EFFECT OF OBJECTIVE FUNCTION ON VIRTUAL TOPOLOGY DESIGN IN OPTICAL NETWORKS

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ABSTRACT

The virtual topology overlay on a physical topology optimises a performance measure like, for example, lightpath congestion on a physical link, traffic congestion on a lightpath, and maximum (or average) hops on the virtual topology between a source destination pair in the network. The virtual topology is typically obtained as solution to a mixed integer linear program (MILP) that optimises any of the above performance measures. In this paper we investigate the effect of optimising one measure on the other measures.

1. INTRODUCTION

In wavelength routed optical networks, a virtual topology is overlaid on the physical network that minimises the electro optical conversions of the traffic flows. The network topology seen by the optical layer is the physical topology. The virtual topology may be represented by a graph whose vertex set is the network nodes and an edge between two vertices of corresponds to having a lightpath between them in the virtual topology. Thus the virtual topology in an optical network is the topology seen by the higher layers of the communication stack and may not be a complete graph. Node pairs not directly connected by lightpaths must use a sequence of lightpaths through intermediate nodes to communicate.

Typically, virtual topology design involves determining source and destinations of the lightpaths, the traffic carried on them, their routes in a physical topology and the wavelengths assigned to them. Ramaswami and Sivarajan [1] describe the designing a virtual topology and routing when physical topology and a traffic matrix are given. The virtual topology is obtained as a solution to a mixed integer linear program (MILP) with the objective function minimising a *congestion* measure, the maximum offered load to a lightpath. No constraints on the number of wavelengths per fibre are prescribed. In [1] several heuristics for designing virtual topology are also presented. Banerjee and Mukherjee [2] give an exact mixed integer linear programming (MILP) formulation for virtual topology design. Average hop distance in the network is used as a performance measure and the objective function minimises this through a simplified and computationally tractable problem formulation that assumes wavelength converters at all nodes. The survey by Dutta and Rouskas [3] of virtual topology design algorithms is a good reference detailing variations and other performance measures considered in literature. In this paper our interest is studying the effects that optimising one performance measure has on the other measures. In the next section we briefly describe the objective functions that have been used in virtual topology design and the motivation for each of these. We also describe the mathematical programming problem for each of these objective functions. Section 3 presents numerical results of the effect of optimising one measure on the other measures. Finally, in Section 4 we discuss our results and propose that virtual topology be designed us-

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ing multiobjective optimisation techniques.

2. OBJECTIVE FUNCTIONS IN VIRTUAL TOPOLOGY DESIGN

As mentioned above, different performance measures may be optimised in the design of the virtual topology and the virtual topology is a solution to a MILP. The physical topology and the traffic matrix are given as inputs and the constraints to the MILP are multicommodity flow based equations governing routing of lightpaths from source to destination and flow of traffic through virtual topology. Typically, the number of transmitters and receivers at a node will constrain the out-degree and in-degree respectively of the virtual topology nodes. Also, the number of wavelengths, and hence lightpaths, that a physical link can carry will typically have an upper bound.

The following is an enumeration of the performance measures that form the objective functions for the MILP.

- The average hop distance on the virtual topology for traffic between any source-destination pair corresponds to the number of electro optic conversions for the traffic and is a measure of the delay and also a measure of the resources that will be required at the electro optic bottleneck. One obvious optimisation would be to minimise the average hop count in the virtual topology and we will denote the MILP corresponding to this by VT-HD. This is the objective used in [2].

- Each lightpath corresponds to a transceiver. Thus minimising the total number of lightpaths in the network will minimise the cost of the transceivers and is a legitimate objective function and we will denote the MILP corresponding to this by VT-NL.

- The traffic in a lightpath corresponds to the utilisation of the lightpath and if packet multiplexing is used in a lightpath is a measure of the packet delays that will be possible on that lightpath. Alternatively, if circuit multiplexing is used the utilisation is a measure of the blocking probability on the lightpath. Thus minimising traffic congestion, the maximum flow (congestion) in a lightpath, may also be used as a design criteria. This is the objective function in [4, 1] and we will denote the MILP corresponding to this by VT-TC.

- The maximum number of lightpaths on any physical link is a measure of the expandability of the virtual topology and minimising it could also be used as an objective in the MILP and this will be denoted by VT-LC.

It is obvious that these performance measures are interrelated and in the virtual topology design, the effect of optimising one measure on the others needs to be studied. For example, minimising the average hop distance (VT-HD) can maximise the total number of lightpaths and increase lightpath congestion. If lightpath congestion is minimised (VT-LC), more traffic is packed on each lightpath and the traffic congestion increases. Thus it is better to decide the objective function based on its effect on the performance measures as there is a tradeoff between these measures.

