Distributed Topology Control of Wireless Networks

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Abstract— **We propose and analyze a distributed control law that will maintain prescribed local properties of a wireless ad hoc network in the presence of node mobility, MAC layer power control and link fades. The control law uses a simple and intuitive power adaptation mechanism. We consider as an example the topology requirement of maintaining the out degrees of each node at prescribed values and keeping the in degree close to the out degree. The topology objective is achieved by adapting the transmission power based only on local information. This power adaptation algorithm is analyzed using the o.d.e. approach to stochastic approximation. Simulation results verify the analysis and demonstrate its effectiveness. We also study the ability of the proposed objective to maintain connectivity. Although many heuristics are described in the literature to maintain local topological properties, the algorithm proposed in this paper is the first one that has proven convergence properties.**

Keywords:- control theory, system design, topology control, power control, network connectivity, mobility.

I. Introduction

Power control in ad hoc wireless networks can be used by different layers of the communication stack, possibly simultaneously. At the physical layer it can be used to maintain channel quality (for example, bit error rates through signal to interference ratio) at acceptable values by adjusting the transmission power. This has been addressed extensively in the context of cellular or infrastructure based networks. See, e.g., [8]. Power control can be used to reduce MAC latency [17] by using, if possible, lower power levels to connect to neighbors without disturbing ongoing transmissions that have forced it to not transmit by a CTS of the CSMA/CA MAC layer protocol. Transmission power determines the radio range of a node thereby defining its connectivity and hence the topology of the network. Thus power control can also be used for topology control by the network layer [22], [21], [25], [9], [13], [14]. Power control can even be mapped to the transport layer where it can impact the congestion control algorithms [18]. [22], [21], [25], [9] describe a local topology objective to achieve a connected network. The topology so obtained is further optimized by eliminating the links to nodes that can be reached in multiple hops with less power.

Our goal in this paper is not to propose a new local property objective. Rather, it is to obtain a control law that will maintain the prescribed objective to obtain the 'working topology' in the presence of changing network conditions. We describe and analyze such a control law based on power, and hence range, adaptation. This control law is aimed at maintaining the prescribed local property objective while the number of active nodes and their locations change, due possibly to MAC layer energy conservation algorithms that switch off a node's radio when it is inactive [23], node movement and channel conditions. We note that there are many heuristics described in the literature to maintain local properties, e.g., [21], but, to the best of our knowledge, this is the first algorithm that has proven convergence properties.

The power control algorithm that we propose is in the spirit of adaptive 'learning' schemes of adaptive signal processing and makes incremental corrective adjustments to the transmission power and hence the transmission range, based on a noisy 'error' signal that is calculated using only local information. It does not aim to find an exact or near-exact optimum transmission range. Thus our treatment of the problem is along the lines of a 'regulation' or 'tracking' problem in control where the objective is to stay close to a prescribed operating point (here the prescribed local property objective) rather than to achieve an optimum. The optimality is built into the choice of the local property objective. The simple scheme can be seen to be totally distributed and extremely easy to implement. The scheme has overtones of the transmission power adaptation scheme of [8] that is aimed at maintaining a specified SIR at the receivers using a distributed control scheme. In our analysis we use only some very gross properties of the scheme such as some natural monotonicities that arise, and do not require the detailed structure of the adaptive scheme. This makes it very robust with respect to the modeling assumptions and hence a potentially powerful tool that may find applications well beyond the present scheme.

The rest of the paper is organized as follows. In the next section we review existing local topology objectives and in Section III we describe the power adaptation scheme with reference to one topology objective and the analysis of the algorithm. Section IV presents some numerical results from simulations. We conclude with a discussion in Section V where we discuss some implementation issues, performance implications of the topology control scheme and application of the controller to achieve other local topology objectives.

II. Local Topology Objectives

There are several recent proposals that determine the transmission power for each node to achieve a local topological property. In the following we summarize these algorithms and the key assumption in these algorithms. [22] determines the geographical area in which each node should search for neighbors. This requires that each node be able to obtain the location information of the nodes in its neighborhood. [25] requires that the transmission range of each be such that it has a neighbor in every sector of angle α. This requires that each node be capable of obtaining the directional information of its neighbors. [9] describes the power aware routing optimization (PARO) algorithm in which a node asks its neighbors to reduce their transmission range if it sees them transmitting to nodes that are further than itself in the same direction, since such nodes can be reached through it with lesser total power. This is similar in principle to that of [22] in that the power is decreased if there are neighbors that can relay. In [9] power reduction is initiated by the "relay node" while in [22] it is initiated by the transmitter. Clearly, like the algorithm of [22], this algorithm also requires that a node know the location of its neighbors. [21] describes a centralized algorithm to obtain the transmission range of each node to achieve k-connectivity in a static ad hoc network. While a centralized algorithm is clearly not very practical, an interesting practical suggestion in [21] is to maintain the degree of each node in a prespecified range. This imposes minimal requirements on the node and is clearly easy to implement. We will discuss this objective in some detail because our topology control algorithm will be described with reference to this objective.

[26] provides an interesting recent result in support of the objective of [21] to keep the degree of each node at a prescribed value. [26] shows that for a static network obtained by deploying the nodes randomly, if each node is connected to $5.1774 \log n$ nodes, then the network is asymptotically connected. Earlier, [15] had reported that if the nodes in a multihop slotted Aloha network are distributed according to a Poisson process in two dimensional space, simulations indicated that when the nodes transmit with a range such as to have an average degree of five, the network almost always connected. [15] also claims that maintaining an average degree of 5.89 maximizes the throughput of the network.

