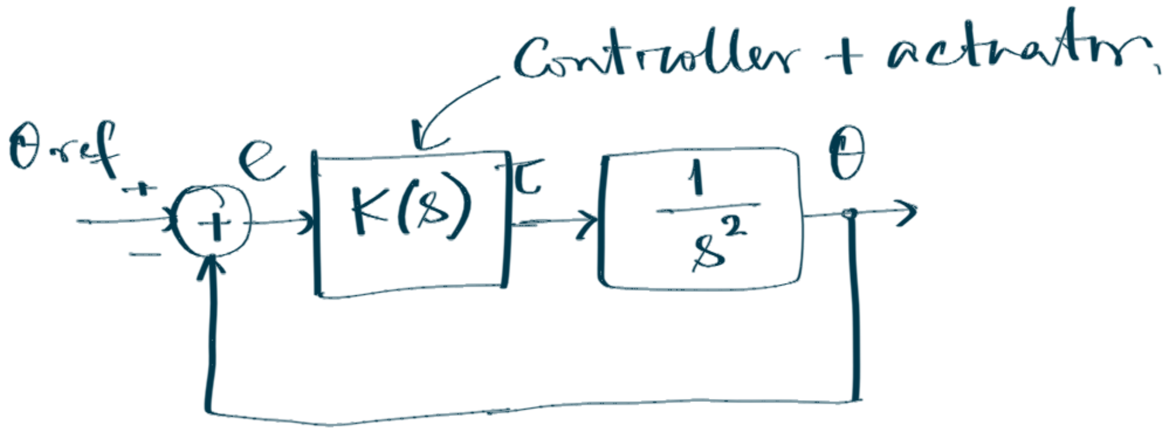
23 March:

Root locus based controller design.

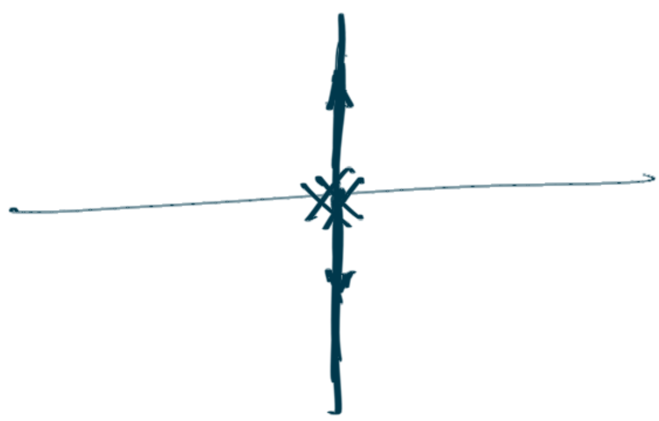


Control spec:

1. Zero steady-state error to step input.
2. 10% overshoot in step response.
3. Settling time = 3 sec.

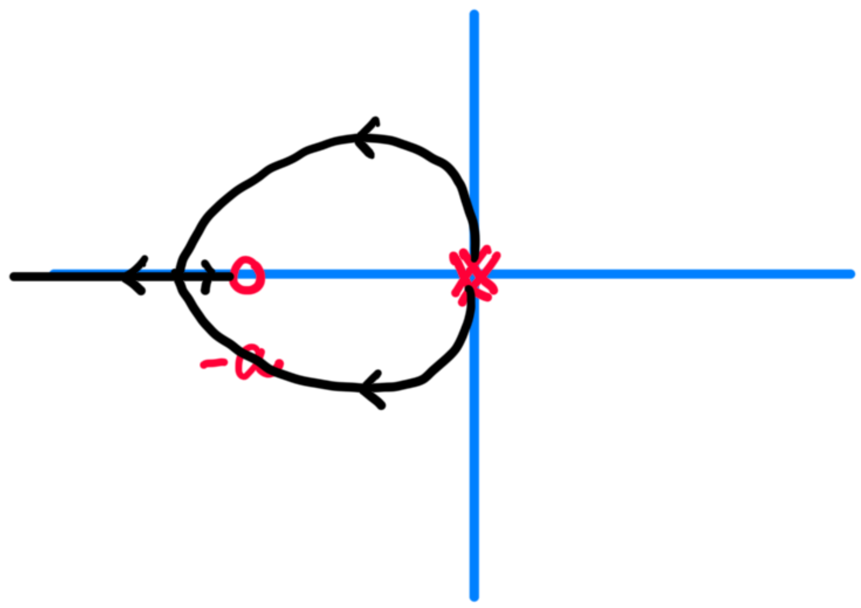
P-controller:

$$K(s) = K$$



PD controller:

$$K(s) = K_P + K_D s$$
$$= K(s+a)$$



Desired pole locations from the specifications:

