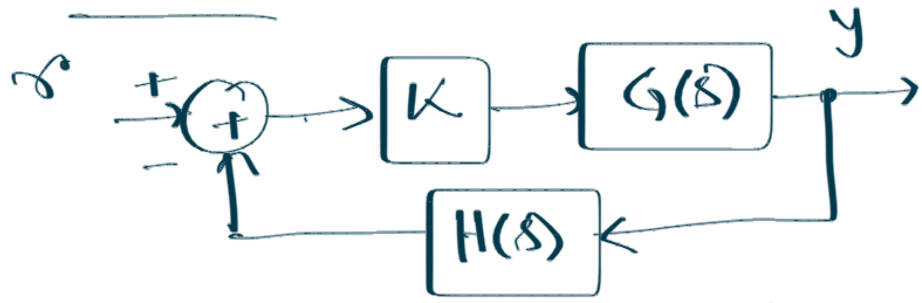
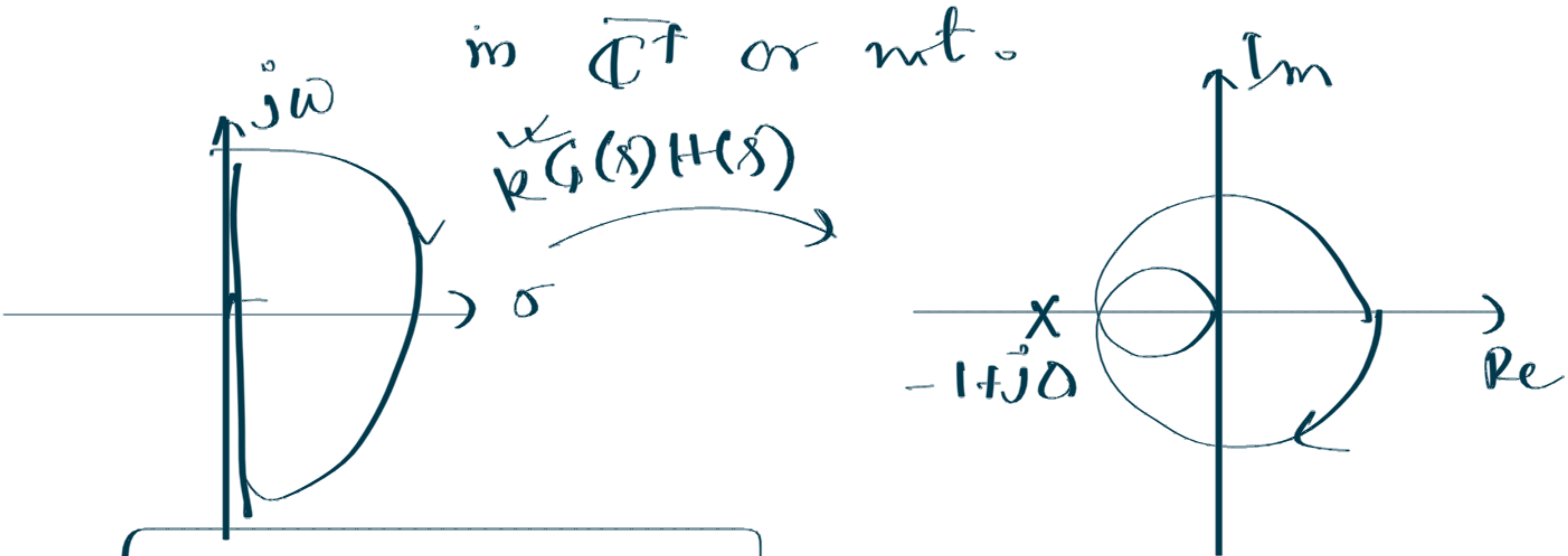
5 March:



