Feb 2: Time domain specs.

$$y(t) = \left[1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin(\omega_d t+\theta)\right]1(t)$$

Peak overshoot:

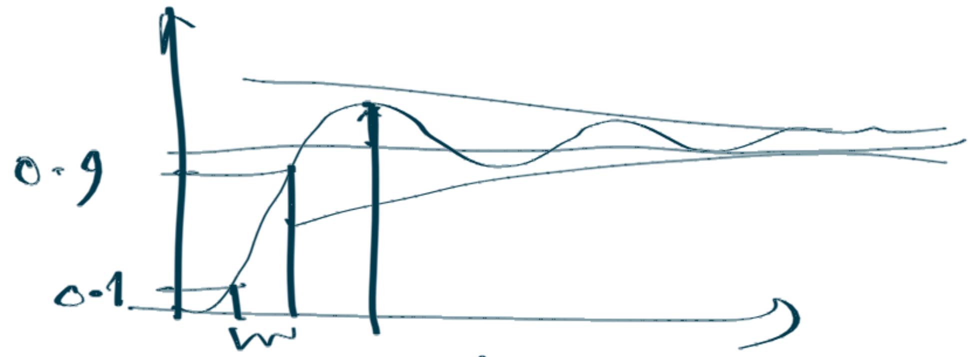
$$\frac{y_{max}-y(\infty)}{y(\infty)}\times 100\% = \%$$

Peak overshoot

%OS.

$y(\infty) = 1$

$y_{max} = ?$



$$\frac{dy}{dt} = \frac{d}{dt}\left[-\frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\sin(\omega_d t+\theta)\right]$$

$$= \frac{\xi\omega_n}{\sqrt{1-\xi^2}} \cancel{e^{-\xi\omega_n t}}\sin(\omega_d t+\theta) - \frac{\cancel{\omega_n\sqrt{1-\xi^2}}}{\cancel{\sqrt{1-\xi^2}}}\cancel{e^{-\xi\omega_n t}}\cos(\omega_d t+\theta) = 0$$

$$\Rightarrow \frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_d t+\theta) - \cos(\omega_d t+\theta) = 0$$

$\Rightarrow$

$$\rho\cos\varphi := \frac{\xi}{\sqrt{1-\xi^2}}, \quad \rho\sin\varphi := 1$$

$$\Rightarrow \rho\cos\varphi\sin(\omega_d t+\theta) - \rho\sin\varphi\cos(\omega_d t+\theta) = 0$$

$$\Rightarrow \cancel{\varphi} \sin(\omega_d t + \theta - \varphi) = 0$$

$$\tan\varphi = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\Rightarrow \varphi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \theta$$

$$\Rightarrow \boxed{\sin(\omega_d t) = 0}$$

$$\omega_d t = n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$$

$$\Rightarrow t_{peak} = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}.$$

$$Y_{max} = Y(t_{peak})$$

$$\Rightarrow Y_{max} = \left[1(t) - \left[e^{-\sigma t}\cos\omega_d t + \left(\frac{\xi}{\sqrt{1-\xi^2}}\right)e^{-\sigma t}\sin\omega_d t\right]1(t)\right]_{t = \frac{\pi}{\omega_d}}$$

$$= 1 - e^{-\xi\omega_n\frac{\pi}{\omega_d}}\left[-1 + 0\right]$$

$$= 1 + e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\boxed{\%OS = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100}$$