From the preceding discussion, we can argue that each node maintaining a prescribed degree would be a good objective for a local topology control algorithm. Thus we describe our topology control algorithm with reference to maintaining the degree of node i around a prescribed level, say k_i^*. Non-homogeneous k_i^* may be useful when the nodes are heterogeneous and/or the node distribution is nonuniform in the operational area. Other local topology objectives are discussed in Section V-C.

If the transmission range of the nodes in a wireless ad hoc network is not homogeneous, some of the links in the network will be unidirectional. Thus k_i^* above is actually the out-degree of a node. Although most well known ad hoc network routing protocols like the dynamic source routing (DSR) protocol [12] or the adaptive on-demand distance-vector (AODV) routing protocol [20] operate correctly in the presence of unidirectional links, the MAC layer performance can be affected. In fact, the 802.11 CSMA/CA protocol is expected to work less efficiently with asymmetric links because the handshaking required before transmission, the acknowledgments at the end of a packet transmission and the collision avoidance algorithm assume bidirectional links. However, there is no known study of the effect of asymmetric links on its performance. Thus we propose that the power adaptation algorithm should also attempt to minimize the asymmetry between the in- and out-degrees and make the links bidirectional. This requirement is in addition to keeping the degree at a prescribed level. We reiterate that the choice of k_i^* is not being investigated in this paper although some pointers to its selection are discussed above. We remark that if the node distribution is nonuniform, then the common transmission range required to keep the network connected may be quite high and can therefore reduce the capacity of the network [13], [14].

We mention here that a different approach to power and topology control is the COMPOW protocol [18], [13], [14]. It prescribes that each node obtain a routing table for each possible power level and the lowest power that makes the network connected is chosen as the operating power.

III. The Control Law

We assume that N nodes are deployed in an operational area. Let the range of node i be $r_i(t)$ at time t. Since the transmission range is a direct function of the transmission power, we will consider range adaptation to imply power adaptation and deal only with the range. Let $K^i(t) = [k_1^i(t), \cdots, k_N^i(t)]$ be the vector representing the in-degrees of the nodes and $K^o(t) = [k_1^o(t), \cdots, k_N^o(t)]$ the vector representing the out-degrees of the nodes at time t. Let k_i^* be the prescribed out-degree for node i which is to be achieved through transmission power adaptation. We will assume that the transmission range $r_i(t)$ can be varied continuously in the interval $[\alpha_i, \beta_i]$. Node i adjusts the power at times $t_1, t_2, \cdots, t_s, \cdots$ according to the adaptation equation

$$\begin{aligned} r_i(t_{s+1}) \quad = \quad & \Gamma\Big(r_i(t_s) + a r_i(t_s)\left[[k_i^* - k_i^o(t_s)]^{\theta_1} + \right. \\ & \left. \gamma\left[k_i^i(t_s) - k_i^o(t_s)\right]^{\theta_2}\right]\Big) \end{aligned} \tag{1}$$

In this equation, $a > 0$ is a small scalar and is called the 'learning parameter', θ_1, θ_2 are odd integers and $\Gamma(\cdot)$ is the projection onto the interval $[\alpha_i, \beta_i]$, $\beta_i > 0$ being the maximum range which the node can achieve and $\alpha_i > 0$ a small number corresponding to the minimum range which the node should have. Thus $\Gamma(x) = \min(\max(x, \alpha_i), \beta_i)$. $\gamma > 0$ is a weight factor.

The idea is as follows. The adaptation equation will strive to maintain the term inside the outer square brackets in Eqn. (1) near 0 on the average. The term in the first inner square bracket will increase the transmission power if the out-degree is less than the prescribed value and decrease it otherwise. The term in the second inner square bracket will increase the transmission power if the out-degree is less than the in-degree and decrease it otherwise. Using an odd integer for $\theta_1 > 1$ will enhance the difference between the actual and desired values and will make the convergence faster, likewise for θ_2.

Ideally, the terms in both the square brackets are to be maintained near 0, i.e., the out-degree should be equal to the prescribed value and the in-degree should be equal to the out-degree. The proposed scheme does not ensure this a priori. However, we have arranged the two terms so that an excessively low, respectively high, out-degree should give the same sign to both the terms and thus we may expect both to remain low on the average. As we see later, simulations confirm this intuition. Note that increasing the transmission power can cause interference at other nodes thus decreasing their SIR. However, we ignore this effect and assume that the only source of interference is through collisions due to simultaneous transmissions.

Assuming a stationary environment, let $F(\cdot) \triangleq [F_1(\cdot), \cdots, F_n(\cdot)]^T$ be defined by:

$$F_i(r) \triangleq E\left[r_i\left[[k_i^* - k_i^o(t_s)]^{\theta_1} + \gamma\left[k_i^i(t_s) - k_i^o(t_s)\right]^{\theta_2}\right]\right],$$

for $1 \leq i \leq n$, where $r = [r_1, \cdots, r_n]^T$ and $E[\cdot]$ stands for the stationary average under $r(t) \equiv r$. (We are implicitly assuming that the ambient random processes such as channel characteristics, mobility patterns, the MAC layer power control behavior, etc., are quasi-stationary.) Then our theoretical analysis below shows that under reasonable hypotheses, $F_i(r)$ is nearly 0 for all i and thus our objective is achieved. The first key assumption is :

A1 $F(\cdot)$ is continuously differentiable and

$$\frac{\partial F_i}{\partial r_j} > 0, \ i \neq j.$$

The continuous differentiability is an assumption for mathematical convenience in our analysis to follow. It will typically hold if the underlying probability distributions have sufficiently nice densities. Note that an increase in the output power of j will tend to cause an increase in the in-degree $k_i^i(\cdot)$ of $i \neq j$ and this affects only the second term in the definition of $F_i(\cdot)$, which will correspondingly tend to increase. Thus this condition is quite natural.