$\zeta = ?$

$\omega_n = ?$

$$e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.1$$

$$\frac{\pi\zeta}{\sqrt{1-\zeta^2}} = -\ln 0.1 = 2.3$$

$$\Rightarrow \left(\frac{\zeta^2}{1-\zeta^2}\right) = 0.537$$

$$\Rightarrow \zeta^2 = \frac{0.537}{1+0.537}$$

$$\Rightarrow \boxed{\zeta = 0.6}$$

$$t_s = \frac{3.912}{\zeta\omega_n} = 3$$

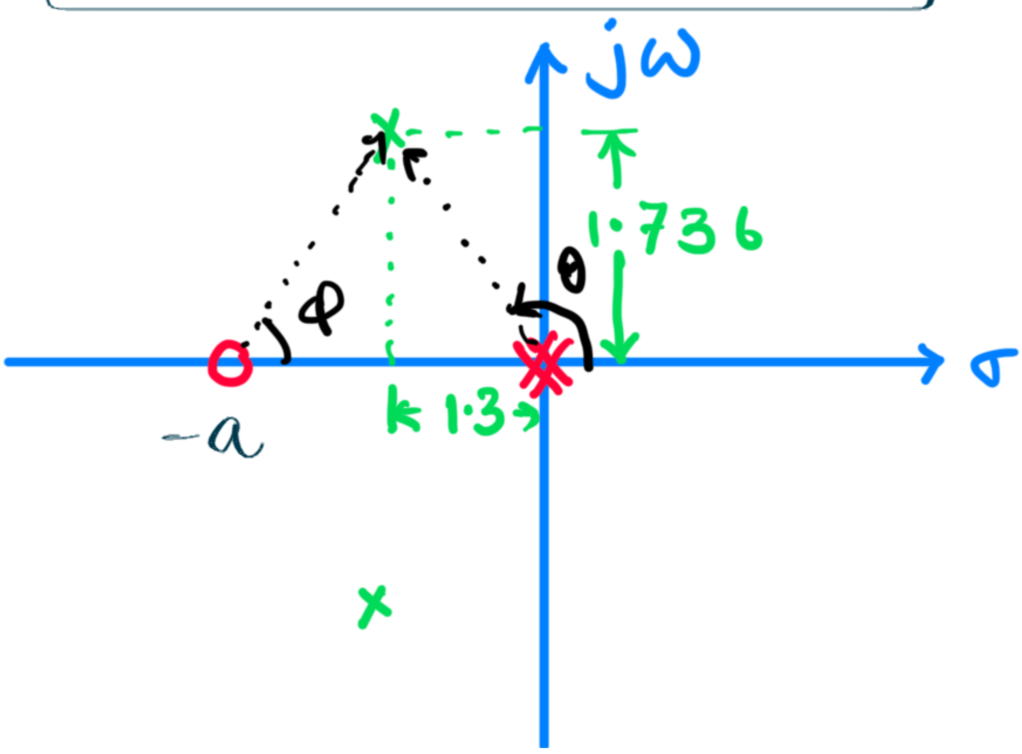
$$\Rightarrow \omega_n = \frac{3.912}{3\times 0.6}$$

$$= 2.17 \text{ rad/s.}$$

Desired closed loop pole locations

$$= -\zeta\omega_n \pm j\left(\sqrt{1-\zeta^2}\right)\omega_n$$

$$\boxed{= -1.3 \pm j\,1.736}$$



$$\theta = 90^\circ + \tan^{-1}\frac{1\cdot 3}{1\cdot 736}$$

$$= 126\cdot 8^\circ$$

$$-2\times 126\cdot 8 + \phi = 180^\circ(2l+1)$$

$$\Rightarrow \phi = -180^\circ + 2\times 126\cdot 8^\circ$$

$$= 73\cdot 6^\circ$$

$\Rightarrow$

$$\tan\phi = \frac{1\cdot 736}{(a-1\cdot 3)}$$

$$\Rightarrow a = 1\cdot 8$$

$\Rightarrow$ We place the OL zero at

$$s = -1\cdot 8 .$$

Thus the OL transfer f^n is

$$\boxed{\frac{K(s+1\cdot 8)}{s^2}} .$$

Let $K = K^*$ be the value at which the CL poles are exactly at the desired locations, $-1\cdot 3 \pm j1\cdot 736$.

