On Packet Processes in Interconnected LANs

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Abstract

[1] We consider devices that interconnect LANs on which the traffic process may be characterized as alternating between trains of packets and idle periods (e.g. Token Ring, Token Bus, FDDI, DQDB). Assuming that the probability that a packet is part of internetwork traffic is independent of all other packets, we characterize the input packet process to the internetwork device as an alternating renewal process of trains of packets and idle times. This is the first step in completely characterizing the packet process through a bridge in an internetworked LAN environment.

1 Introduction

Bridges, gateways and routers interconnect LANs. Typically, bridge capacities are of the order of a few thousand packets per second, orders of magnitude less than the throughputs of the LANs they interconnect [1, 2]. This means that during a sudden burst of internetwork traffic, packets have to be buffered in the bridge causing a transit queueing delay in the bridge. To study the performance of the internetworked system and the bridge, we need to model the passage of packets through the bridge. This involves the characterization of (see Figure 1)

1. Bridge **inflow** to the input buffer or the bridge inflow process. This depends on the packet process and the internetwork traffic on the input LAN.

2. **Processing Delay** at the processor. This depends on the functions of the bridge and speed of the processor.

3. Bridge **outflow** from the output buffer (bridge outflow process). This depends on the protocol and the packet process on the output LAN.

In LANs with controlled access, there is a high correlation in the output packet process. This makes modelling of the traffic inflow into the bridge very difficult. In this paper, as a first step in our attempt to model the passage of traffic through the bridge, we characterize the bridge inflow for correlated packet processes on the input LAN.

Packet process models have been used to study the packet interdeparture processes in various networks. Packet output

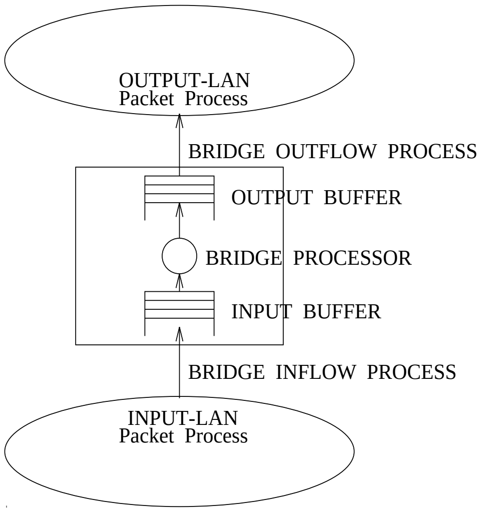

Figure 1: Schematic of a bridge and its queues

process and delay distributions of packets in random access networks have been studied in [3]-[5]. Interconnected random access networks have been studied in [6]-[8].

Interconnected controlled access systems are studied in [9, 10]. In [9] a switch connection approach to interconnect two token ring LANs is proposed and throughput-delay characteristics of the system are studied. This system's operation is substantially different from the bridges considered here, and it does not introduce any queueing or processing delays in the switch. In [10] a Token Ring system interconnected through bridges is analyzed. Input to the bridge is assumed to be a renewal process with average arrival rate corresponding to the intenetwork throughput. The bridge is then modeled as a GI/D/1 queue with vacations. There is no processing at the bridge. This enables the invocation of decomposition theorems to calculate the mean waiting time at the bridge without characterizing the input process.

The studies in [6]-[9] are aimed at specific system configurations and characterize average throughput and delay performance of the system. In [10] the restrictive assumptions of fixed packet lengths and no processing at the bridge allow them to bypass the characterization of the input process to the bridge. To analyze systems with variable packet lengths and significant bridge processing, we need to characterize the three components of the packet process through the bridge outlined above. Also, the assumptions in [10] are such that

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only the average delays may be calculated. Higher moments of the delay cannot be calculated.

In this paper we take the first step in the complete delay analysis of internetwork traffic by analyzing the traffic inflow process to the bridge. We characterize the bridge inflow process in an internetworking environment from one of the input LANs connected to the bridge. In section 2 we explain the assumptions of our model and the justifications for these assumptions. In section 3 we first describe a Markov model of the packet process on the input network, extend it to obtain a Markov model for the input to the bridge and finally, describe the bridge inflow process. In section 4, we provide some examples.