A. Analysis of the Control Law

Our convergence analysis is based on the observation that Eqn. (1) is a special case of the celebrated *stochastic approximation* algorithm [1], [3], [16]. We shall take the 'o.d.e. ' (for '*ordinary differential equations*') approach to the analysis of Eqn. (1) (*ibid.*). The limiting o.d.e. in this case will be

$$\dot{\tilde{r}}_i(t) = F_i(\tilde{r}(t)), \ 1 \leq i \leq n. \tag{2}$$

To be precise, Eqn. (1) is a *projected* stochastic approximation algorithm ([16], Chapter 5) because we restrict the r.h.s. of Eqn. (1) to the bounded interval $[\alpha, \beta]$. Thus the limiting o.d.e. of Eqn. (2) should incorporate additional correction terms at the boundary points that ensure that the trajectory remains in $[\alpha, \beta]^n$. We shall make the perfectly reasonable assumption:

A2 $F_i(r) > 0$ when $r_i = \alpha_i$ and < 0 when $r_i = \beta_i$, for $1 \leq i \leq n$, $r = [r_1, \cdots, r_n]$,

(because it pays to raise the power from α which would be chosen to be 'too low to be optimal', and to lower it from a very high β which would be chosen to be 'too high to be optimal'). In this case, the driving vector field of Eqn. (2) turns out to be transversal to the boundary of $[\alpha, \beta]^n$ and oriented towards its interior, whence the boundary corrections turn out to be zero.

Our assumption (A1) ensures that Eqn (2) is a *cooperative* o.d.e. in the sense of [11], [24]. This allows us to invoke the known results for such o.d.e. , such as:

Lemma 1: For generic initial conditions (i.e., initial conditions belonging to an open dense set) in Eqn. (2), $\tilde{r}(t)$ converges to the set $H \triangleq \{r : F_i(r) = 0 \ \forall i\}$, the set of equilibria of Eqn. (2). Let H_s denote its subset consisting of *stable* equilibria.

To see this, one mimics the proof of Theorem 1.1, page 56, of [24] with weak inequalities in place of strong, in order to verify the strong order-preserving property defined on page 2 of [24] for the flow associated with Eqn. (2). The above lemma then is simply a restatement of Theorem 4.3, page 11, [24]. We strengthen assumption (A1) to:

A3 The Jacobian matrix $[[\frac{\partial F_i}{\partial r_j}]]$ is irreducible.

Intuitively this means that if we draw an 'influence diagram' with a directed arrow from node j to node i when the latter's transmission can be heard by the former, it leads to a path-connected directed graph. If this fails, one can consider this analysis restricted to each path-connected component. Lemma 1 can now be strengthened to:

Lemma 2: For generic initial conditions in Eqn. (2), $\tilde{r}(t)$ converges to some point (depending on the initial condition) in H.

A further restriction is:

A4 The matrix $[[\frac{\partial F_i}{\partial r_j}]]$ is nonsingular at points in H.

In this case, we know from the inverse function theorem that points in H are isolated and thus H is discrete. This assumption can be justified on 'genericity' grounds: nonsingular matrices form an open dense set in the space of all square matrices of a given size. Suppose we also impose:

A5 The symmetric part of the matrix $[[\frac{\partial F_i}{\partial r_j}]]$ is positive definite.

This restriction would imply that the map $G(\cdot) = [G_1(\cdot), \cdots, G_n(\cdot)]$ defined by $G_i(r) = r_i + F_i(r), 1 \leq i \leq N$, is monotone ([19], section 5.4) and thus has a unique fixed point, i.e., H is a singleton, say, $\{r^*\}$. In fact, a straightforward calculation shows that $||r - r^*||^2$ serves as a Liapunov function for Eqn. (2) implying the global asymptotic stability of r^*. Assumption (A5), however, is a strong assumption, so while taking note of the fact that it permits stronger claims about Eqn. (2), we shall assume only (A1)–(A4).

Invariant sets of Eqn. (2) could include, in addition to H, possible invariant sets contained in the set of initial conditions excluded in Lemmas 1 and 2. These lemmas imply that the latter form a set of first category. In addition, the stable manifolds of unstable equilibria will typically also form sets of first category. We shall thus make the following reasonable assumption:

> **A6** There exists an open dense set U such that all trajectories of Eqn. (2) initiated in U converge to some point in H_s.

We also assume:

> **A7** $\{r(t) = [r_1(t), \cdots, r_n(t)]\}$ is asymptotically stationary and the stationary law ν_a of $r(t)$ has a density w.r.t. the Lebesgue measure on $\mathcal{R}^n$.

Assumption (A7) ensures that the stationary probability under ν_a of 'bad' initial conditions for Eqn. (2) is zero. While this has an intuitive appeal, it could be ensured under weaker conditions. In turn, it can be forced by adding to the r.h.s. of Eqn. (1) extraneous i.i.d. noise whose distribution has a positive density w.r.t. the Lebesgue measure and a finite second moment. In simulations, the intrinsic randomness of the system seemed to suffice. The same may be expected in practice.

