29 Jan:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$ Standard 2nd order.

$$Y(s) = \frac{1}{s} G(s)$$ Laplace transform of step-response

$$\boxed{\zeta > 1 \Rightarrow \text{overdamped}}$$

$$Y(s) = \frac{1}{s}\left[\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \zeta^2\omega_n^2 - \zeta^2\omega_n^2 + \omega_n^2}\right]$$

$$= \frac{1}{s}\left[\frac{\omega_n^2}{(s+\zeta\omega_n)^2 - \omega_n^2(\zeta^2 - 1)}\right]$$

$$\underbrace{\zeta^2-1}_{> 0}$$

$$= \frac{1}{s}\left[\frac{\omega_n^2}{\left(s+\zeta\omega_n + \omega_n\sqrt{(\zeta^2-1)}\right)\left(s+\zeta\omega_n - \omega_n\sqrt{(\zeta^2-1)}\right)}\right]$$

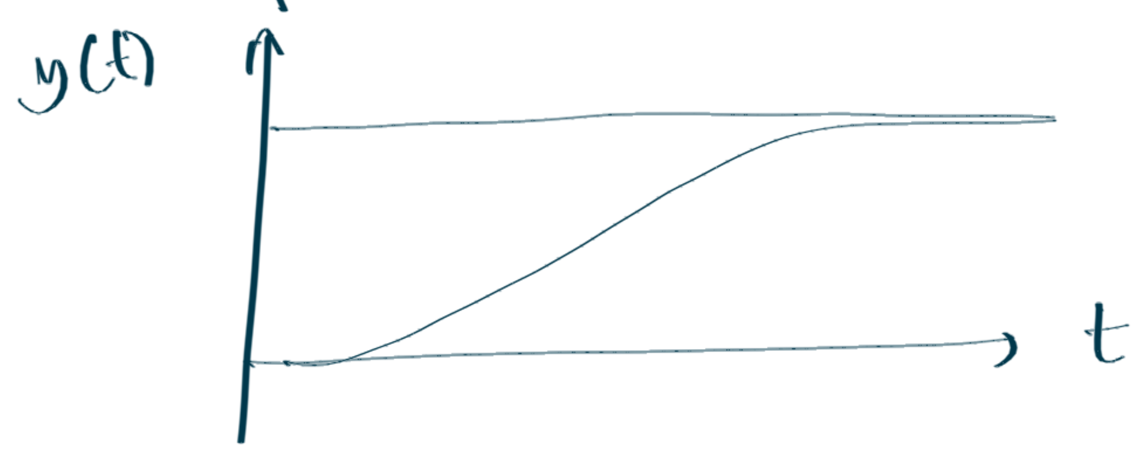
$$= \frac{1}{s} \cdot \frac{\lambda_1\lambda_2}{(s+\lambda_1)(s+\lambda_2)} \qquad \lambda_1, \lambda_2 > 0$$

$$= \frac{A_0}{s} + \frac{A_1}{s+\lambda_1} + \frac{A_2}{s+\lambda_2}$$

$$\Rightarrow y(t) = A_0 1(t) + A_1 e^{-\lambda_1 t} 1(t) + A_2 e^{-\lambda_2 t} 1(t).$$

Complete this analysis.

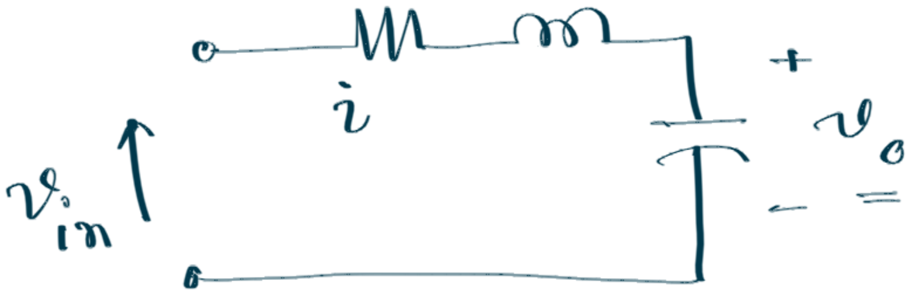
And plot $y(t)$ vs t.



$$\boxed{0 < \zeta < 1 \rightarrow \text{Underdamped}}$$

$$Y(s) = \frac{1}{s} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} \frac{\omega_n^2}{\underbrace{(s+\zeta\omega_n)^2 + (1-\zeta^2)\omega_n^2}_{\text{Roots are @ } -\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2}}}$$



$$V_{in}(s) = R\,I(s) + Ls\,I(s) + \frac{1}{Cs} I(s)$$

$$\Rightarrow \quad I(s) = \frac{Cs\,V_{in}(s)}{RCs + LCs^2 + 1}$$

$$i = C\frac{dv_o}{dt} \Rightarrow I(s) = Cs\,V_o(s)$$

$$\Rightarrow V_o(s) = \frac{1}{Cs} I(s)$$

$$\Rightarrow \quad V_o(s) = \frac{1}{\cancel{Cs}} \cdot \frac{\cancel{Cs}\; V_{in}(s)}{RCs + LCs^2 + 1}$$