2 Preliminaries

Many LANs of today like FDDI, the high speed Token Ring and the Cambridge Fast Ring on ring topologies and DQDB, Fasnet, Expressnet and D-Net on bus topologies, use some form of controlled access. In these networks there is no wastage of bandwidth due to aborted packet transmission attempts. Packets are added to the tail of a train of packets and in the case of source removal they are removed from the head of the train. The network access unit of the bridge node will see a series of alternating packet trains and idle periods (a train and the following idle period). The ratio of the average train length to the average cycle length will be the throughput of the network ρ.

We assume that the packet process on the input network is an alternating renewal process corresponding to the busy and idle periods. The lengths of the busy and idle periods are independent and identically distributed continuous random variables with distributions $B_1(x)$ and $I_1(y)$ respectively. The busy periods are assumed to consist of a train of packets with each packet length being independently distributed according to the continuous distribution function $P(x)$ and the train length having the discrete distribution $T_1(n)$. We also assume that the train length distribution is independent of the packet lengths. The Laplace Stieltjes Transforms (LSTs) of $B_1(x)$, $I_1(y)$ and $P(x)$ will be $\mathcal{B}_1(s)$, $\mathcal{I}_1(s)$ and $\mathcal{P}(s)$ respectively and the generating function for $T_1(n)$ will be $\mathcal{T}_1(z)$. From these definitions it is easily seen that $\mathcal{B}_1(s) = \mathcal{T}_1(\mathcal{P}(s))$.

The probability that a packet in the train on the input network will be a part of the bridge inflow process will be q and the decision that a packet is a part of the bridge inflow is independent of all past packets. The train length, the busy time and the idle time distributions of bridge inflow process will be denoted by $T_2(n)$, $B_2(x)$ and $I_2(y)$ respectively. In the next section we will obtain these from $\mathcal{B}_1(s)$, $\mathcal{P}_1(s)$ and $\mathcal{T}_1(z)$ and q.

3 Packet Process Model

In this section we derive the probability generating function for the train length and the LST of the distribution of the idle times in the packet inflow process to the bridge. The packet process on the input LAN is first described using a one dimensional Markov Chain. This model is then extended to obtain a two dimensional Markov Chain description of the input process to the bridge. This Markov Chain is then solved. A three state Embedded Markov Chain (EMC) approximation is then introduced and solved. Further details of the derivations are found in [11]

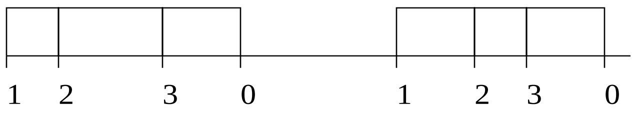

Figure 2: Embedded Points for Packet Process on Input LAN

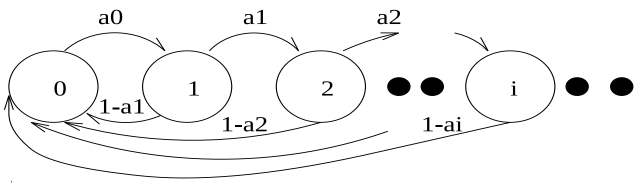

Figure 3: The EMC for Packet Process on Input LAN

3.1 Input LAN Packet Process

We first obtain an EMC model for the packet process on the input network. The embedded points are beginnings and ends of packets on the network (Figure 2). The state of the Markov chain is the sequence number of the packet in the train following the embedded point. When the network goes idle, the Markov chain enters state 0. The corresponding transition diagram is shown in Figure 3. If p_{ij} is defined as the transition probability from state i to state j, we see that $p_{ij} = 0$ for $j \neq i+1$ and $j \neq 0$. So we can define $p_{ij} = a_i$ for $j = i+1$ and $p_{ij} = 1 - a_i$ for $j = 0$, where a_i is given by

$$\begin{aligned} a_i &= \text{Prob(transitioning from state } i \text{ to state } i+1) \\ &= \text{Prob}(\geq i+1 \text{ packets in the train} \mid \geq i \text{ packets}) \\ &= \frac{1 - T_1(i)}{1 - T_1(i-1)} \end{aligned} \tag{1}$$

If we denote the steady state solution of the EMC by π_i then

$$\pi_i = \pi_0 \prod_{k=0}^{i-1} a_k \qquad \text{for } i = 1, 2, \ldots \tag{2}$$