The subscript a in ν_a renders explicit the dependence on the stepsize a in Eqn. (1). Let $B(\epsilon) \stackrel{\Delta}{=} \{R \in \mathcal{R}^n : \inf_{r^* \in S} ||r - r^*|| < \epsilon\}$ (i.e., the $\epsilon-$neighborhood of S) and set

$$\begin{aligned} T(x) &= \sup\{t \geq 0 : q(t) \notin B(\epsilon)\} \\ &= \inf\{t \geq 0 : q(s) \in B(\epsilon)\ \forall\ s > t\}, \end{aligned}$$

where $q(\cdot)$ satisfies Eqn. (2) with initial condition x. $T(x)$ is the least time such that the trajectory starting at x lies in $B(\epsilon)$ thereafter. (Thus $T(x) < \infty$ for $x \in U$.) Conditions (A6), (A7) imply in particular that $\nu_a(\bigcup_n \{x : T(x) \leq n\}) = 1$. Thus given any $\delta > 0$, we can pick an $N(\delta) \geq 1$ such that $\nu_a(\{x : T(x) \leq N(\delta)\}) > 1-\delta$. Consider a stationary process $\{r(t)\}$ governed by Eqn. (1) and the corresponding limiting o.d.e. of Eqn. (2) with $t_0 = 0$ and $T = N(\delta)a$. Then it follows from Theorem 2.3 of [6] that

$$\nu_a(B(\epsilon)) = \mathsf{Prob}(\tilde{r}(T) \in B(\epsilon + O(a))) > 1 - \delta. \quad (3)$$

We have proved:

Theorem 1: The stationary distribution ν_a of (1) concentrates on the stable equilibria of Eqn. (2) in the sense of Eqn. (3).

Remarks:

1) Note that *any* equilibrium point $\hat{r}$ of Eqn. (2) corresponds to the desired equality $F_i(r) = 0\ \forall i$, so they are all equivalent for our purposes.
2) We have given a 'back of the envelope' calculation to justify Theorem 1. A more formal treatment leading to an analogous claim under certain technical conditions may be found in [2].
3) The assumptions that we have made in the analysis are either intuitive or are made for mathematical convenience, typically with no obvious loss of generality. In practice, it could be quite messy to explicitly verify them in most cases.
4) All nodes should use the same value of the learning parameter a. In practice this can be easily enforced by making it an administrative requirement or can be exchanged as part of the routing protocol messages. If they used different 'a', that would induce multiple timescales with the effect that the faster updates (corresponding to larger 'a') see the slower ones as quasi-static, whereas the slower ones see the faster as quasi-equilibrated. This would induce a hierarchical, Stackleberg-like leader-follower behavior. While this can also be analyzed, we do not do so here. (See, e.g., [4].) It is also worth noting here the standard trade-off in choosing stepsizes: a larger 'a' means faster learning, but also higher fluctuations.
5) The nodes may do their power adaptations asynchronously. This would call for an analysis of the scheme as an asynchronous algorithm, which in the stationary case typically modifies the limiting o.d.e. by multiplying its right hand side by a diagonal matrix with positive entries that reflect the relative frequency with which the respective updates are done. (See [5] for an analysis in the case of diminishing stepsizes.) This would not affect the sets H, H_s, or the assumptions.
6) In practice the topology can be learned when the nodes exchange protocol information. In addition, it can also be learned when a node relays traffic or when it hears its neighbors' transmissions. The delays caused by this or otherwise can be shown to lead to another $O(a)$ error term. (This can be shown by adapting the arguments of [5], where for diminishing stepsizes it is shown that the effect of such delays is asymptotically negligible.) In particular, their effect may be neglected for small 'a'. The frequency of updates depends on the specific routing protocol which can be bounded and also the traffic pattern.
   Also, there may be errors in the knowledge of the instantaneous values of the in and out degrees.

IV. Simulation Results

We now report some results from the extensive simulations carried out to test the efficacy of the proposed topology control algorithm. The focus here is the effectiveness of our power adaptation algorithm in achieving the desired topology control objective. We will compare the performance measures from a network using the power adaptation algorithm of Eqn. (1) and another network operating with all nodes having the same transmission range, and hence the same transmission power. Also, we will consider transmission ranges in our simulations rather than transmission power. This only means that we will not model the effects of multipath fading and noise while helping us focus on the performance of the basic topology control algorithm.

The simulation model is as follows. The N nodes of the ad hoc network are distributed in $[0, 1]^2$ such that each of the coordinates of the node location is chosen independently according to a prescribed probability density function, $f(\alpha)$.

We experimented with following three density functions

$$f_u(\alpha) = \begin{cases} 1 & \text{for } 0.0 \le \alpha \le 1.0 \\ 0 & \text{otherwise} \end{cases}$$

$$f_e(\alpha) = \begin{cases} 2 - 4\alpha & \text{for } 0.0 \le \alpha \le 0.5 \\ 4\alpha - 2 & \text{for } 0.5 \le \alpha \le 1.0 \\ 0 & \text{otherwise} \end{cases}$$

$$f_m(\alpha) = \begin{cases} 4\alpha & \text{for } 0.0 \le \alpha \le 0.5 \\ 4 - 4\alpha & \text{for } 0.5 \le \alpha \le 1.0 \\ 0 & \text{otherwise} \end{cases}$$

Choosing the coordinates according to $f_u(\alpha)$ distributes the nodes uniformly in $[0,1]^2$, $f_e(\alpha)$ distributes the nodes such that the nodes are biased toward being near the periphery of $[0,1]^2$ while using $f_m(\alpha)$ to choose the coordinates will bias the nodes toward being near the center of $[0,1]^2$.

Mobility and the resulting topology change are modeled by a modified *random waypoint* model. A node when it decides to move chooses a new location according to the prescribed density function $f(\alpha)$ and then moves towards that point at a steady speed. It pauses at the new point for a random amount of time and then repeats the procedure. All nodes move independently of each other. The pause times are assumed to be i.i.d. with exponential distribution. We assume that the speed of all the mobile nodes is the same. In the simulation we assume that given the transmission power, every node knows its transmission radius and also its in-degree and out-degree. The mean pause time is assumed to be ρ and the speed of movement 5ρ per unit time. The power adaptation is performed at regular intervals. Time is normalized to the power adaptation interval.

