$$G(s) = \frac{s+4}{s^3+6s^2+11s+6}$$

$$Y(s) = G(s)\,U(s)$$

$$= \frac{s+4}{(s^3+6s^2+11s+6)} \cdot \frac{1}{s}$$

$$= \frac{s+4}{(s+1)(s+2)(s+3)\,s}$$

Distinct real roots

$$= \frac{A_0}{s} + \frac{A_1}{s+1} + \frac{A_2}{s+2} + \frac{A_3}{s+3}$$

$$= \frac{A_0(s+1)(s+2)(s+3) + A_1 s(s+2)(s+3) + A_2 s(s+1)(s+3) + A_3 s(s+1)(s+2)}{s(s+1)(s+2)(s+3)}$$

$$A_0 = \lim_{s\to 0} sY(s) = \frac{2}{3}$$

$$A_1 = \lim_{s\to -1} (s+1)Y(s) = -\frac{3}{2}$$

$$A_2 = \lim_{s\to -2} (s+2)Y(s) = 1$$

$$A_3 = \lim_{s\to -3} (s+3)Y(s) = -\frac{1}{6}$$

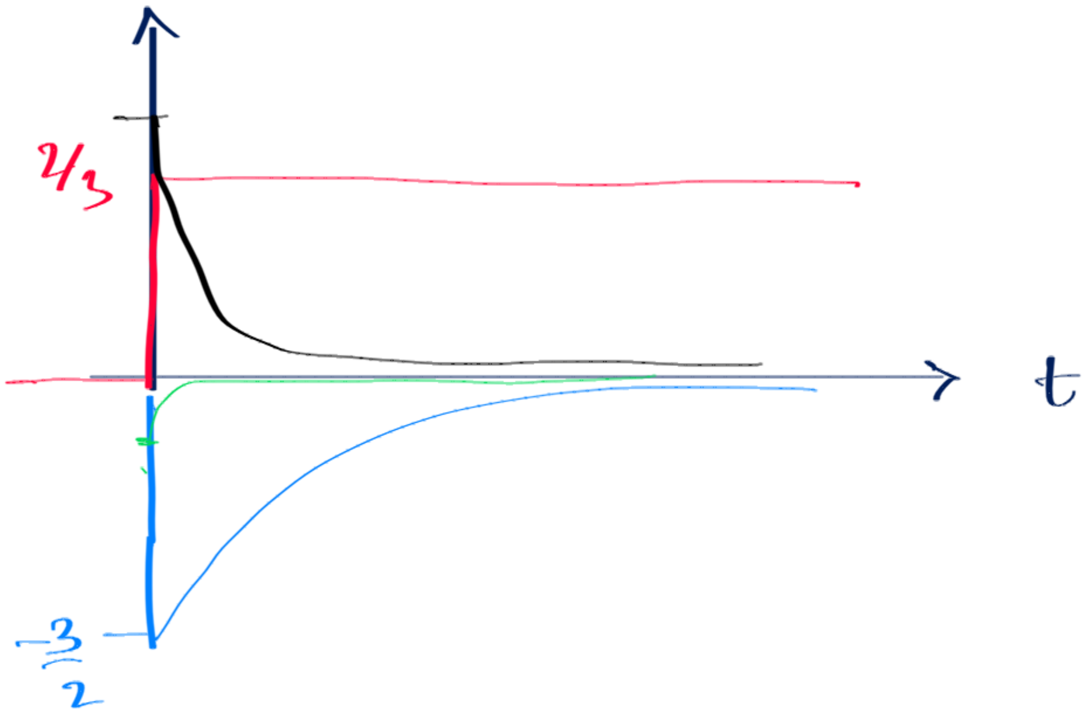
$$Y(s) = \frac{2}{3}\cdot\frac{1}{s} - \frac{3}{2}\cdot\frac{1}{s+1} + \frac{1}{s+2} - \frac{1}{6}\cdot\frac{1}{s+3}$$

$$y(t) = \mathcal{L}^{-1}(Y(s))$$

$$= \frac{2}{3}\cdot 1(t) - \frac{3}{2}e^{-t}1(t) + e^{-2t}1(t) - \frac{1}{6}e^{-3t}1(t).$$

$$\boxed{y(t) = \left(\frac{2}{3} - \frac{3}{2}e^{-t} + e^{-2t} - \frac{1}{6}e^{-3t}\right)1(t)}$$

Unit step response of $G(s)$.



$$G(s) = \frac{s^m + b_{m-1}s^{m-1} + \cdots + b_0}{\left(s^n + a_{n-1}s^{n-1} + \cdots + a_0\right)}$$

$$\Rightarrow \left(\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}}{dt^{n-1}}y + \cdots + a_0 y\right) = \frac{d^m u}{dt^m} + b_{m-1}\frac{d^{m-1} y}{dt^{m-1}} + \cdots + b_0 u$$

- The case of an "improper" transfer fn.

$$\frac{s^3 + 6s^2 + 11s + 6}{s+4} = \left(s^2 + as + b\right) + \frac{K}{s+4}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$\ddot{\delta}(t) + a\dot{\delta}(t) + b\delta(t) + Ke^{-4t}1(t)$$

- Assumption: "Proper" transfer fn

(i.e., deg num $\leq$ deg den)

- Poles and zeros have been introduced (read up from the textbook).