To obtain π_0 we note that in any train with n packets there are $n+1$ embedded points and the last point corresponds to a transition to state 0. Thus, if $\overline{T_1}$ is the average train length $\pi_0 = \frac{1}{\overline{T_1}+1}$

3.2 Bridge Input Process: Markov Model

We now look at the same embedded points but describe the state with two variables, $[i, j]$. The first variable, i, corresponds to the sequence number of the packet in the train on

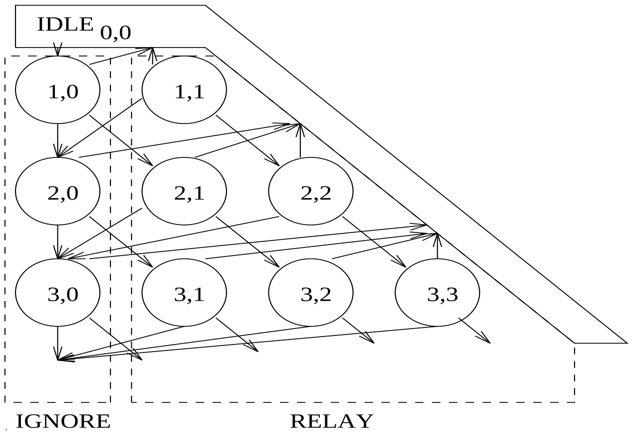

Figure 4: The two dimensional Markov Chain representation of the bridge inflow process

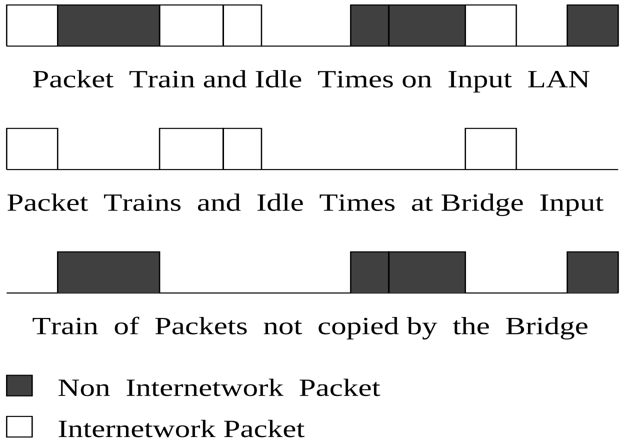

Figure 5: The Train of Packets that is not copied or relayed from the Network

the input network as before and the second variable, j, corresponds to the sequence number of the packet for the train in the bridge inflow. It is easy to see that $j \leq i$. The state $[0,0]$ will correspond to the network, and hence the bridge inflow, going idle. Also, the state $[i,0]$ means that the current packet on the input LAN is not part of the bridge inflow. The probability that a packet on the input network is part of the bridge inflow is given by q. The corresponding EMC is shown in Figure 4. The transition probabilities are

$$p_{ij \to mn} = \begin{cases} qa_i & \text{for } m = i+1,\ n = j+1 \\ (1-q)a_i & \text{for } m = i+1 \text{ and } n = 0 \\ 1 - a_i & \text{for } m = n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The steady state probabilities π_{ij} for the EMC are now obtained as follows.

$$\pi_{ij} = \begin{cases} \pi_{00} q^j (1-q)(1 - T_1(i-1)) & \text{for } i \neq j \\ \pi_{00} q^i (1 - T_1(i-1)) & \text{for } i = j \end{cases} \tag{4}$$

The train length distribution generating function and idle time distributions LST of the bridge inflow process are derived next.

3.3 Busy and Idle Periods

We now obtain the probability mass function for the train length, $t_2(n)$, and the probability density functions for the busy periods, $b_2(x)$, and idle periods, $i_2(y)$, of the bridge inflow. We denote the generating function of $t_2(n)$ by $\mathcal{T}_2(z)$ and the Laplace Transforms of $b_2(x)$ and $i_2(y)$ by $\mathcal{B}_2(s)$ and $\mathcal{I}_2(s)$ respectively.