We remark that each node can estimate its in and out degrees from most routing protocols that are being proposed for ad hoc networks. We will discuss this further in Section V-A. The estimates will, however, be noisy because the information used in the estimation may be old. In the simulation we assume perfect knowledge of the in and out degrees at all times.

The performance measures are obtained from simulations as follows. The network is simulated for 5000 time units and then time averages of the performance measures of interest are collected for this duration. This gives us one sample of the performance measures. We then repeat the simulation for the same parameters with different seeds 50 times. The sample mean and sample variance from these 50 repetitions are reported. We would like to mention here that running the simulations for longer than 5000 time units per sample did not give us appreciably different results. Note that 'steady state' is not achieved quickly in the simulations because of the considerable dependence on the initial topology. Further, in the context of ad hoc networks that work on batteries and are typically expected to be 'short term' networks, we argue that running simulations many times for short periods of time and taking sample averages and variances as above is more appropriate than running the simulations for a long time till 'steady state' is achieved.

First we consider the transient and short term average behavior of the topological properties due to the power adaptation

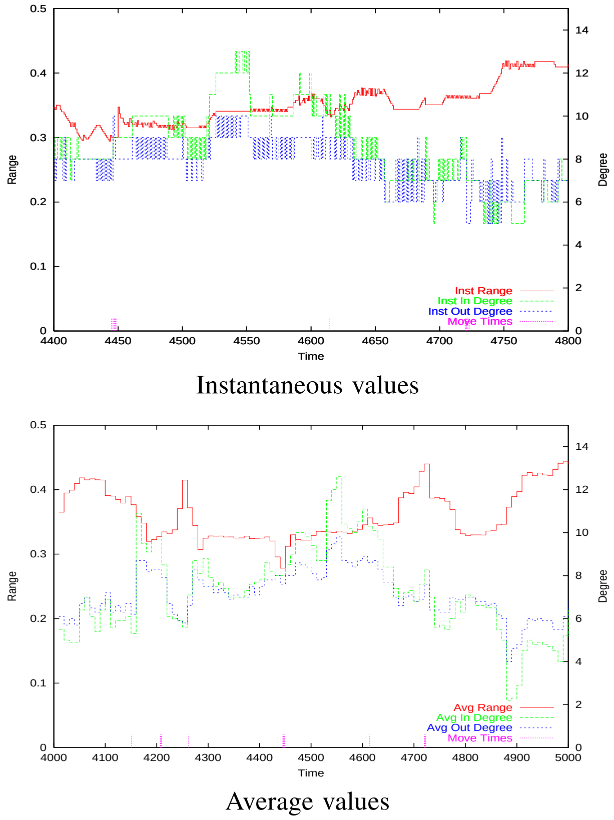

Fig. 1. A sample of the instantaneous and average values of the range and the degrees of an arbitrary node during a simulation. Also marked are instants at which the node moved causing a potential change in its in and out degrees.

algorithm. In Figure 1 we show the average and transient degree of an arbitrarily selected node during 5000 units of time with $k^* = 7$. The average degree is obtained by averaging over the previous ten intervals. Observe that the fluctuations in the degree are quite small and can be easily smoothed using techniques described in Section V. Also, the average degree is seen to be fairly smooth except when the topology changes due to a movement of this node or another node coming into the area (not shown in the figure) and the node has to adjust its transmission range.

Table I shows the performance measures for a 25 node network with $\rho = 5.0$ and for different node distributions. The nodes are always available. The sample mean and sample standard deviation of the average range and the average degree for different k^* are shown. We also simulate a network with no power adaptation in which the transmission radius is equal to the sample average for each of the k^* and obtain the statistics on the degree of the nodes. Observe that for all node distributions, the average degree with power adaptation is very close to k^* with a very low sample standard deviation. This implies that the power control does indeed achieve its objective of maintaining the degree of each node at the prescribed value. The sample standard deviation of the transmission range in the presence of power control is small—less than 2% of the sample mean in most cases. This implies that the variation in

Uniform node distribution; coordinates have density $f_u(\alpha)$							
	With power control				**With no power control**		
	Avg Range		Degree		Range	Degree	
k^*	Mean	StDv	Mean	StDv		Mean	StDv
4	0.281	0.008	4.143	0.072	0.281	4.609	0.114
5	0.314	0.010	5.072	0.039	0.314	5.574	0.142
6	0.345	0.014	6.010	0.012	0.345	6.528	0.159
7	0.374	0.017	6.954	0.021	0.374	7.464	0.185
8	0.403	0.019	7.902	0.043	0.403	8.408	0.201
9	0.432	0.021	8.855	0.065	0.432	9.360	0.207
10	0.460	0.022	9.812	0.087	0.460	10.31	0.216

Edge biased node distribution; Coordinates have density $f_e(\alpha)$							
	With power control				**With no power control**		
	Avg Range		Degree		Range	Degree	
k^*	Mean	StDv	Mean	StDv		Mean	StDv
4	0.285	0.020	4.148	0.096	0.285	4.585	0.602
5	0.337	0.025	5.046	0.069	0.337	5.419	0.513
6	0.396	0.040	5.948	0.079	0.396	6.182	0.477
7	0.454	0.045	6.849	0.101	0.454	6.924	0.581
8	0.513	0.050	7.763	0.130	0.513	7.736	0.744
9	0.562	0.047	8.692	0.158	0.562	8.627	0.917
10	0.604	0.047	9.636	0.181	0.604	9.601	1.014