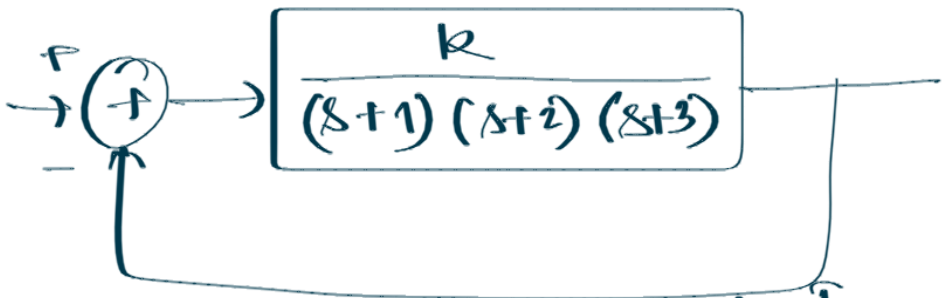

$$CLTF = \frac{K\,G(s)}{(1 + K\,G(s)\,H(s))}$$

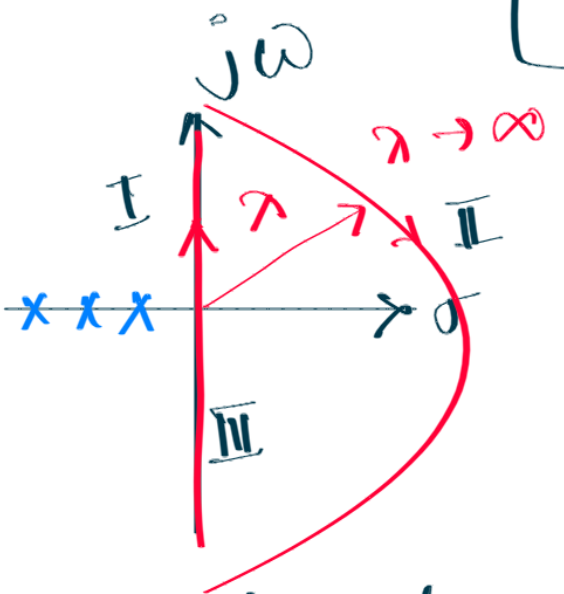
Nyquist plot tells us whether $1 + K\,G(s)\,H(s)$ has zeros in $\mathbb{C}^{+}$ or not.

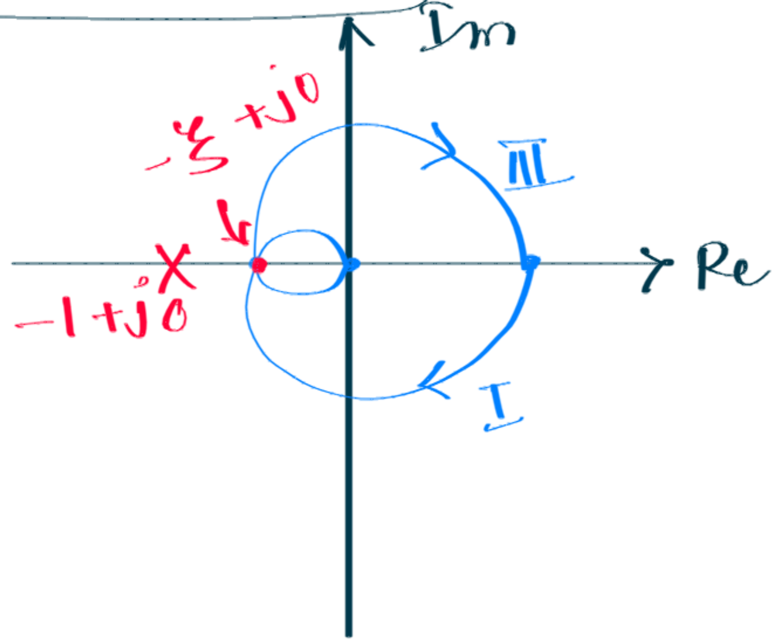



$$\boxed{N = \underline{P} - \underline{Z}}$$

Example:




Nyquist contour

segment I: $j\omega$, $\omega = 0$ to $\omega = \infty$

@ $\omega = 0$ $\quad kG(s) = \dfrac{k}{6}$

@ $\omega = \infty$ $\quad kG(s) = 0\angle{-270^\circ}$

Segment II : $s = \lambda e^{j\varphi}$ where $\lambda \to \infty$

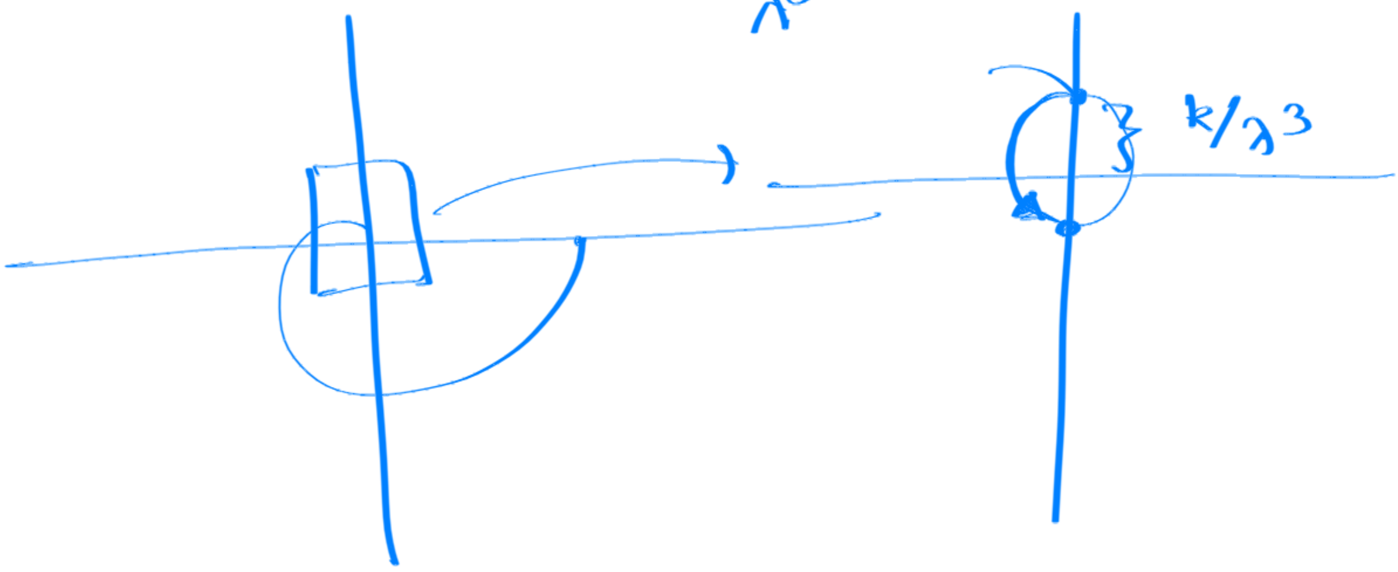
$\varphi$ goes from $90°$ to $-90°$ in

CW motion.

$$kG(s)\Big|_{s=\lambda e^{j\varphi}} = \frac{k}{(\lambda e^{j\varphi}+1)(\lambda e^{j\varphi}+2)(\lambda e^{j\varphi}+3)}$$

$$\approx \frac{k}{\lambda^3 \; e^{j3\varphi}}$$

$$= \frac{k}{\lambda^3} e^{-j3\varphi}$$




Segment III : $s = j\omega$ $\omega = -\infty$ to $0$.

Mirror image of the plot for

segment I (about the real axis)

[$\because$ The TF has only real coefficients]

Real axis intersection @ $-\xi + j0$.

$\therefore$ CL stability $\Leftrightarrow$ $\xi < 1$.