$$= \frac{V_{in}(s)}{LCs^2 + RCs + 1}$$

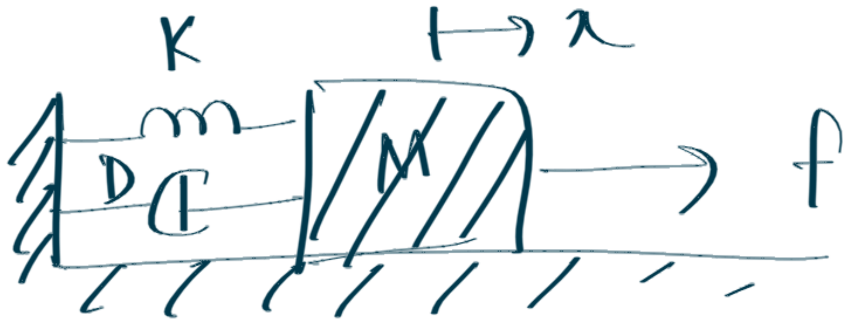
$$= \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \cdot V_{in}(s)$$

$$2\zeta\omega_n = \frac{R}{L}$$

$$\Rightarrow 2\zeta \frac{1}{\sqrt{LC}} = \frac{R}{L}$$

$$\Rightarrow 2\zeta \frac{1}{\sqrt{C}} = \frac{R}{\sqrt{L}}$$

$$\Rightarrow \zeta = \frac{1}{2} R \sqrt{\frac{C}{L}}$$



$$M\ddot{x} + D\dot{x} + Kx = f$$

$$\Rightarrow X(s) = \frac{F(s)}{Ms^2 + Ds + K}$$

$$= \frac{1}{M} \cdot \frac{F(s)}{\left(s^2 + \frac{D}{M}s + \frac{K}{M}\right)}$$

$$Y(s) = \frac{1}{s} \frac{\omega_n^2}{(s + \zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}$$

$$= \frac{A_0}{s} + \frac{A_1 s + A_2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

To find A_0, A_1, A_2.

$$A_0 s^2 + 2\zeta\omega_n A_0 s + A_0\omega_n^2 + A_1 s^2 + A_2 s = \omega_n^2$$

$$\boxed{A_0 = 1} \qquad 2\zeta\omega_n \cancel{s} + A_2 \cancel{s} = 0$$

$$\Rightarrow \boxed{A_2 = -2\zeta\omega_n}$$

$$\cancel{s} + A_1 \cancel{s} = 0 \quad \Rightarrow \boxed{A_1 = -1}$$

$$\Rightarrow Y(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}$$

$\omega_d := \omega_n\sqrt{1-\zeta^2} \rightarrow$ damped freq. of oscillation.

$\sigma := \zeta\omega_n$

$$= \frac{1}{s} - \frac{s+2\sigma}{(s+\sigma)^2 + \omega_d^2}$$

$$= \frac{1}{s} - \frac{s+\sigma}{(s+\sigma)^2+\omega_d^2} - \frac{\sigma}{(s+\sigma)^2+\omega_d^2}$$

$$= \frac{1}{s} - \frac{s+\sigma}{(s+\sigma)^2+\omega_d^2} - \frac{\zeta\cancel{\omega_n}\cdot\omega_n\sqrt{1-\zeta^2}}{(s+\sigma)^2+\omega_d^2}\,\frac{1}{\cancel{\omega_n}\sqrt{1-\zeta^2}}$$

$$= \underline{\frac{1}{s}} - \frac{s+\sigma}{(s+\sigma)^2+\omega_d^2} - \frac{\zeta}{\sqrt{1-\zeta^2}}\frac{\omega_d}{(s+\sigma)^2+\omega_d^2}$$

$$\Rightarrow \boxed{y(t) = 1(t) - \left[e^{-\sigma t}\cos\omega_d t + \left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)e^{-\sigma t}\sin\omega_d t\right]1(t)}$$

$$= 1(t) - e^{-\sigma t}\left[\underline{r\sin\theta}\cos\omega_d t + \underline{r\cos\theta}\sin\omega_d t\right]1(t)$$

$$= 1(t) - e^{-\sigma t}\, r\sin(\omega_d t + \theta)\,1(t)$$

$$r^2 = 1 + \frac{\zeta^2}{1-\zeta^2} = \frac{1-\cancel{\zeta^2}+\cancel{\zeta^2}}{1-\zeta^2}$$

$$\Rightarrow r = 1/\sqrt{1-\zeta^2}$$

$$\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\Rightarrow \theta = \tan^{-1}\left(\sqrt{\frac{1-\xi^2}{\xi^2}}\right)$$

$$y(t) = \left[1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t + \theta)\right] 1(t)$$