Center biased node distribution; Coordinates have density $f_m(\alpha)$							
	With power control				**With no power control**		
	Avg Range		Degree		Range	Degree	
k^*	Mean	StDv	Mean	StDv		Mean	StDv
4	0.207	0.010	4.331	0.164	0.207	5.160	0.198
5	0.228	0.011	5.271	0.134	0.228	6.084	0.227
6	0.248	0.013	6.218	0.109	0.248	7.024	0.250
7	0.266	0.014	7.170	0.086	0.266	7.941	0.262
8	0.285	0.016	8.127	0.066	0.285	8.856	0.280
9	0.302	0.017	9.087	0.048	0.303	9.750	0.287
10	0.320	0.018	10.05	0.030	0.320	10.63	0.297

TABLE I

RESULTS FOR A 25 NODE NETWORK

the power consumption is not significant.

When we compare the average degree in the network with power control and without power control, we see that for the same average transmission range the average degree in a network without power control is marginally higher, 5–10%, than that with power control. This means that we could achieve similar average topological properties with lower transmission power. However, observe that the standard deviations are higher—usually 5-6 times, and even as high as 10 times than that with power control. This means that the deviations from the average characteristics could also be significant than that with power control.

We next consider another source of topology changes in addition to node mobility. Recall that a proposed energy saving mechanism (and even recover some battery energy) is for a node to turn off its radio for extended periods of time. Further, it is also possible that the radio environment around a node is so harsh that it cannot communicate with its neighbors and is effectively 'unavailable' to the network. We now consider topology changes due to these factors. To be able to model the use of topology control in a network in which the nodes become inaccessible for extended periods of time, we consider a 50 node network in which a node alternates between 'available' and 'unavailable' states each of which last for an exponentially distributed time with a mean of 25 time units. Nodes are also mobile and follow the random waypoint model described earlier with $\rho = 5.0$. Movement and non availability are assumed to occur independently.

Sample mean and standard deviation of the average range in the network and the average degree are obtained as before. Table II shows these results. Also shown are the results for a network operating without power control where all the nodes have the same range as the average range in the network with power control. Observe that the network with power control is able to maintain the node degree at the specified level with very low standard deviation. However the average degree of the nodes in the network without power control is considerably lower than that in the network with power control and could be as low as nearly half of the latter. Observe also that the standard deviation is very bad and can be comparable to the mean. Thus in this case power control achieves topology control very effectively.

One of the requirements of our power adaptation algorithm was to also make the in-degree of the nodes close to their out degrees. To see how effectively this is achieved, we also measured the asymmetries in the degrees of the nodes when power control was used. In Table III the root mean square of the difference between the in and out degrees of every node during each sample are shown for the various node distribution models in the case of the 25 node network. Observe that the asymmetry is not significant and is in the range of 1-2 even for large k^*. We remark here that such difference can arise naturally even in the presence of constant transmission powers due to asymmetries in the channel characteristics.

A. *Local Topology Control and Network Connectivity*

Although the topology control algorithm of Equation 1 is independent of how k^*, the local topology objective, is obtained, it is interesting to study the effect of a specified k^* on the connectivity of the network.

For nodes uniformly distributed over a unit volume in d-dimensions area, along the lines of [10], it can be shown that the critical transmission radius for asymptotic connectivity of the network is $r_c := (\log n/n)^{\frac{1}{d}}$. What this means is that if *every* node transmits with a power such that its transmission radius is r_c, then in the limit as n becomes large, the network is connected with probability 1. However, the asymptote is approached very slowly and for finite networks, the transmission radius required to make the network connected with a high probability is significantly higher than r_c defined above. We illustrate this for the one dimensional network using the exact expression for the probability of connectivity from [7]. Table IV shows the probability that the network is connected when the transmission range of every node is $r = \log n/n$ and also the r required to make the network connected with probability 0.95. Observe that the range required for a small network can be nearly 50% higher than that suggested by the asymptotic result.

Uniform node distribution; Coordinates have density $f_u(\alpha)$							
	With power control				**With no power control**		
	Avg Range		Degree		Range	Degree	
k^*	Mean	StDv	Mean	StDv		Mean	StDv
5	0.310	0.004	5.066	0.012	0.310	3.386	3.467
6	0.341	0.005	6.013	0.013	0.341	3.436	1.040
7	0.372	0.005	6.968	0.015	0.372	3.963	1.197
8	0.402	0.006	7.931	0.016	0.402	4.488	1.350
9	0.432	0.007	8.898	0.017	0.432	5.006	1.509
10	0.461	0.007	9.869	0.019	0.461	5.525	1.666
11	0.490	0.008	10.84	0.020	0.490	6.046	1.828
12	0.519	0.009	11.82	0.021	0.519	6.555	1.979

Edge biased node distribution; Coordinates have density $f_e(\alpha)$							
	With power control				**With no power control**		
	Avg Range		Degree		Range	Degree	
k^*	Mean	StDv	Mean	StDv		Mean	StDv
5	0.329	0.011	5.022	0.031	0.329	3.447	3.604
6	0.390	0.016	5.906	0.039	0.390	3.397	1.100
7	0.452	0.023	6.808	0.048	0.452	3.823	1.264
8	0.509	0.027	7.734	0.049	0.509	4.224	1.435
9	0.558	0.027	8.683	0.045	0.558	4.691	1.616
10	0.601	0.027	9.647	0.043	0.601	5.152	1.791
11	0.638	0.026	10.62	0.041	0.638	5.654	1.960
12	0.671	0.026	11.60	0.041	0.671	6.152	2.132

