Behavioral Theory of Systems (EE 714)

Problem Set 5

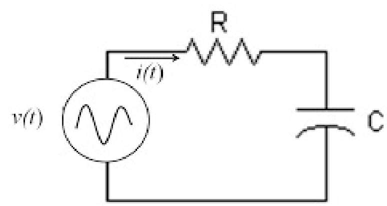

Figure 1: RC Circuit

1. Consider the circuit in Figure 1. Write down $\mathfrak{B} = \ker R(\frac{d}{dt})$. Check that it's of the form $R_0 + \xi R_1$. Verify $\mathfrak{B}$ is *Markovian* by applying the definition of *Markovianity.*

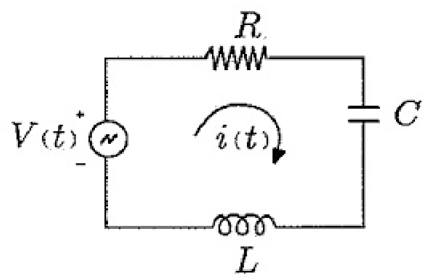

Figure 2: RLC Circuit

2. Consider the circuit in Figure 2. Write down $\mathfrak{B} = \ker R(\frac{d}{dt})$ and $\mathfrak{B}_{\text{full}}$ where $(V, i, V_c) \in \mathfrak{B}_{\text{full}}$. Verify $\begin{bmatrix} i \\ V_c \end{bmatrix}$ satisfies *axiom of state.*

3. This exercise shows the importance of going modulo the equation module $\mathcal{R}$. Consider the following kernel representation matrix:

$$R(\xi) = \begin{bmatrix} 1 & \xi^2 \\ 0 & \xi \end{bmatrix}.$$

Suppose $X(\xi)$ is the matrix obtained by doing shift and cut on $R(\xi)$. Show that the rows of $X(\xi)$ are linearly independent over $\mathbb{R}$. Further, show that the state map defined by $X(\xi)$ is not minimal because the resulting state space is not trim. What is the dimension of the minimal state space?

Come up with two more examples, where an $\mathbb{R}$-linearly independent set of generators of the vector space Ξ (the space generated by the rows of $X(\xi)$ that is obtained from $R(\xi)$ by shift and cut) does *not* give a minimal state space.

4. Consider the following behaviors in *kernel representation* $\mathfrak{B} = \{w : R(\frac{d}{dt})w = 0\}$ where $R(\frac{d}{dt})$ are given by

   (a) $\begin{bmatrix} -1+\xi & 1-\xi+\xi^2 \\ -1-\xi+\xi^2 & 1+\xi-\xi^2+\xi^3 \end{bmatrix}$

   (b) $\begin{bmatrix} \xi^2+\xi+1 & \xi^2+\xi+1 & \xi^2+3\xi+3 \\ 2\xi+1 & \xi^2+2\xi & \xi^2+4\xi+2 \end{bmatrix}$.

   (c) $\begin{bmatrix} \xi^2+3\xi+2 & \xi+3 & \xi^2+5\xi+6 \\ \xi+1 & \xi+2 & \xi^2+3\xi+2 \end{bmatrix}$

   Answer the following questions for each of the behaviors mentioned above.

   (a) How many input-output partitions do these behaviors have?

   (b) How many input-output partitions correspond to a minimal state space representation?

   (c) Find a minimal state space representation using shift and cut.

   (d) Find an i/s/o representation.

5. We have learned the axiom of state. With this notion, prove that the behavior given by

$$\mathfrak{B} = \{(x,u) \in \mathfrak{L}_1^{\text{loc}} \mid \dot{x} = Ax + Bu\}$$

   is behaviorally controllable (that is, patchability of trajectories) if and only if the pair (A, B) is state controllable (that is, reachability from every initial state to every final state in finite time).