# Feb 5:

## Cascade



$u_1 \longrightarrow \boxed{G_1(s)} \xrightarrow{Y_1 = u_2} \boxed{G_2(s)} \xrightarrow{Y_2}$

$$G_1 G_2$$

## Feedforward

$$u = u_1$$



$$G_1 + G_2$$

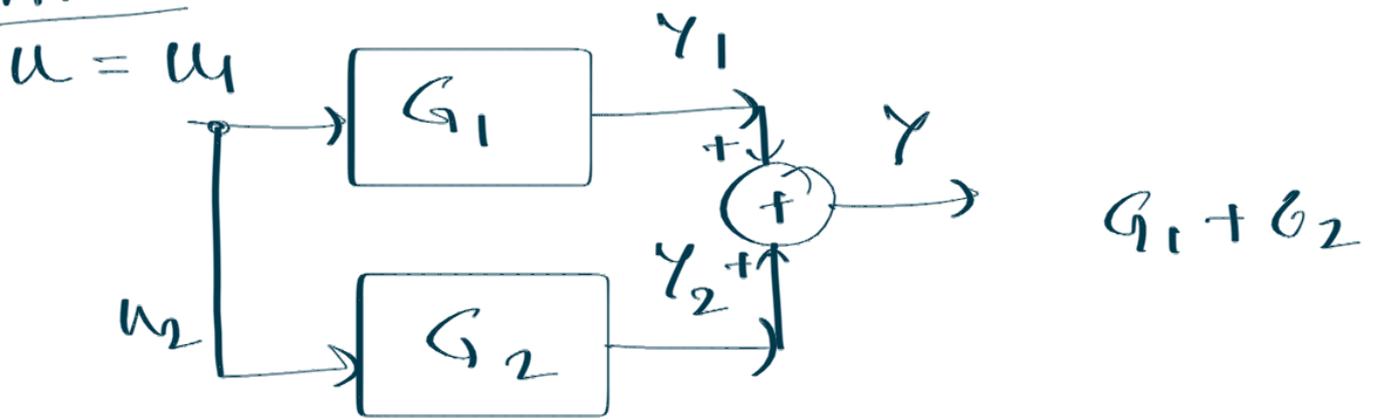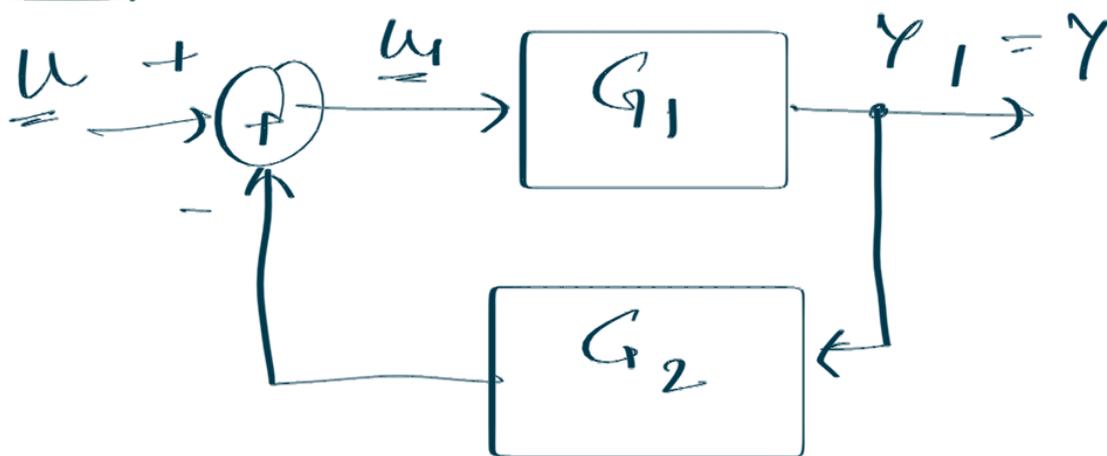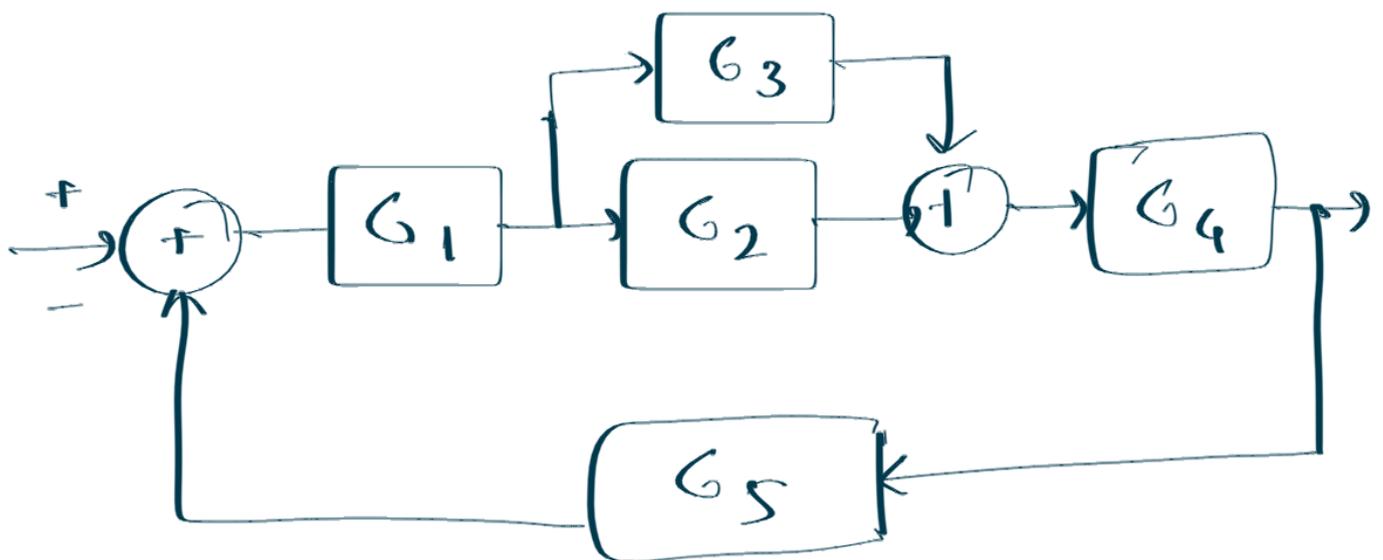## Feedback:



$$G_1(u - G_2 Y) = Y$$

$$\Rightarrow \quad G_1 u - G_1 G_2 Y = Y$$

$$\Rightarrow \quad (1 + G_1 G_2) Y = G_1 u$$

$$\Rightarrow \quad \frac{Y(s)}{U(s)} = \frac{G_1(s)}{1 + G_1(s) G_2(s)}$$



Homework.

## Stability

BIBO : Bounded input bounded output.

$$u : [0, \infty) \to \mathbb{R} \quad \exists M > 0 \text{ s.t.}$$
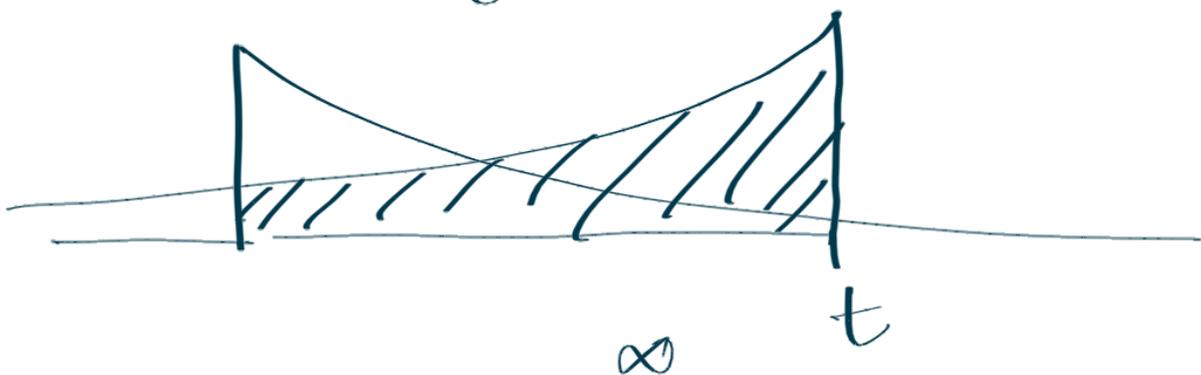
$$|u(t)| \leq M \quad \forall t \in [0, \infty)$$

$u$ is said to be bounded.