To find the real axis intersection

(value of $\xi$).

$$G(s) = \frac{K}{(s+1)(s+2)(s+3)} = \frac{K}{s^3+6s^2+11s+6}$$

$$G(j\omega) = \frac{K}{-j\omega^3 - 6\omega^2 + j11\omega + 6}$$

$$= \frac{K}{(-6\omega^2+6) + j\underline{(-\omega^3+11\omega)}}$$

At real axis intersection

$$-\omega^3 + 11\omega = 0$$

$$\Rightarrow \omega = \pm\sqrt{11}$$

The location of the real axis intersection is then given by

$$G(j\sqrt{11}) = \frac{K}{-6(\sqrt{11})^2+6}$$

$$= \frac{K}{-66+6}$$

$$= -\frac{K}{60}.$$

$$\Rightarrow \boxed{\xi = \frac{K}{60}}$$

CL Stability $\Leftrightarrow$ $\xi < 1$

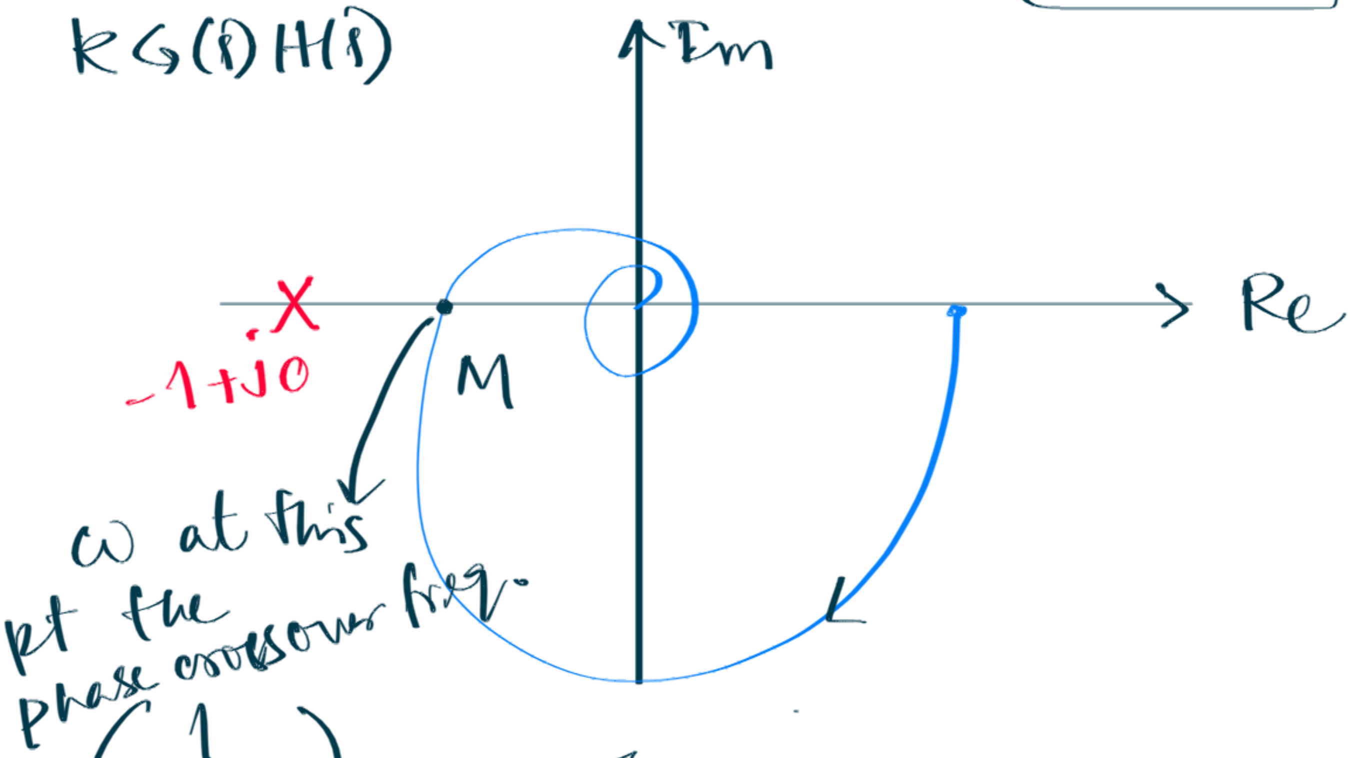
$$\Rightarrow \frac{K}{60} < 1$$

$$\Rightarrow K < 60$$

## (B) Stability margin:

We assume that the Loop TF is stable

Then CL stability $\Leftrightarrow$ $\boxed{N = 0}$

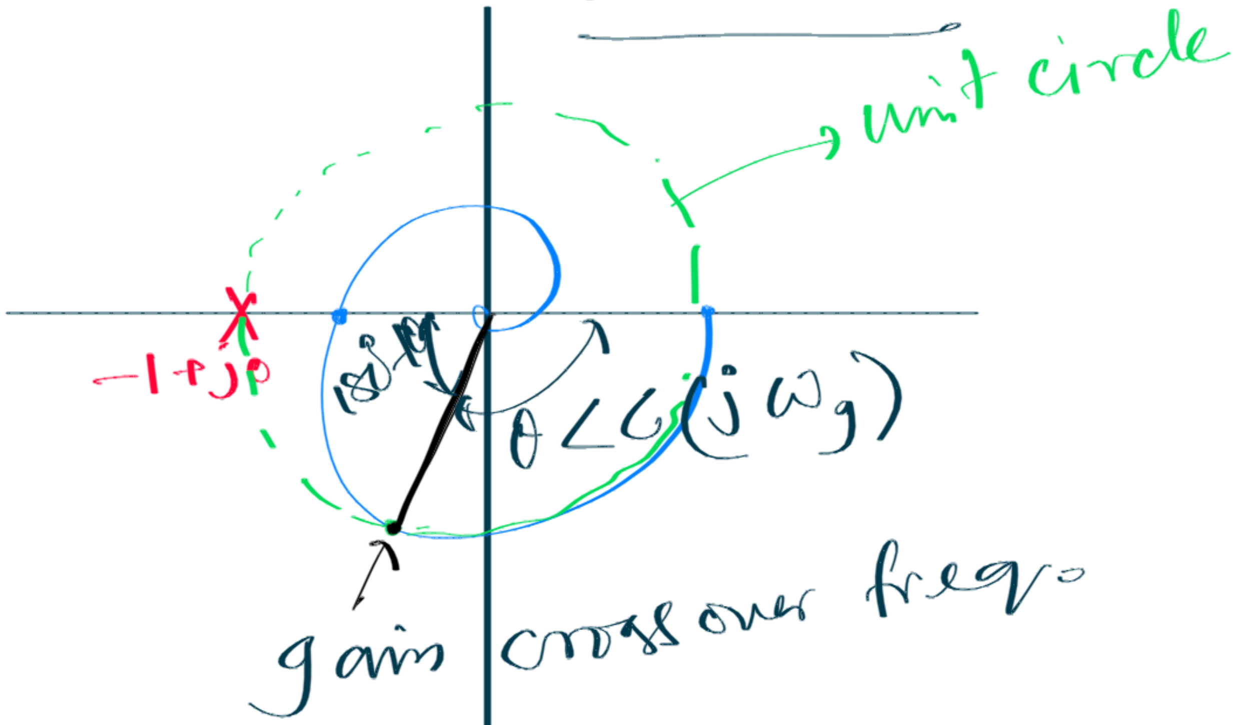



$\left(\frac{1}{M}\right)$ Gain margin

$$GM = (-20 \log M) \text{ dB}$$

$$= (-20 \log |G(j\omega_p)|)$$

$\omega_p$ is the PCF.




GCF.

$$|G(j\omega_g)| = 1$$

$$\text{Phase margin} = 180^\circ - |\angle G(j\omega_g)|.$$