$$\Rightarrow 1 + K^*\frac{(s+1\cdot 8)}{s^2}\Bigg|_{s=-1\cdot 3+j1\cdot 736} = 0$$

$$\Rightarrow K^* = -\frac{s^2}{s+1\cdot 8}\Bigg|_{s=-1\cdot 3+j1\cdot 736}$$

$$= (-2\cdot 6 + j0\cdot 012)$$

Take $K^* = |-2.6 + j0.072|$

$\simeq 2.6$

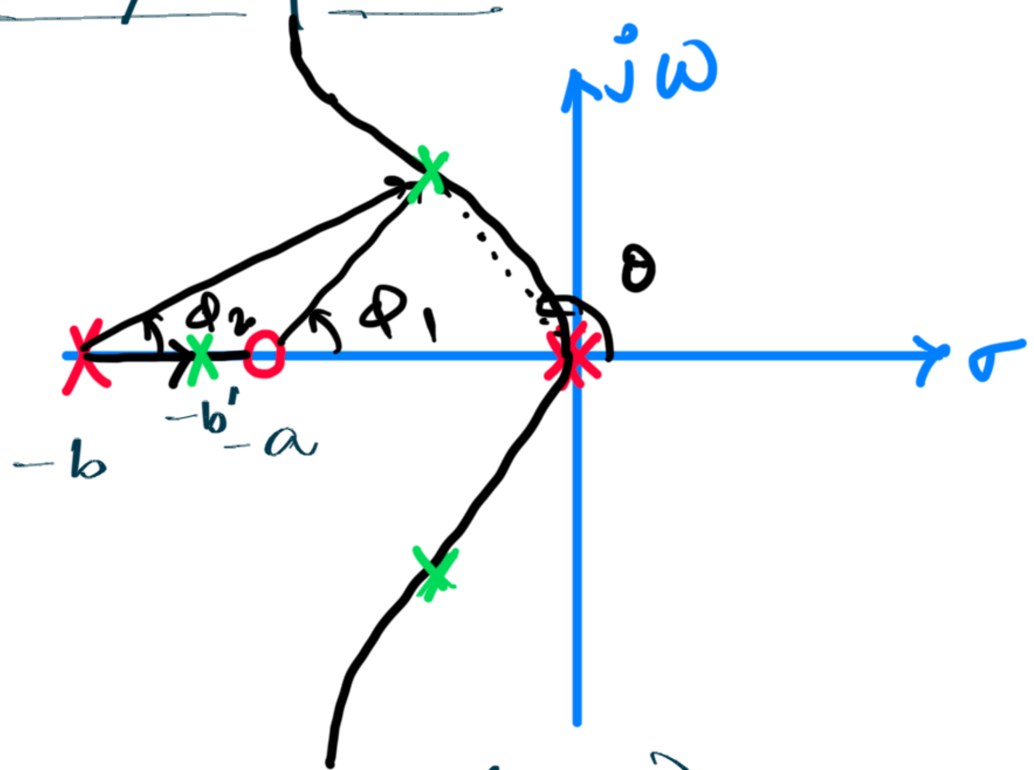
The controller is taken to be

$$\boxed{2.6(s+1.8)}$$

$$CLTF = \frac{\frac{2.6(s+1.8)}{s^2}}{1+\frac{2.6(s+1.8)}{s^2}}$$

$$= \frac{2.6(s+1.8)}{s^2+2.6s+2.6\times1.8}$$

(ii) lead/lag compensator:



$$K(s) = \frac{K(s+a)}{(s+b)}$$

$$CLTF = \frac{K^*(s+a)}{(s+b')(s^2+2.6s+2.6\times1.8)}$$

$$1 + KG(\lambda) = 0$$

$$\Rightarrow KG(\lambda) = -1$$

$$\Rightarrow K = -\frac{1}{G(\lambda)}$$

$$\Rightarrow K = \frac{1}{|G(\lambda)|}$$