2 April

Bode plot:

$$(\sin \omega t)\,u(t) \longrightarrow \boxed{G(s)} \xrightarrow{y(t)}$$

Assume $G(s)$ is BIBO STABLE

$$y(t) = \mathcal{L}^{-1}\left[G(s)\,\frac{\omega}{s^2+\omega^2} \right]$$

$$= \mathcal{L}^{-1}\Bigg[\underbrace{\frac{A_0}{s+\lambda_0} + \frac{A_1}{s+\lambda_1} + \cdots + \frac{A_n}{s+\lambda_n}}_{\text{Transient}} + \underbrace{\frac{A}{s-j\omega} + \frac{A^*}{s+j\omega}}_{\text{Steady-state}} \Bigg]$$

$$y_{ss}(t) = |G(j\omega)| \sin(\omega t + \angle G(j\omega))$$

(Homework)

$$e^{j\omega t}u(t) \longrightarrow \boxed{G(s)} \xrightarrow{ss} G(j\omega)e^{j\omega t}$$

$$\mathcal{L} = \frac{1}{s-j\omega}$$

$$\mathcal{L}(y(t)) = \left[\frac{G(s)}{s-j\omega} \right]$$

$$\Rightarrow \mathcal{L}(y(t)) = \left(\frac{A_0}{s+\lambda_0} + \cdots + \frac{A_n}{s+\lambda_n} \right) + \frac{G(j\omega)}{s-j\omega}$$

$$\Rightarrow y_{ss}^{(1)}(t) = G(j\omega)e^{j\omega t}u(t)$$

$$y_{ss}^{(2)}(t) = \mathcal{L}^{-1}\left[\frac{G(s)}{s+j\omega}\right]_{ss}$$

$$= G(-j\omega)\, e^{-j\omega t}\, u(t)$$

$$e^{-j\omega t}u(t) \longrightarrow \boxed{G(s)} \xrightarrow{ss} G(-j\omega)\, e^{-j\omega t}\, u(t)$$

$$\underline{\underline{\sin\omega t}} = \frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right)$$

$\downarrow$ Superposition

$$y_{ss}(t) = \frac{1}{2j}\left[\underline{\underline{G(j\omega)}}\, e^{j\omega t} - G(-j\omega)\, e^{-j\omega t}\right]u(t)$$

$$= \frac{1}{2j}\left[M e^{j\phi} e^{j\omega t} - \underline{\underline{M e^{-j\phi}}}\, e^{-j\omega t}\right]u(t)$$

$$M := |G(j\omega)|, \quad \phi := \angle G(j\omega)$$

$$= \frac{M}{2j}\left[e^{j(\omega t+\phi)} - e^{-j(\omega t+\phi)}\right]u(t)$$

$$= \boxed{M \sin(\omega t+\phi)\, u(t).}$$

Bode magnitude plot:

$$G(s) = (s\tau + 1)$$

$$|G(j\omega)| = |j\omega\tau + 1| = (1+\omega^2\tau^2)^{\frac{1}{2}}$$

$$20\log_{10}|G(j\omega)|\ \text{dB} = 10\log(1+\omega^2\tau^2)\ \text{dB}$$

$(\omega\tau =: x)$

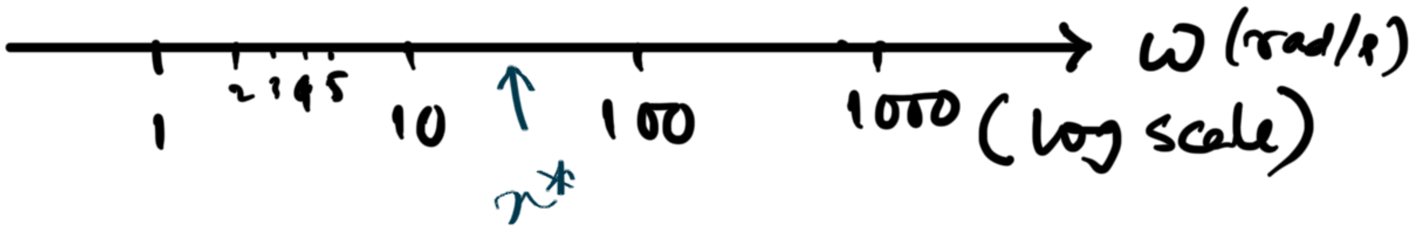
$$|G(j\omega)| = 10\log(1+x^2)\ \text{dB}.$$