In the following we formally describe the MILP and the objective functions enumerated above. The constraints are assumed to be like that described in [2] and we assume wavelength converters at the routing nodes. The following notation is used: s and d as subscript or superscript denote *source* and *destination*, respectively; i and j denote *originating* and *terminating* nodes respectively, in a lightpath; m and n denote *endpoints of a physical link* that might occur in a lightpath. Let

- N be the number of nodes in the network.

- W be the maximum number of wavelengths per physical link.

- The physical topology with at most one fibre link connecting between a node pair is given. The physical network will have a total of M fibre links in the network. $P_{mn} = 1$ is the $1/0$ variable indicating the presence of a fibre link between nodes m and n.

- We will assume that a lightpath between i and j will be routed on one of the alternate shortest paths in the physical topology.

- The number of transmitters and receivers at node T_i and R_i respectively, are given. We assume $T_i = R_i$, although this is not a strict requirement.
- The traffic matrix TM is given where Λ_{sd} denotes traffic flow from node s to node d, in units of lightpath capacity.

The MILP will assign the following variables.

- Virtual topology variables, V_{ij} that denote the number of unidirectional lightpaths from node i to node j in the virtual topology. V_{ij} is a nonnegative integer.
- Traffic routing variables, λ_{ij}^{sd} that denote the traffic flowing from node s to node d on a lightpath between i and j. The total traffic between a source destination pair may split with different components flowing through different sets of lightpaths.
- Physical topology route variables, p_{mn}^{ij} denoting the number of lightpaths between nodes i and j routed through physical link mn.

As discussed earlier, the following objective functions will be used in the MILP.

- VT-HD: Minimise the average hop distance.

$$\text{Minimise: } \frac{1}{\sum_{s,d}\Lambda_{sd}} \sum_{i,j}\sum_{s,d} \lambda_{ij}^{sd} \tag{1}$$

- VT-NL: Minimise total number of lightpaths.

$$\text{Minimise : } \sum_{i,j} V_{ij} \tag{2}$$

- VT-TC: Minimise congestion on lightpaths.

$$\text{Minimise :} F_{max} \tag{3}$$

$$\text{such that } F_{max} \geq \sum_{s,d} \lambda_{ij}^{sd} \quad \forall\ ij \tag{4}$$

- VT-LC: Minimise number of lightpaths on a physical link.

$$\text{Minimise :} F_{max} \tag{5}$$

$$\text{such that } F_{max} \geq \sum_{i,j} p_{mn}^{ij} \quad \forall\ mn \tag{6}$$

These MILPs will have the following constraints.

- On virtual topology connection matrix V_{ij}: Constraints to upper bound the number of lightpaths emerging from a node is by the number of transmitters at that node and the number terminating at a node by the number of receivers. Also, V_{ij} are integers.

$$\sum_j V_{ij} \leq T_i \quad \forall\ i \tag{7}$$

$$\sum_i V_{ij} \leq R_j \quad \forall\ j \tag{8}$$

$$\text{int } V_{ij} \tag{9}$$

- On the physical route variables p_{mn}^{ij}: There will be two kinds of constraints – multicommodity flow equations for routing of lightpaths on the physical topology (Eqns 10 and 11), and a constraint to upper bound the number of lightpaths flowing through a fibre link to W (Eqn 12). Since we assumed that the nodal architecture provides wavelength conversion there will be no wavelength continuity constraint. We will also need integrality constraints on p_{mn}^{ij}.

$$\sum_m p_{mk}^{ij} = \sum_n p_{kn}^{ij} \quad \text{if } \ k \neq i,j \tag{10}$$

$$\sum_n p_{in}^{ij} = V_{ij} \quad ; \quad \sum_m p_{mj}^{ij} = V_{ij} \tag{11}$$

$$\sum_{i,j} p_{mn}^{ij} \leq W \ \ P_{mn} \tag{12}$$

$$\text{int } p_{mn}^{ij} \tag{13}$$

- On virtual topology traffic variables λ_{ij}^{sd}: We will need multicommodity flow constraints for each of the lightpaths (Eqns 14 and 15). We will also need to constrain the traffic to an existing lightpath (Eqn 16) and a capacity constraint on the flow through a lightpath (Eqn 17).

$$\sum_i \lambda_{ik}^{sd} = \sum_j \lambda_{kj}^{sd} \quad \text{if } \ k \neq s,d \tag{14}$$

$$\sum_j \lambda_{sj}^{sd} = \Lambda_{sd} \;\; ; \;\; \sum_i \lambda_{id}^{sd} = \Lambda_{sd} \tag{15}$$

$$\lambda_{ij}^{sd} \leq \Lambda_{sd}\, V_{ij} \tag{16}$$

$$\sum_{s,d} \lambda_{ij}^{sd} \leq V_{ij} \tag{17}$$

The following two simplifications reduce the total number of equations and variables making the problem tractable on easily available programming packages like lpsolve. First, we reduce the number of constraints by pruning the search space by restricting the lightpath routes to only the alternate shortest paths, i.e., lightpaths are routed only through one of a few permissible routes and all combinations of p_{mn}^{ij} are not enumerated. Second, we restrict the possible lightpaths to those between the set of nodes present in the alternate shortest paths obtained above. This means that all possible values of λ_{ij}^{sd} are not enumerated and only a some are possible candidates for the lightpaths.