Center biased node distribution; Coordinates have density $f_m(\alpha)$							
	With power control				**With no power control**		
	Avg Range		Degree		Range	Degree	
k^*	Mean	StDv	Mean	StDv		Mean	StDv
5	0.227	0.004	5.323	0.015	0.227	3.743	3.782
6	0.247	0.004	6.282	0.015	0.247	3.744	1.152
7	0.266	0.005	7.249	0.016	0.266	4.249	1.300
8	0.285	0.005	8.221	0.015	0.285	4.739	1.457
9	0.303	0.006	9.198	0.016	0.303	5.232	1.610
10	0.322	0.006	10.17	0.017	0.322	5.722	1.756
11	0.340	0.007	11.15	0.018	0.340	6.223	1.903
12	0.359	0.007	12.13	0.018	0.359	6.718	2.048

TABLE II

RESULTS FOR A 50 NODE NETWORK IN WHICH NODES RANDOMLY ALTERNATE BETWEEN BEING 'AVAILABLE' AND 'UNAVAILABLE.' AT ANY TIME, ON AN AVERAGE, 50% OF THE NODES ARE AVAILABLE. AVERAGE TIME BETWEEN STATE CHANGE IS 25 TIME UNITS FOR EACH NODE.

RMS of Asymmetry						
	Uniform		Edge Bias		Center Bias	
k^*	Mean	StDvn	Mean	StDvn	Mean	StDvn
5	1.1456	0.0772	0.9887	0.1652	1.5342	0.1417
6	1.2742	0.0845	1.0596	0.1603	1.6684	0.1217
7	1.4479	0.0923	1.2376	0.1625	1.8278	0.0927
8	1.6398	0.0793	1.4763	0.1547	1.9853	0.0847
9	1.8228	0.0998	1.6863	0.1819	2.1464	0.0883
10	2.0055	0.0901	1.8799	0.2121	2.2829	0.1113

Network with 25 nodes that are always 'available'.

RMS of Asymmetry						
	Uniform		Edge Bias		Center Bias	
k^*	Mean	StDvn	Mean	StDvn	Mean	StDvn
5	1.1955	0.0257	0.9636	0.0839	1.5235	0.0400
6	1.3770	0.0313	1.0965	0.0911	1.6955	0.0435
7	1.5775	0.0357	1.3341	0.0769	1.8811	0.0474
8	1.7888	0.0393	1.6118	0.0898	2.0698	0.0498
9	2.0050	0.0406	1.8820	0.1124	2.2532	0.0504
10	2.2166	0.0411	2.1283	0.1287	2.4257	0.0495

Network with 50 nodes randomly toggling between sleep and 'on' states.

TABLE III

THE RMS VALUE OF THE DIFFERENCE BETWEEN THE IN AND OUT DEGREES OF THE NODES IN THE NETWORK FOR THE 25 NODE NETWORK WHERE THE NODES ARE ALWAYS LIVE THE 50 NODE NETWORK WHERE THE NODES RANDOMLY SWITCH OFF THEIR TRANSCEIVERS.

n	$\frac{\log n}{n}$	$P_c(n, \frac{\log n}{n})$	r for $P_c > 0.95$
5	0.322	0.461	0.584
10	0.230	0.417	0.405
20	0.150	0.392	0.257
50	0.078	0.376	0.129
100	0.046	0.372	0.073
200	0.026	0.370	0.041
500	0.012	0.368	0.018
1000	0.0069	0.368	0.00983

TABLE IV

PROBABILITY OF CONNECTIVITY WITH RANGE $\log n/n$ AND THE RANGE REQUIRED TO MAKE THE NETWORK CONNECTED FOR DIFFERENT n

Table V shows sample mean and sample standard deviation of the probability of the network being connected for different k^* and node distributions in a 25-node network with nodes being always available. Observe that the k^* required to keep the network connected is significantly lower than that obtained from the asymptotic analysis of [26]—asymptotic analysis suggests that each node should have a degree of $5.17 \log 25 = 16.64$ whereas we see that a degree of 10 suffices to make the network connected with very high probability. Also observe that the probability that the network is connected is almost always higher with power control than without. However, the sample standard deviation seems higher with power control except in the case of nodes distributed with an edge bias.

V. DISCUSSION AND CONCLUSION

A. Discovering In and Out Degrees

The topology control algorithm that we describe above needs information about the node's in-degree and out-degree. Knowing the in-degree is fairly straightforward. It has to keep track of the number of distinct neighbors whose transmissions it has been receiving. Appropriate aging of the information will need to be done to keep this information current. Further, it is obvious that any link state routing protocol or its variants where local connectivity information is flooded into the network will provide the power control algorithm with the necessary information. We will now discuss this problem with reference to the two more popular ad hoc network routing protocols: Dynamic Source Routing (DSR) [12] and Adaptive On Demand Distant Vector (AODV) routing [20] protocols. We mention here that if accurate information is not possible, it is necessary to have an unbiased estimate of the out-neighbors.