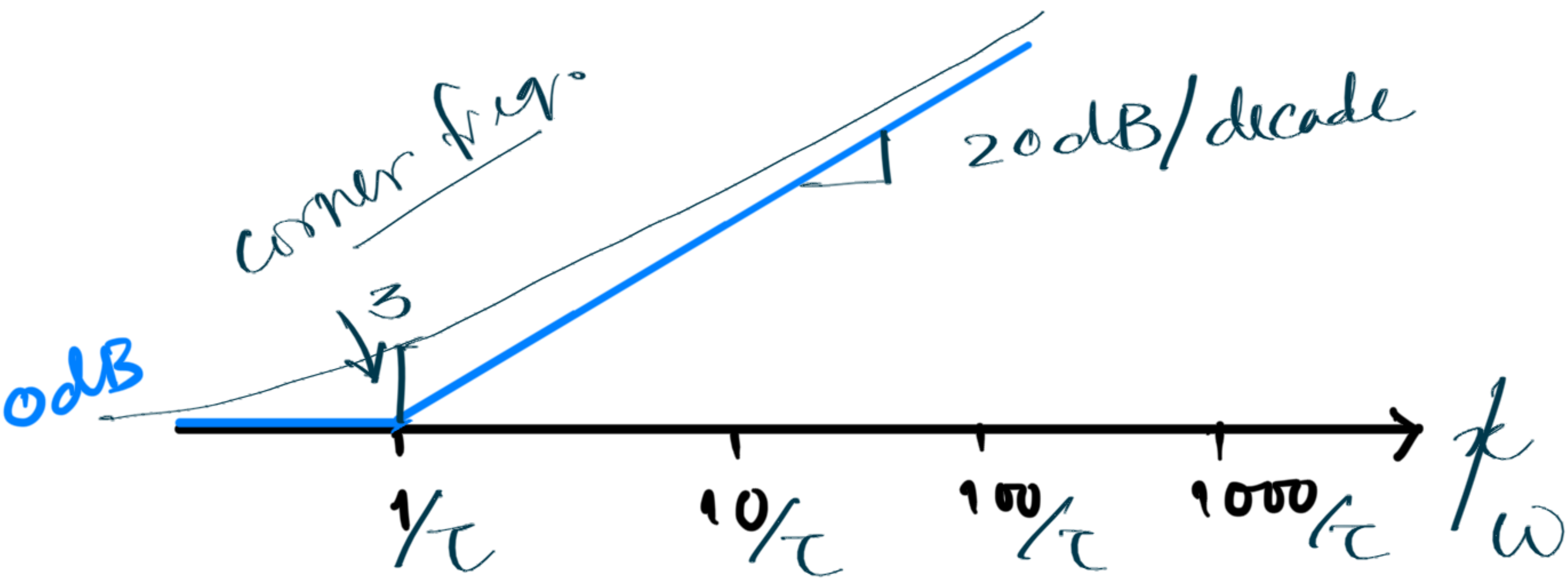
$$\approx 0 \quad \text{for} \quad x \ll 1$$

$$\approx 10\log x^2 = \underline{20\log x} \quad x \gg 1$$

$$|G(j\omega)|\Big|_{\omega = x^*} = 20\log x^*\ \text{dB}, \quad |G(j\omega)|\Big|_{\omega = 10x^*} = 20\log 10x^*\ \text{dB}$$

$$= 20\log x^* + 20\ \text{dB}$$





Bode asymptotic plot.

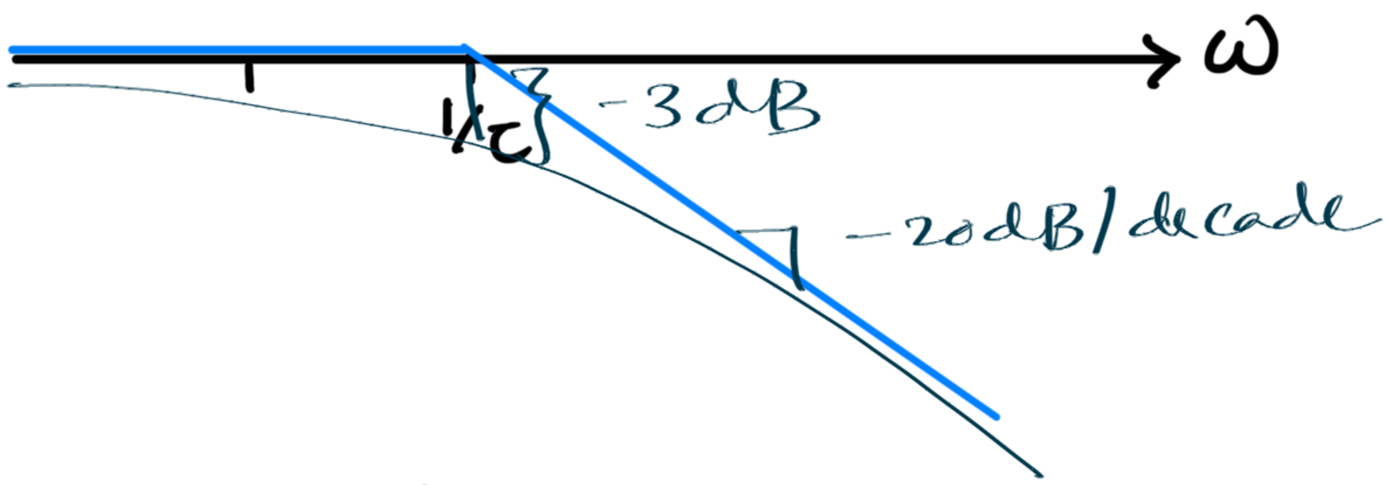
Exact value of $|G(j\omega)|$ @ $\omega = \frac{1}{\tau}$.

