Nonlinear dynamical systems (3-0-0-6)

Preamble

In many engineering applications, the to-be-controlled plant turns out to be nonlinear, *i.e.*, described by ordinary nonlinear differential equations. For example, electrical cricuits with nonlinear active/passive elements, electric drives, flexible beams, robotic manipulators etc. The huge ensemble of various types of nonlinearities makes determination of general control strategies for larger classes of nonlinear systems a challenging task. However, the mathematical theory of analyses of nonlinear dynamical systems is very reach and has a long and fascinating history. One of the most significant contribution to the theory was Lyapunov's pioneering work on stability theory. Over the past few decades a significant development has been carried out surrounding Lyapunov theory, and state-of-the-art methods for control of nonlinear systems have been devised. The aim of this course is twofolds: first, to get a thorough mathematical basis for analyzing nonlinear dynamical systems. Secondly, to go through various methods for control of such systems, and to understand their working principles.

Syllabus

Introduction: state-space representation of dynamical systems, phase-portrait of 2^{nd} order systems, types of equilibrium points: stable/unstable node, stable/unstable focus, saddle; **Existence and uniqueness of solutions:** Lipschitz continuity, Picard's iteration method, proof of existence and uniqueness theorem, continuous dependence of solutions on initial conditions; **Features of nonlinear dynamical systems:** multiple disjoint equilibrium points, limit cycles, Bendixson criterion, Poincaré-Bendixson criterion, negative resistance circuits for designing stable oscillators; **Linearization:** linearization around an equilibrium point, Validity of linearization: hyperbolic equilibrium points, linearization around a solution; **Stability analysis:** Lyapunov stability of autonomous systems, Lyapunov theorem of stability, Lyapunov stability applied to linear systems: Lyapunov equation/inequality, converse theorems of Lyapunov theorem, input/output stability of non-autonomous systems, $\mathcal{L}$-stability; **Control of nonlinear systems:** describing functions method, passivity theorem, small gain theorem, Kalman-Yakubovich-Popov lemma, circle/Popov criteria, methods of integral quadratic constraints and quadratic differential forms for designing stabilizing linear controllers, Aizermann conjecture, multiplier techniques.

Books

Textbook

1. H.K. Khalil, *Nonlinear systems*, Prentice Hall, 2002.

2. M. Vidyasagar, *Nonlinear systems analysis*, Society of Industrial and Applied Mathematics, 2002.

Reference books

1. H. Márquez, *Nonlinear control systems: analysis and design*, Wiley, 2003.

2. A. Isidori, *Nonlinear control systems*, Springer, 1995.

3. F. Verhulst, *Nonlinear differential equations and dynamical systems*, Springer, 1990.