In the EMC of Figure 4, it is seen that for $j > 0$, if the system is in state $[i,j]$ it means that the packet train of the bridge inflow is at least j packets long. If the system makes a transition to either state $[i+1, 0]$ or state $[0,0]$ the train will be exactly j packets long. The probability of making either of these transitions is $(1 - qa_i)$. From this observation the probability mass function for the train length in the bridge inflow, $t_2(n)$ is,

$$t_2(n) = \frac{\sum_{i=n}^{\infty} \pi_{in}(1 - qa_i)}{\sum_{i=1}^{\infty} \sum_{j=1}^{i} \pi_{ij}(1 - qa_i)} \tag{5}$$

This can be simplified to yield a closed form expression for $t_2(n)$ (details are in [11]).

Let $\mathcal{T}_2(z)$ be the generating function of this probability mass function. If we regard the state of the bridge inflow when there is a train of packets at the input as a busy state then the LST of the busy period, $\mathcal{B}_2(s)$, is given by

$$\mathcal{B}_2(s) = \sum_{n=1}^{\infty} \mathcal{P}^n(s) t_2(n) = \mathcal{T}_2(\mathcal{P}(s)) \tag{6}$$

We have now characterized the packet trains in the bridge inflow. We now look at the train that was not part of the bridge inflow. If the packets meant for the bridge were removed from the input network, this is the train that would have remained on the network. To put it another way, if the packets on the input network meant for the bridge were to be marked, we now look at the train of unmarked packets on the network. (See Figure 5.)

If we replace q in the equations for $\mathcal{T}_2(z)$ and $\mathcal{B}_2(s)$ by $\tilde{q} = 1 - q$ we obtain the corresponding functions $\mathcal{T}_N(z)$ and $\mathcal{N}_2(s)$ for the train that is not part of the bridge inflow.

We have now obtained the probability generating function for the train lengths in the bridge inflow process and also that of the *ignored-train*. The ignored-train is part of the idle period in the bridge inflow. The state of the bridge inflow may now be represented by an equivalent three state process corresponding to the states Idle(I), Relay(R) and Ignore(N). These states correspond to the state $[0,0]$, $[i, j \neq 0]$ and $[i, 0]$ respectively in Figure 4. We now consider an approximation. We assume that the system loses memory at the transition instants in the three state process and the time spent in each state is as derived above. This gives us a three state EMC with state transition probabilities as shown in Figure 6. To test the approximation we built a simulation model using

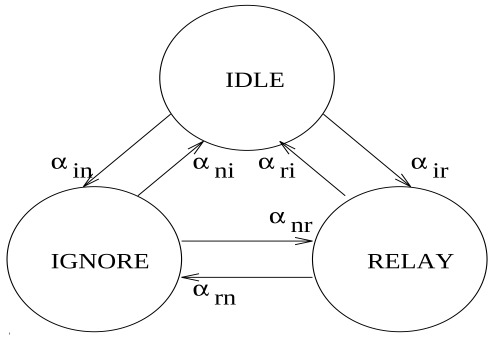

Figure 6: Three state representation of input to bridge

various distributions of input train length, packet length and idle times and also various values of q. As expected, the first moments of the busy and idle periods in the analytical and simulation model matched exactly. The difference in the second moments from the two models was of the order of 2% when the average train length on the input LAN was not very high.

We now use the approximate three state process of Figure 6. The transition probabilities (α_{ir}, α_{in}, α_{ni}, α_{nr}, α_{ri} and α_{rn}) of the three state process of Figure 6 may now be calculated from the two-dimensional EMC of Figure 4. These probabilities correspond to the probability of crossing the boundaries marked by dotted lines in Figure 4. We have
$\alpha_{ir} = q$,
$\alpha_{rn} = \frac{(1-q)(\overline{T_1}-1)}{1+(1-q)(\overline{T_1}-1)}$,
$\alpha_{ni} = \frac{1}{1+q(\overline{T_1}-1)}$,
and $\alpha_{ri} = 1-\alpha_{rn}$, $\alpha_{nr} = 1-\alpha_{ni}$ and $\alpha_{in} = 1-\alpha_{ir}$.