$$= 20\log(1+1)^{\frac{1}{2}} = 10\log 2.$$

$$= 3\text{dB}$$

$$G(s) = \frac{1}{(s\tau+1)}$$

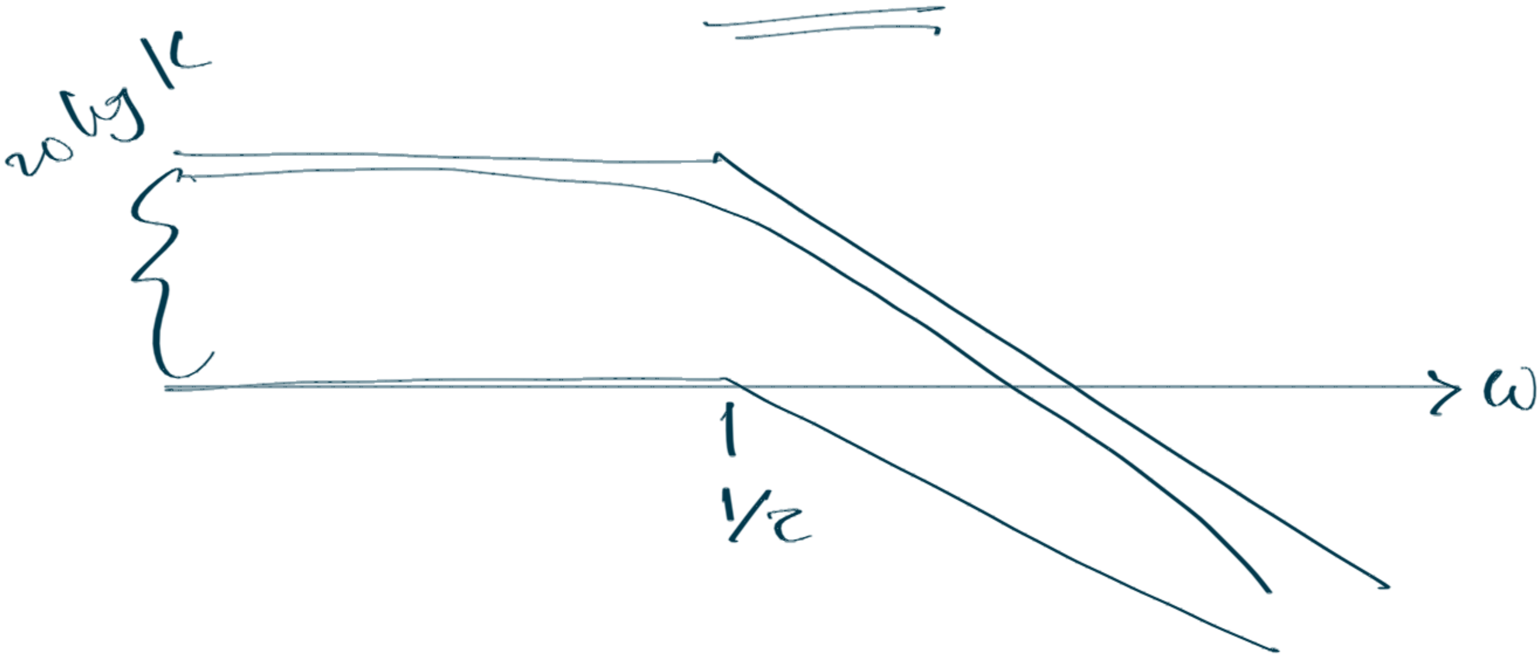
$$|G(j\omega)| = -10\log(1+\omega^2\tau^2)$$



$$G(s) = \frac{K}{s\tau + 1}$$

$$|G(j\omega)| = 20\log \frac{K}{(1+\omega^2\tau^2)^{\frac{1}{2}}}$$

$$= \underline{\underline{20\log K}} - 10\log(1+\omega^2\tau^2)$$



$$G(s) = \frac{K}{(s\tau_1+1)(s\tau_2+1)}$$