LTI systems :

$$|y(t)| = \left| \int_0^t u(\tau) \, h(t-\tau) \, d\tau \right|$$

$$\leq \int_0^t |u(\tau)| \, |h(t-\tau)| \, d\tau$$

$$\leq M \int_0^t |h(t-\tau)| \, d\tau$$



$$\leq M \int_0^\infty |h(\tau)| \, d\tau$$

$$\int_0^\infty |h(\tau)| \, d\tau \leq \gamma \implies \text{BIBO}$$

STABLE.

Consider t.f. $G(s)$. Then

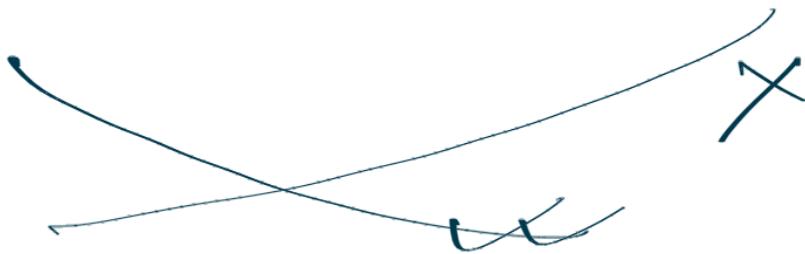$$h(t) = \mathcal{L}^{-1} \left( G(s) \right)$$

$$G(s) = p(s) + \frac{A_0}{s} + \frac{A_1}{s+\lambda_1} + \cdots +$$

Assume $\boxed{p(s) = 0}$ $\quad \frac{A_n}{s+\lambda_n} + \frac{B_1}{(s+\lambda_1)^2} + \cdots$

$$G(s) = \sum \frac{A_i}{(s+\lambda_i)^{k_i}}_{\mathcal{U}}$$

$$\lambda_i = \sigma_i + j\omega_i$$

$$\mathcal{L}^{-1}\left(\frac{A_i}{s + \sigma_i + j\omega_i}\right) = A_i e^{-\sigma_i t}_{\mathcal{U}} \underbrace{(e^{-j\omega_i t})}$$

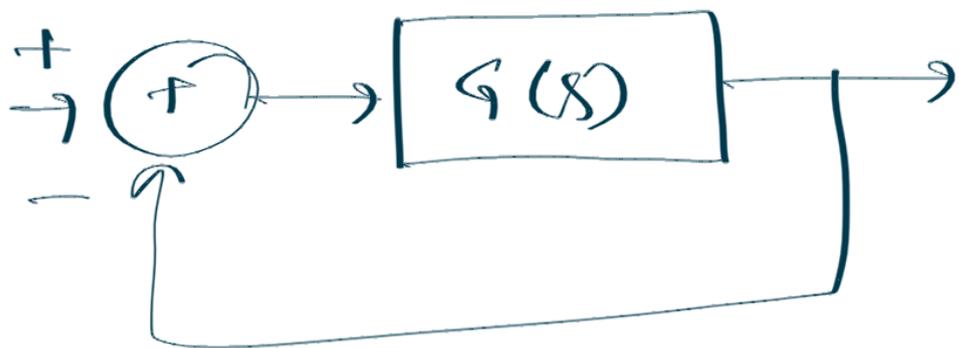$G(s)$. Poles in $\overline{\mathbb{C}^-}$ (open left-half of $\mathbb{C}$).

$\Rightarrow$ BIBO stable.

TF: $G(s)$ is BIBO stable

$(\Longleftrightarrow)$ 1. $G(s)$ is proper (deg Num $\leq$ deg Den)

2. Poles of $G$ are in $\overline{\mathbb{C}^-}$.

$$G(s) = \frac{K}{s^3 + 6s^2 + 11s + 6}$$

$$\Rightarrow CLTF = \cfrac{G}{1+G}$$

$$= \cfrac{\dfrac{K}{s^3+6s^2+11s+6}}{1+\dfrac{K}{s^3+6s^2+11s+6}}$$

$$= \cfrac{K}{s^3+6s^2+11s+(6+K)}$$



$s^3 :$   1    11

$s^2 :$   (6)    6+K

$s^1 : \dfrac{6\times 11 - 1\times(6+K)}{6}$    0

$\left(= 10 - \dfrac{K}{6}\right)$

$s^0 :$   6+K

That is      Routh-Hurwitz table.

$s^3 :$   1 ✓    11

$s^2 :$   6 ✓    6+K

$s : 10 - \dfrac{K}{6}$    0

$1 : 6+K$ ✓