From the three state representation, we can observe that the bridge inflow process is idle when the system is either in the Ignore(N) or Idle(I) state. The distribution of the time spent in the idle state has LST $\mathcal{I}_1(s)$ and the distribution of the time spent in the Ignore state has the LST $\mathcal{N}_2(s)$. The LST of the distribution of the time that the bridge inflow process is idle, $\mathcal{I}_2(s)$, can now be obtained.

$$\begin{aligned}\mathcal{I}_2(s) = \textstyle\sum_{k=0}^{\infty} & [\alpha_{ri}\mathcal{I}_1(s)]\,[\alpha_{in}\alpha_{ni}\mathcal{I}_1(s)\mathcal{N}_2(s)]^k\,[\alpha_{ir}] \\ & + [(\alpha_{ri}\mathcal{I}_1(s)]\,[\alpha_{in}\alpha_{mi}\mathcal{I}_1(s)\mathcal{N}_2(s)]^k\,[\alpha_{in}\mathcal{N}_2(s)\alpha_{nr}] \\ & + [\alpha_{rn}\mathcal{N}_2(s)]\,[\alpha_{in}\alpha_{ni}\mathcal{I}_1(s)\mathcal{N}_2(s)]^k\,[\alpha_{nr}] \\ & + [\alpha_{rn}\mathcal{N}_2(s)]\,[\alpha_{in}\alpha_{ni}\mathcal{I}_1(s)\mathcal{N}_2(s)]^k\,[\alpha_{ni}\mathcal{I}_1(s)\alpha_{ir}]\end{aligned}$$

We simplify this equation to get

$$\mathcal{I}_2(s) = \frac{A_2\mathcal{I}_1(s) + B_2\mathcal{N}_2(s) + C_2\mathcal{I}_1(s)\mathcal{N}_2(s)}{1 - D_2\mathcal{I}_1(s)\mathcal{N}_2(s)} \qquad (7)$$

where $A_2 = \alpha_{ri}\alpha_{ir}$, $B_2 = \alpha_{rn}\alpha_{nr}$, $C_2 = \alpha_{rn}\alpha_{ni}\alpha_{ir} + \alpha_{ri}\alpha_{in}\alpha_{nr}$ and $D_2 = \alpha_{in}\alpha_{ni}$

4 Examples and Numerical Results

In this section we take some examples of the train length, packet length and idle time distributions for the packet process on the input network and see what the bridge inflow will be for each of these cases. We also demonstrate that three state approximation is quite accurate.

The train lengths, packet lengths and idle periods are assumed to have independent distributions. To reduce the number of possible variables characterizing the traffic, we use a traffic description similar to an M/G/1 queueing system. All lengths will be normalized to the average packet length. We use ρ to characterize the traffic on the input LAN. The mean train length distribution will be $\frac{1}{1-\rho}$ and mean idle time will be $\frac{1}{\rho}$. The input network throughput will be ρ and the internetwork throughput will be $q\rho$.

We first consider a simple case of geometrically distributed train lengths and exponentially distributed packet lengths and idle periods. In this case we see that the train lengths and the idle periods of the bridge inflow process will be of the same type as those on the input network, except that the parameter will be factored by q. Thus, the input functions are $t_1(n) = \rho^{n-1}(1-\rho)$, $i_1(x) = \rho e^{-\rho x}$, and $p_1(y) = e^{-y}$.

From these distributions and equations for $\mathcal{T}_2(z)$ and $\mathcal{I}_2(s)$ we obtain the generating functions for $t_2(n)$ and the characteristic function of $i_2(x)$. Inverting them we obtain, $t_2(n) = (q\rho)^{n-1}(1-q\rho)$, and $i_2(x) = (q\rho)e^{-q\rho}$.

In this case the approximation using the three state EMC of Figure 6 (that the system loses memory when it transitions between states) was not an approximation but an exact result.

We now give examples using fixed length packets. We use three different models for the input network packet process, i.e.three distributions for the train lengths

- a geometric distribution with parameter ρ, i.e.,
  $t_1(n) = \rho^{n-1}(1-\rho) \qquad$ for $n = 1, 2, ...$
- a modified Poisson distribution, i.e.,
  $t_1(n) = \frac{\lambda^n}{n!}\frac{e^{-\lambda}}{1-e^{-\lambda}} \qquad$ for $n = 1, 2, 3, ...,$
  where λ is the solution of $\frac{\lambda}{1-e^{-\lambda}} = \frac{1}{1-\rho}$
- the traffic on the input LAN resembles the output process from an M/D/1 queue, i.e.,
  $t_1(n) = \frac{(n\rho)^n}{n!}e^{-n\rho} \qquad$ for $n = 1, 2, 3...$

In all these examples we assume that the packet length distribution and the idle time distributions are given by $p(x) = \delta(x-1)$ and $i_1(y) = \rho e^{-\rho y}$

For each of these traffic models we calculated the first and the second moments of the idle periods and the busy periods in the bridge inflow process using the three state approximation.