In DSR, nodes cache routes to other nodes in the network every time they participate or overhear the transmissions in a route discovery. Thus the nodes maintain one or more routes to other nodes in the network. From these routes is possible

Sample mean of the probability of connectivity						
	Uniform		Edge Bias		Center Bias	
k^*	PC	No PC	PC	No PC	PC	No PC
3	0.1147	0.1056	0.0041	0.0000	0.0762	0.0774
4	0.4346	0.3785	0.0637	0.0000	0.2720	0.2247
5	0.6970	0.6642	0.1464	0.0600	0.4745	0.4073
6	0.8586	0.8419	0.4410	0.4200	0.6236	0.5692
7	0.9333	0.9302	0.7306	0.7400	0.7278	0.7048
8	0.9702	0.9684	0.9437	0.9200	0.8232	0.8046
9	0.9837	0.9847	0.9740	0.9600	0.8829	0.8725
10	0.9911	0.9931	0.9967	1.0000	0.9197	0.9231

Sample standard deviation of the probability of connectivity						
	Uniform		Edge Bias		Center Bias	
k^*	PC	No PC	PC	No PC	PC	No PC
3	0.1353	0.0362	0.0255	0.0000	0.0900	0.0301
4	0.2163	0.0695	0.2005	0.0000	0.1873	0.0578
5	0.2113	0.0679	0.3184	0.2375	0.2231	0.0777
6	0.1241	0.0511	0.4383	0.4936	0.2229	0.0790
7	0.0717	0.0306	0.3966	0.4386	0.2148	0.0740
8	0.0236	0.0205	0.2021	0.2713	0.1853	0.0602
9	0.0157	0.0110	0.1169	0.1960	0.1424	0.0498
10	0.0094	0.0086	0.0157	0.0000	0.1361	0.0376

TABLE V

SAMPLE MEAN AND SAMPLE STANDARD DEVIATION OF THE TIME AVERAGE OF THE PROBABILITY THAT THE NETWORK IS CONNECTED FOR DIFFERENT k^* AND NODE DISTRIBUTIONS

to find the number of out-neighbors by obtaining the number of distinct nodes that follow it in the set of routes that it has cached. Of course, information could be stale and out neighbors could have moved away. Since the opposite is also possible with new neighbors having moved in, the expectation of the error in the estimate will be zero.

In AODV, the routing table entries contain the address of the next hop node on the route to every node in the network much like the traditional distance vector routing protocols. The number of distinct next hops in the routing table can be used to obtain the out degree of the node.

B. Variations

In addition to the original scheme of Eqn. (1), the following adaptation, dubbed the 'momentum method', can also be used.

$$\begin{aligned} q_i(t_{s+1}) &= \Gamma\Big(q_i(t) + a\Big[[k_i^* - k_i^o(t_s)]^{\theta_1} + \\ &\qquad \gamma\left[k_i^i(t_s) - k^o(t_s)\right]^{\theta_2}\Big] - bq_i(t_s)\Big) \\ r_i(t_{s+1}) &= r_i(t_s) + ar_i(t_s)q_i(t_s), \end{aligned}$$

for some $b > 0$. This scheme replaces the 'error signal' driving the adaptation scheme by an exponentially weighted average thereof. This is a purely ad hoc suggestion, guided by intuition or experience from other domains rather than by rigorous theory. The earlier mathematical analysis does not apply to this in the strict sense. It is inspired by the eponymous scheme from neural networks literature. Since this is essentially a small variation of the original scheme, we expect this scheme would do as well, except that the additional averaging will lead to a more graceful finite time behavior at the expense of speed of convergence.

Another, a more practical modification to the basic scheme of Eqn. (1) would be to change the transmission power only of it changes by a predetermined threshold, i.e., discretize the steps of power change that are allowed. Once again, we expect that this would do as well. This proposal is motivated by the expectation that power control in ad hoc networks will probably be like in cellular networks with transmission power changing in discrete steps rather than continuously.

C. Other Objective Functions

We now describe how to modify the power adaptation equation of Eqn. (1), to achieve the local topology objective of [25]. Consider a node i that has n_i neighbors. Let λ_j, $\lambda_j \in [-\pi, \pi]$, be the angle of arrival of the signal from neighbor j, $j = 1, \cdots, n_i$ such that $\lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_{n-1}$. Let the requirement be to have a neighbor in every sector of angle λ_i^*. Define

$$\hat{\lambda}_i \triangleq \max\left(\max_{j=1,\cdots,n_i-1}(\lambda_{j+1} - \lambda_j), \quad 2\pi - (\lambda_n - \lambda_1)\right)$$

Rewrite Eqn. (1) as follows.

$$\begin{aligned} r_i(t_{s+1}) &= \Gamma\Big(r_i(t_s) + ar_i(t_s)\Big[\Big[\hat{\lambda}_i(t_s) - \lambda_i^*\Big]^{\theta_1} + \\ &\qquad \gamma\left[k_i^i(t_s) - k^o(t_s)\right]^{\theta_2}\Big]\Big) \end{aligned} \tag{4}$$

Eqn. (4) will strive to keep $\hat{\lambda}_i$ around λ_i^* in exactly the same manner as Eqn. (1) was keeping the out degree around k_i^*.

In addition to the objectives used in Eqns. (1) and (4), a more detailed 'ideal' could also be defined as long as it satisfies assumptions A1–A7 defined in Section III and the Appendix. For example, if each node has a directional antenna, each node could control its power in different sectors by prescribing an ideal degree for each sector. Another example would be for the case of a network that has two classes of nodes – relay and non-relay nodes with the former being responsible for routing in the network. In this case, the ideal for the non-relay node could be to connect to at least k relay nodes.

D. Conclusion

In this paper we have described a power adaptation scheme for use in an ad hoc network at the network layer that can be used to achieve topology control objective. We have analyzed the scheme to show that under fairly general assumptions, the adaptation scheme will converge. Extensive simulation results show that the topology control objectives are indeed achieved fairly efficiently and accurately. The adaptation scheme is also fairly general and can be easily adapted to achieve other, more detailed, topology objectives. We remark once again that Eqn. (1) can be used to obtain the 'working topology' and further optimization mechanisms can be used alongside to fine-tune the topology.

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