[4] $p(s) = s^6 + s^5 + 4s^4 + 3s^3 + 2s^2 + s + 1$

R-H table:

s^6 :	1	4	2	1
s^5 :	1	3	1	—
s^4 :	$\frac{1\times4 - 1\times3}{1} = 1$	$\frac{1\times2 - 1\times1}{1} = 1$	$\frac{1\times1}{1} = 1$	
s^3 :	2	0	—	
s^2 :	1	1		
s :	-2	—		
1 :	1			

Inference: 4 roots in $\mathbb{C}^-$
2 roots in $\mathbb{C}^+$ ← No of sign changes in the 1st col of the RH table.
0 roots in $j\mathbb{R}$

Singular cases:

– Case 1: 1st entry in a row = 0.

$\Rightarrow$ Proceed by replacing 0 by ε and then take limits wrt s.

$$s^3 + 6s^2 + 11s + 66 = p(s)$$

s^3 :	1	11
s^2 :	$\cancel{6}$ 1	$\cancel{66}$ 11
s^1 :	$\cancel{0}$ ε	
s^0 :	11	

← $(s^2 + 11)$ is a factor of $p(s)$

$\varepsilon \downarrow 0 \Rightarrow$ No sign change

$\varepsilon \uparrow 0 \Rightarrow$ 2 sign changes.

Case 2: Entire row = 0.

$p(s) = s^5 + s^4 + s^3 + s^2 + s + 1$

$$\begin{array}{lccc} s^5: & 1 & 1 & 1 \\ s^4: & 1 & 1 & 1 \rightarrow \underline{(s^4 + s^2 + 1)} \\ s^3: & 0 & 0 & 0 \end{array}$$

$$\begin{array}{r|l} & \quad (s+1) \\ \hline s^4+s^2+1 & s^5 + s^4 + s^3 + s^2 + s + 1 \\ & s^5 \qquad\ \ + s^3 \qquad\ \ + s \\ \hline & \qquad s^4 + s^2 + 1 \\ & \qquad s^4 + s^2 + 1 \\ \hline & \qquad\quad \times \end{array}$$

$$s^4 + s^2 + 1 \xrightarrow{\frac{d}{ds}} 4s^3 + 2s + 0$$

$$\begin{array}{lccc} s^4: & 1 & 1 & 1 \\ s^3: & \cancel{0} & \cancel{0} & \cancel{0} \\ & 4 & 2 & \\ s^2: & \frac{1}{2} & 1 & \\ s^1: & -6 & & \\ s^0: & 1 & & \end{array}$$

What if ε method does not work.

$$p(s) = s^2 + s + 1 \quad \leftarrow \lambda$$

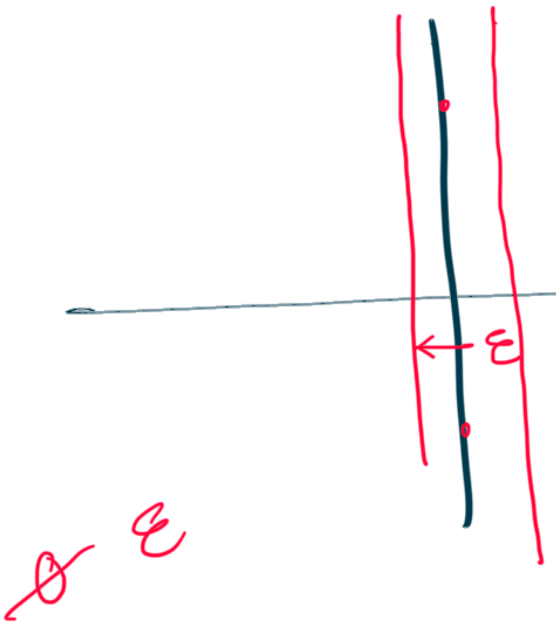
$$p_\varepsilon(s) := p(s-\varepsilon) = (s-\varepsilon)^2 + (s-\varepsilon) + 1$$

