

6 April:

$$G(s) = \frac{(s+a)(s+b)}{(s+c)(s+d)(s+e)}$$

$$= \left(\frac{ab}{cde}\right) \frac{\left(\frac{s}{a}+1\right)\left(\frac{s}{b}+1\right)}{\left(\frac{s}{c}+1\right)\left(\frac{s}{d}+1\right)\left(\frac{s}{e}+1\right)}$$

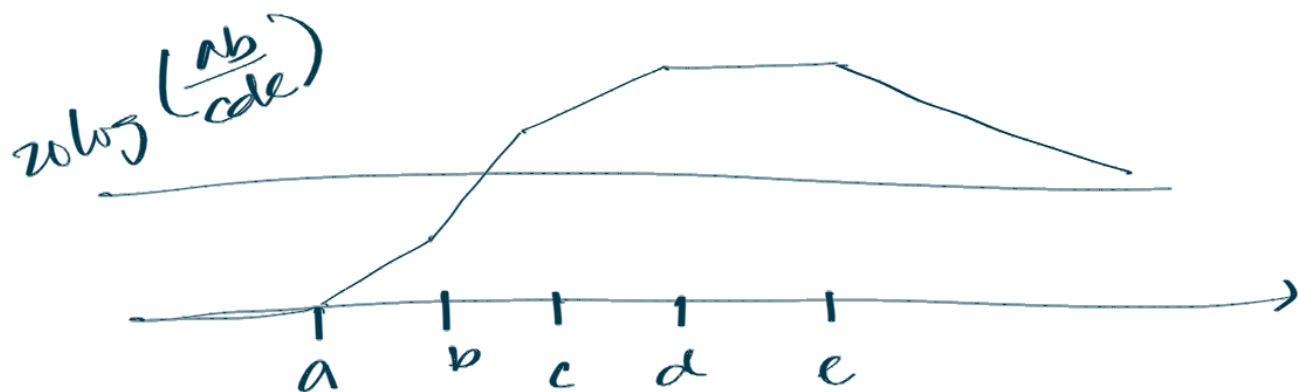
$$20 \log |G(j\omega)| \text{ dB} = 20 \log \left| \frac{ab}{cde} \right| +$$

$$10 \log \left( 1 + \left( \frac{\omega}{a} \right)^2 \right) + 10 \log \left( 1 + \left( \frac{\omega}{b} \right)^2 \right)$$

$$- 10 \log \left( 1 + \left( \frac{\omega}{c} \right)^2 \right) - 10 \log \left( 1 + \left( \frac{\omega}{d} \right)^2 \right)$$

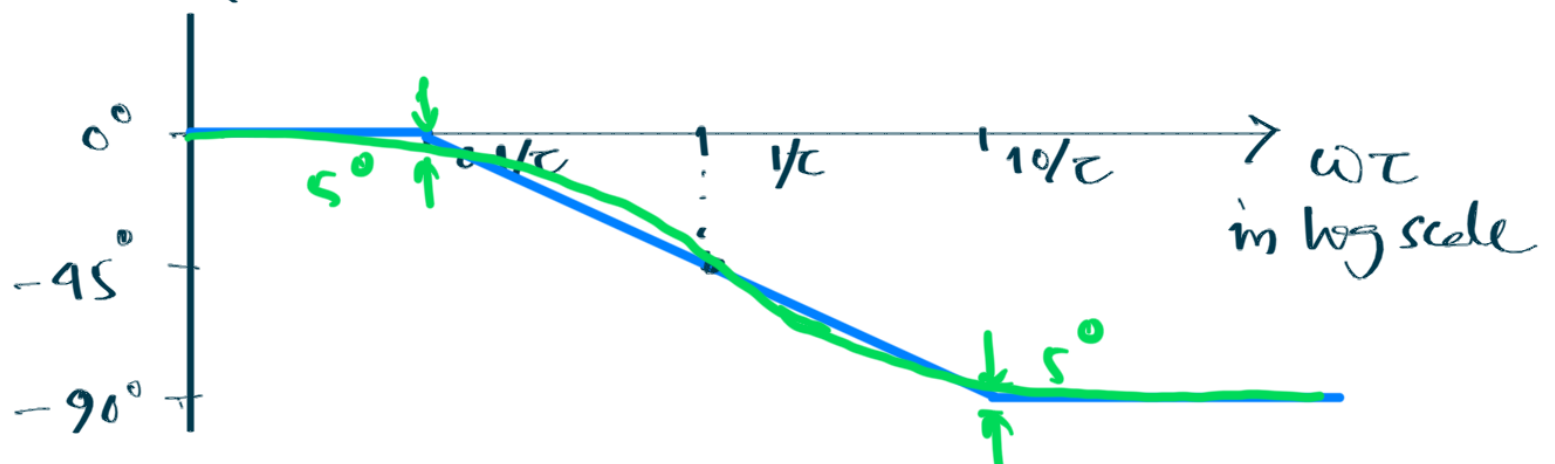
$$- 10 \log \left( 1 + \left( \frac{\omega}{e} \right)^2 \right)$$

$$0 < a < b < c < d < e$$



Bode angle/phase plot:

$$G(s) = \left( \frac{1}{s\tau + 1} \right) \Rightarrow \angle G(j\omega) = -\tan^{-1} \frac{\omega\tau}{1}$$



$$-\tan^{-1} \frac{x}{1} \Big|_{x=1} = -45^\circ$$

$$-\tan^{-1} x \Big|_{x=0.1} \approx -5^\circ$$

$$-\tan^{-1} x \Big|_{x=10} \approx -85^\circ$$

$$x > 1$$

$$\tan^{-1} x - \tan^{-1} 1 = \tan^{-1} \frac{x-1}{1+x}$$

$$\begin{aligned} \tan^{-1} 1 - \tan^{-1} \frac{1}{x} &= \tan^{-1} \frac{1 - \frac{1}{x}}{1 + \frac{1}{x}} \\ &= \tan^{-1} \frac{x-1}{1+x} \end{aligned}$$

Example:

$$G(s) = \frac{(s+100)^3}{s^3 + 6s^2 + 11s + 6}$$