- DC gain of $G(s)$ (under stability assumption)

$$:= G(0).$$

- Time constant:

$$G(s) = \frac{K}{s+\lambda} \qquad K, \lambda > 0$$

Step response?

$$Y(s) = G(s) \cdot \frac{1}{s}$$

$$= \frac{K}{s+\lambda} \cdot \frac{1}{s} = \frac{A_0}{s} + \frac{A_1}{s+\lambda}.$$

$$A_0 s + A_0 \lambda + A_1 s = K$$

$$\Rightarrow (A_0 + A_1) s + A_0 \lambda = K.$$

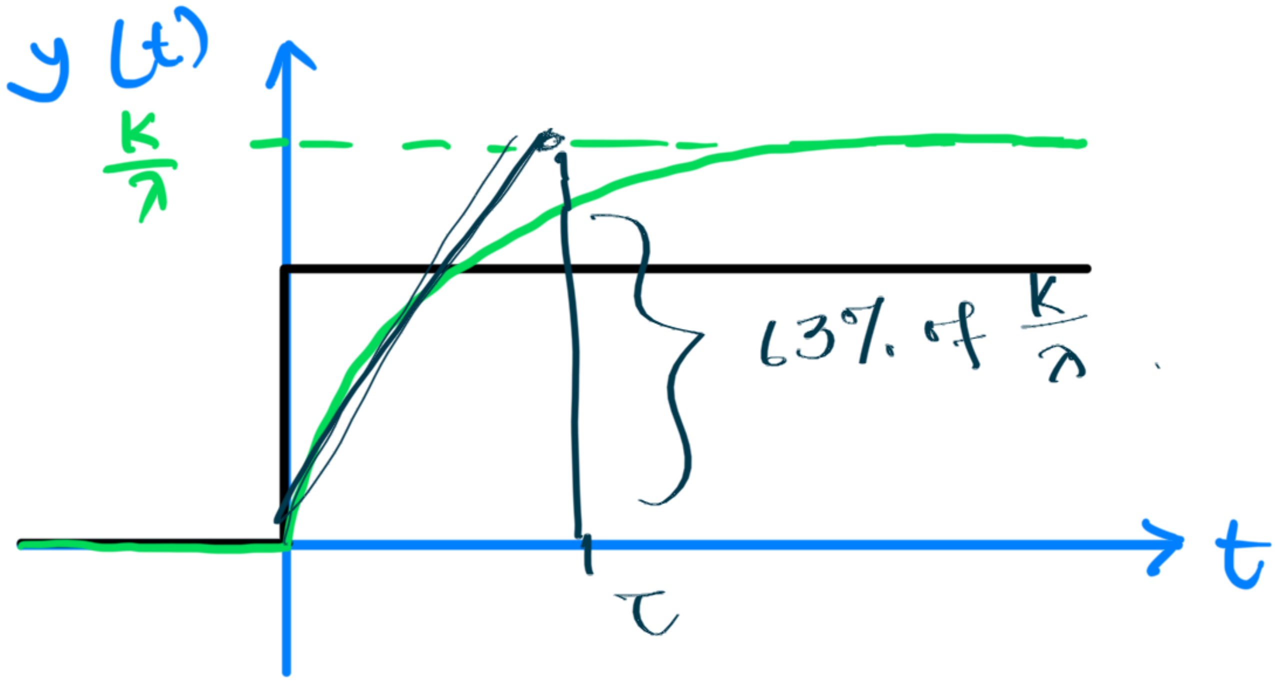
$$\Rightarrow A_0 + A_1 = 0 \quad \text{--- (1)}$$

$$\lambda A_0 = K \Rightarrow A_0 = \frac{K}{\lambda}.$$

$$\Rightarrow A_1 = -\frac{K}{\lambda}.$$

$$\Rightarrow Y(s) = \frac{K}{\lambda}\left[\frac{1}{s} - \frac{1}{s+\lambda}\right]$$

$$\Rightarrow \boxed{y(t) = \frac{K}{\lambda}\left(1 - e^{-\lambda t}\right) 1(t).}$$



$$\tau = \frac{1}{\lambda}.$$

$$\dot{y} = K e^{-\lambda t}$$

$$\dot{y}\big|_{t=0^+} = \underline{\underline{K}}.$$

$\underline{\text{2nd order system}}$:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\underline{\underline{\zeta}}\omega_n s + \omega_n^2}$$

$\omega_n \to$ natural freq. of oscillation

$\zeta \to$ damping ratio.

Discriminant

$$\cancel{4}\zeta^2 \cancel{\omega_n^2} - \cancel{4}\cancel{\omega_n^2} \overset{?}{\geq} 0$$
$$< 0$$

$$\zeta^2 - 1 \geq 0 \; ?$$
$$< 0$$

$\zeta \geq 0$

$\zeta = 1$, $\zeta < 1$, $\zeta > 1$

$\zeta = 1 \downarrow$ Critically damped

$\zeta < 1 \downarrow$ underdamped

$\zeta > 1 \downarrow$ Overdamped