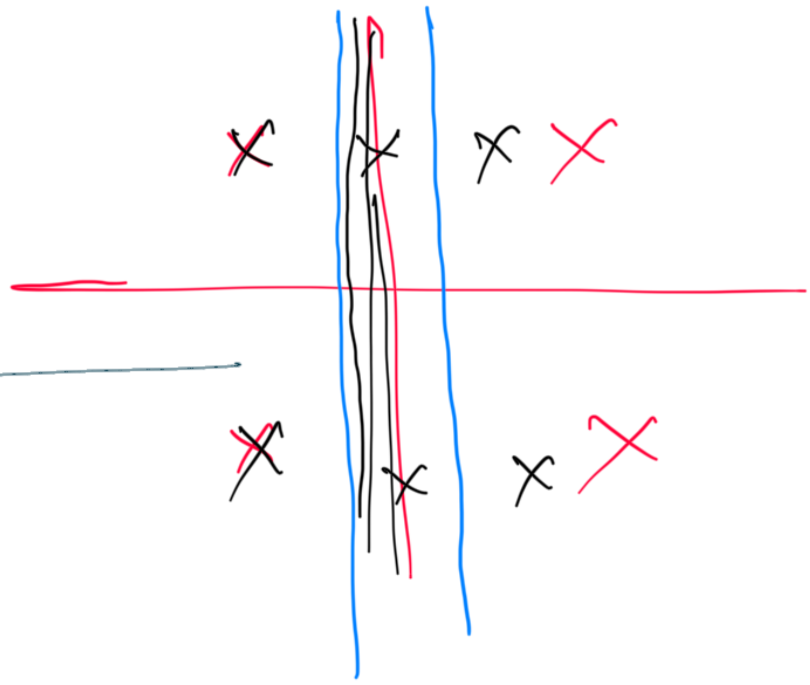
$\lambda + \varepsilon$

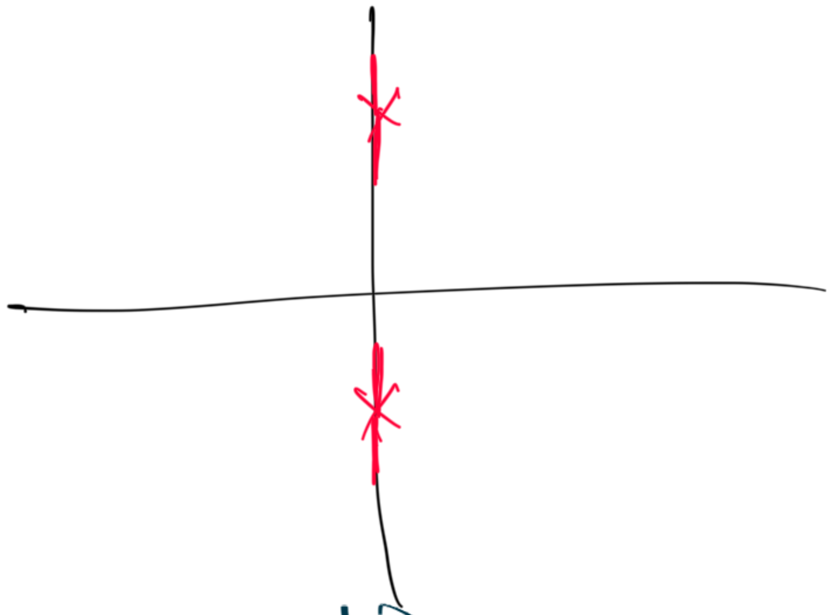
$$= s^2 - 2\varepsilon s + \varepsilon^2 + s - \varepsilon + 1$$

$$= s^2 + \underline{(1-2\varepsilon)} s + (\underline{\varepsilon^2 - \varepsilon + 1})$$

$$\begin{array}{lcc} s^2: & 1 & \varepsilon^2 - \varepsilon + 1 \\ s: & 1 - 2\varepsilon & \end{array}$$







$p(s)$

$q(s) :=$ num $p\left(\frac{1}{s}\right)$

$s^2+s+1 \longrightarrow \frac{1}{s^2}+\frac{1}{s}+1$

$= \frac{s^2+s+1}{s^3}$