3. NUMERICAL EXAMPLES

We now describe the numerical results to investigate the effect of optimising one performance measure on the other measures. The physical topology, traffic matrix, bound on the number of transmitters, T and receivers, R at each node and number of wavelengths supported by any fibre link, W are inputs to the MILP. The physical topology is obtained as a random graph by specifying the number of nodes in the graph, N, and the probability of an edge, P_e between any node pair. Each entry in the traffic matrix is chosen randomly with uniform distribution. The MILP is solved to optimise one performance measure and the other measures are obtained from the solution to the MILP. The performance measures are average hop distance (HD), total number of lightpaths (NL), traffic congestion (TC), and lightpath congestion (LC).

We first describe one example in detail and then summarise the results from other examples. The six node example physical topology is shown in Fig 1 and the traffic matrix with traffic normalised to the lightpath capacity in Table 1. The results of MILP for this example in terms of the performance measures are as shown in Table 2. Observe that while optimising average hop distance (VT-HD) there is maximum lightpath congestion which is three times the optimal value. Similarly, when lightpath congestion is optimised (VT-LC), the traffic congestion is 0.9 as opposed to the optimal value of 0.3. Similar observations can be made about the other measurements. More importantly, we see that these observations carry over into other examples. We performed similar experiments for different random topologies with random traffic matrices and the results are shown in Tables 3– 5.

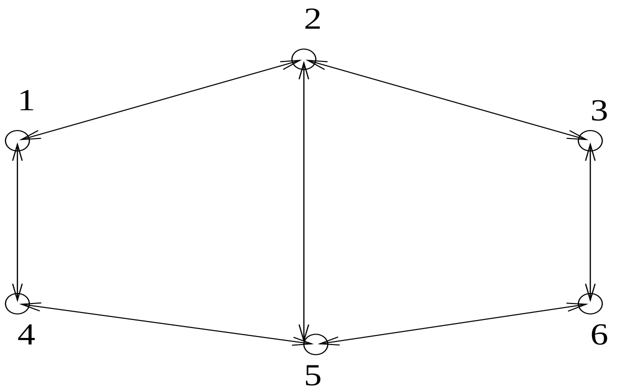

Figure 1: A sample six node network

Nodes	1	2	3	4	5	6
1	0	0	0.5	0	0	0.2
2	0	0	0	0	0	0.2
3	0	0	0	0	0	0
4	0	0	0.3	0	0.3	0.2
5	0	0	0	0	0	0
6	0	0	0	0	0	0

Table 1: Traffic matrix for six node topology shown in figure 1

4. DISCUSSION

In this paper we investigated the virtual topology design with different objective functions such as average logical hop distance, lightpath congestion on any physical link, traffic congestion on any log-

Measures	VT-HD	VT-NL	VT-TC	VT-LC
$W = 3$				
HD	1	1.411	1.294	2.117
NL	6	6	9	7
TC	0.5	0.7	0.3	0.9
LC	3	3	3	1
$W = 2$				
HD	1.117	1.588	1.676	2.117
NL	6	6	11	7
TC	0.7	0.7	0.375	0.9
LC	2	2	2	1

Table 2: For 6 node topology shown in Figure 1, traffic given by Table 1, $T = R = 3$.

Measures	VT-HD	VT-NL	VT-TC	VT-LC
HD	1.108	1.228	1.182	1.428
NL	29	26	31	34
TC	2	2	1.7	5
LC	8	8	8	8

Table 3: For 6 node random topology with $P_e = 0.4$, uniform traffic in $(0, 1)$, $T = R = 8, W = 8$

ical link and total number of lightpaths. Specifically, we have investigated the effect of different objective functions on the performance measures. We observe that optimising one measure can take the other measure(s) significantly away from the optimal value. Since it is not always obvious as to what should be optimised, we believe, it is useful to consider virtual topology design using multiobjective combinatorial optimisation techniques [5].

Measures	VT-HD	VT-NL	VT-TC	VT-LC
HD	1.007	1.325	1.090	1.295
NL	29	22	26	25
TC	0.9	2	0.9	1.9
LC	6	4	5	4

Table 4: For 6 node random topology with $P_e = 0.6$, uniform traffic in $(0, 1)$, $T = R = 6, W = 6$

Measures	VT-HD	VT-NL	VT-TC	VT-LC
HD	1	1.219	1.103	1.219
NL	41	36	38	40
TC	1.8	2.7	1.8	3.2
LC	7	5	5	4

Table 5: For 6 node random topology with $P_e = 0.7$, uniform traffic in $(0, 2)$, $T = R = 9, W = 10$

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