We constructed an exact simulation model to verify the approximations. The analytical results for the first moments were exactly equal to that from simulation. The second moments of the busy period were very close to the results obtained from the simulation model (within 0.5%) and the second moments of the idle periods obtained from the analytical model and the simulation model are within 2.5% of each

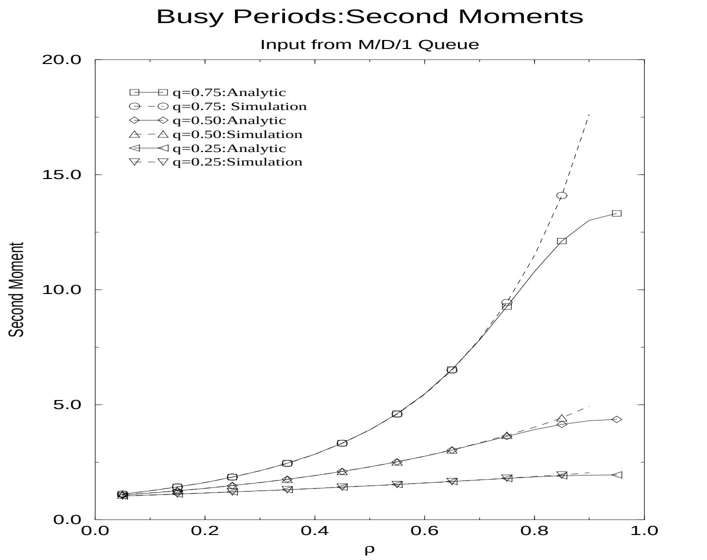

Figure 7: Second Moment of the Busy Periods in the Bridge Inflow Process: Train Lengths from the output process of an M/D/1 queue. Analytical and Simulation Results

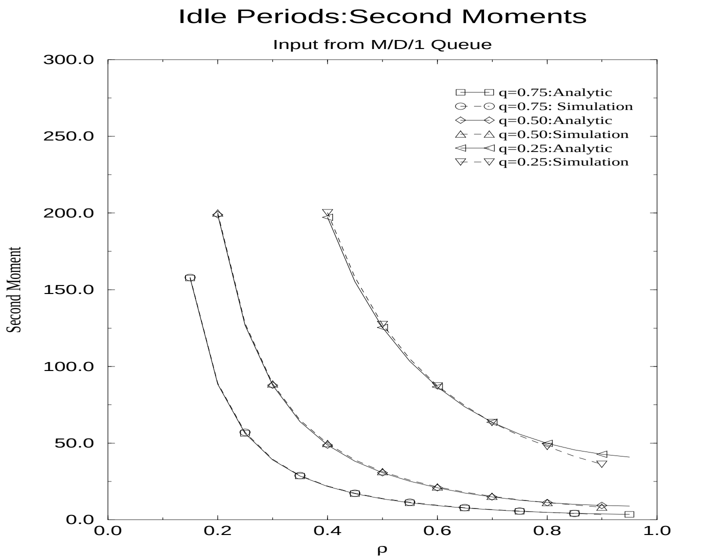

Figure 8: Second Moment of the Idle Periods in the Bridge Inflow Process: Train Lengths from the output process of an M/D/1 queue. Analytical and Simulation Results

other. Two examples illustrating the accuracy of these results for second moments results are plotted in Figures 7 and 8. More results may be found in [11].

5 Conclusion and Future Work

We have characterized the bridge inflow process in an internetworking environment. This process is described as alternating trains of packets and idle periods. Some examples have also been provided. Our approximation of the bridge inflow by a three state process turned out to be a very good one in light of the simulation results. Also, our assumption of the train length being independent of the packet length is not a limitation in the constant-packet-length environments of ATM and DQDB type of networks.

We are currently modeling the bridge inflow process when there is correlation between internetwork packets of the same train. This would mean that the copy/relay probability q would depend on the state of the Markov chain. We then use techniques on correlated queues to characterize the output process from the bridge processor and the bridge outflow process.

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