Settling time: $t_s :=$ is such that

$$e^{-\zeta\omega_n t_s} = 0.02$$

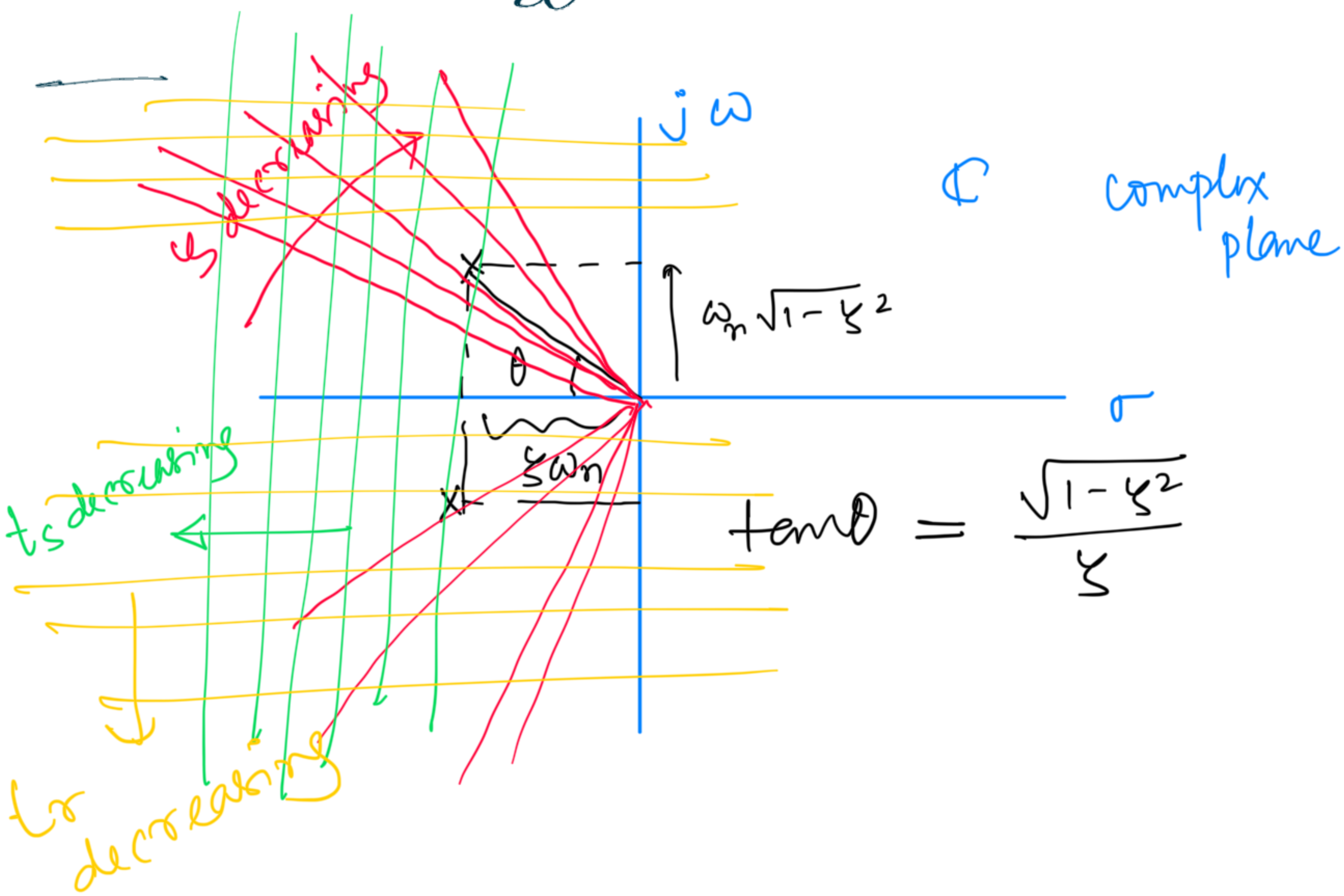
$$\Rightarrow \cancel{-}\zeta\omega_n t_s = \ln 0.02$$

$$= \cancel{-}3.912$$

$$\Rightarrow \boxed{t_s = \frac{3.912}{\zeta\omega_n}.}$$

Rise time:

$$t_r = \frac{4}{\omega_d}\ ?$$ **Homework**



$$\tan\theta = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

Convolution: $u(t)$, $h(t)$

$$(u * h)(t) := \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau$$

If input $u(t) = e^{\lambda t}$ (defined over $\mathbb{R}$)

$$y(t) = \int_{-\infty}^{\infty} e^{\lambda\tau} h(t-\tau)\, d\tau$$

$$t - \tau =: \xi .$$

$$= -\int_{\infty}^{-\infty} e^{\lambda t - \lambda\xi} h(\xi)\, d\xi$$

$$= e^{\lambda t} \int_{-\infty}^{\infty} e^{-\lambda\xi} \underline{h(\xi)}\, d\xi .$$

Causal

$$= e^{\lambda t} \int_{0}^{\infty} e^{-\lambda\xi} h(\xi)\, d\xi$$

$$= \boxed{e^{\lambda t} H(\lambda)}$$

$$e^{\lambda t} \longrightarrow \boxed{} \longrightarrow H(\lambda) e^{\lambda t}$$