EE659: A First Course in Optimization

Course Overview

Course Objectives

The objectives of this course are to identify and formulate optimization problems, and to master the theory behind solving such problems.

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Location: IIT Bombay

Core Curriculum Modules

Module ID	Module Title	Primary Topics
Module 1	**Basics**	Linear algebra, matrix theory, calculus fundamentals, Taylor's theorem, convergence criteria, and limit points.
Module 2	**Unconstrained Optimization**	Necessary and sufficient conditions for optimality, local vs. global minimum identification. Convexity and its implication.
Module 3	**Algorithms for Unconstrained Optimization**	Steepest descent, line search (exact and

Module ID	Module Title	Primary Topics
		inexact/Wolfe/backtracking), convergence analysis, Zoutendijk's & Luenberger's Theorems, Newton's method.
Module 4	**Trust Region Algorithms**	Trust region methodology, Cauchy point, dogleg methods, 2D subspace minimization, and global convergence analysis.
Module 5	**Conjugate Gradient Method**	Linear CG method, convergence rates, preconditioning techniques, and non-linear CG variants.
Module 6	**Quasi-Newton Methods**	BFGS and SR1 methods (descriptions only; convergence analysis for self-study).
Module 7	**Theory of Constrained Optimisation**	Examples, difficulties, quadratic cost with linear/affine constraints, optimality conditions (1st/2nd order), Farkas' Lemma, Lagrange multipliers, duality.
Module 8	**Linear Programming**	Weak/strong duality, geometry of the feasible set, simplex method. Interior point methods.

Detailed Syllabus Content

Module 1: Foundations

- **Linear Algebra & Matrix Theory:** Review of subspaces, eigenvalues, and positive definiteness.
- **Calculus Review:** Multi-variable calculus, gradients, and Hessians.
- **Taylor's Theorem:** Expansion for multi-variable functions and error bounds.
- **Sequences:** Concepts of convergence and limit points in $\mathbb{R}^n$.

Module 2: Optimality Conditions

- **First-Order Necessary Conditions:** Gradient-based criteria.
- **Second-Order Conditions:** Necessary and sufficient conditions involving the Hessian.
- **Global Optimality:** Conditions under which local minima are global (convexity).

Module 3: Iterative Descent Methods

- **Steepest Descent:** Implementation and performance characteristics.
- **Line Search Strategies:**
  - Exact line search.
  - Inexact methods: Wolfe conditions and backtracking.
- **Theoretical Foundations:**
  - Zoutendijk's Theorem.
  - Luenberger's Theorem for rate of convergence.
- **Newton's Method:** Derivation, implementation, and local convergence analysis.

Module 4: Trust Region Frameworks

- **Methodology:** Introduction to the trust region subproblem.
- **Cauchy Point:** Computation and its role in ensuring global convergence.
- **Minimization Techniques:** Dogleg method and 2D subspace minimization.
- **Convergence:** Analysis of the trust region radius and step acceptance.

Module 5: Conjugate Gradient Methods

- **Linear CG:** Derivation for quadratic forms and expanding subspace property.
- **Convergence:** Rate of convergence and dependence on condition number.
- **Preconditioning:** Improving CG performance through preconditioning matrices.

- **Non-Linear CG:** Extension to general objective functions (Fletcher-Reeves, Polak-Ribière).

Module 6: Quasi-Newton Methods

- **BFGS and SR1 methods:** Descriptions provided; convergence analysis is reserved for self-study.

Module 7: Theory of Constrained Optimisation

- Examples and difficulties in constrained problems.
- Special case: Quadratic cost with linear/affine constraints.
- Optimality Conditions: First-order necessary, Farkas' Lemma, second-order necessary and sufficient conditions.
- Lagrange multipliers and Duality.

Module 8: Linear Programming

- Duality: Weak and strong duality theory.
- Geometry: Properties of the feasible set.
- Methods: The Simplex method.
- Interior point methods: Outline, the central path, central path neighbourhood and path following.

Reference Materials

- **Primary Text:** Numerical Optimization by Jorge Nocedal and Stephen J. Wright
- **Primary Reference:** Convex Optimization by Stephen Boyd and Lieven Vandenberghe
- **Course Webpage:** Place