Routing and Wavelength Assignment in Optical Networks from Edge Disjoint Path Algorithms

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Abstract— **Routing and wavelength assignment (RWA) problems in wavelength routed optical networks are typically solved using a combination of integer programming and graph coloring. Such techniques are complex and make extensive use of heuristics. We explore an alternative solution technique in the well known maximum edge disjoint paths (EDP) problem which can be naturally adapted to the RWA problem. Maximum EDP is NP-hard, but now it is known that simple greedy algorithms for it are as good as any of the more complex heuristic solutions. In this paper we investigate the performance of a simple greedy maximum edge disjoint paths algorithm applied to the RWA problem and compare it with a previously known solution method.**

Keywords— **Optical Networks, Routing and Wavelength Assignment, Edge Disjoint Path Algorithms.**

I. Introduction

Routing and wavelength assignment (RWA) for the lightpaths of the virtual topology in wavelength routed optical networks is typically solved in two parts – lightpath routing, to determine the route of the lightpaths on the physical topology, and wavelength assignment, to assign the wavelengths to each lightpath in the virtual topology such that wavelength constraints are satisfied for each physical link. The RWA problem for a given physical network and virtual topology is known to be NP-complete. Further, routing and wavelength assignment problems are each known to be individually NP-complete [1] and many heuristic solutions have been proposed. See [1] for a recent survey of the various solutions to the RWA problem.

Lightpath routing is done by first selecting the lightpaths and then assigning routes to them. The order in which the lightpaths are assigned can be selected by different heuristic schemes such as random, fixed, longest-first or shortest-first. Routes can be found by techniques such as shortest path algorithms, weighted shortest path and $K-$shortest path algorithms. The route is selected from the available candidates using different schemes such as random, first-fit or probabilistic. The RWA problem is also solved by formulating it as an integer linear program (ILP) optimization problem [2] with one of two objective functions – minimize the required number of wavelengths to establish the given set of lightpaths and maximize the number of established lightpaths with a fixed number of wavelengths. A hard version of the problem is when no wavelength conversion is used at the routing nodes of the physical network and this requires that the wavelength continuity constraint be satisfied, i.e., a lightpath should use the same wavelength on all the links in its path. Extensive and complex heuristics are used to solve these ILPs. One solution considers a fractional relaxation with rounding techniques to obtain an integral solution [3]. In this solution method a multicommodity flow problem where the maximum flow on any physical link is minimized subject to flow, capacity and resource constraints is solved. Fractional relaxation for multicommodity flow formulation combined with randomized rounding is used to calculate the routes for lightpaths.

Wavelength assignment (WA) is solved as the coloring of the path graph $P(G)$ in which each lightpath of the virtual topology is a node in $P(G)$ and two nodes in $P(G)$ are connected by an undirected edge if the corresponding paths share a common edge in the physical topology. Thus wavelength assignment is equivalent to coloring the vertices of $P(G)$ such that adjacent edges do not have the same color. Each color is a wavelength.

From above, solving the RWA problem using mathematical programming is quite complex in view of the large number of variables and constraints and complex problem size reduction heuristics. Further, it will not be known how close the solution is to the optimal. Given this, it would be interesting to look for simple and intuitive alternative methods for RWA with performance comparable to that from mathematical programming techniques. In the next section we describe one such method based on the greedy solution of a well known graph theory problem. We then investigate its performance in comparison with the solution method of [3].

Pallavi Manohar was supported by a Photonic Switching Project from the Ministry of Human Resources Development, Govt of India. Author Address: Department of Electrical Engineering, Indian Institute of Technology, Bombay, Powai Mumbai 400 076 India. Email: pallavim, dmanju, rks@ee.iitb.ac.in

II. RWA Using Edge Disjoint Paths

In this section we develop a simple and intuitive solution to the RWA problem. Observe that the lightpaths that are assigned the same wavelength will not traverse the same physical link, i.e., these paths are *edge disjoint*. In graph theory, given a graph and a set of source-destination pairs and the requirement that a path be found for as many of the pairs as possible, finding the maximum set of edge disjoint paths is a well known problem and is formally defined as follows. Let $G = (V, E)$ be the graph of the physical network, V the vertex set and E the edge set. Connection request i is specified by a pair (s_i, t_i), $s_i, t_i \in V$. Let τ be the set of connections for whom edge disjoint paths need to be found, $\tau = \{(s_1, t_1), \ldots, (s_k, t_k)\}$. τ is said to be realizable in G if there exist mutually edge-disjoint paths $P_1, \ldots, P_k$ in G such that P_i has endpoints s_i and t_i. The *maximum edge disjoint paths problem* is to find a maximum size realizable subset of τ, given G and τ. Let the maximum size of a realizable subset of τ in G by $\alpha(\tau)$. It is easy to see that the maximum edge disjoint paths (EDP) problem is a combinatorial optimization problem and is known to be NP-hard [4].

Given G and τ, let $\tau_1, \cdots \tau_i$ be a partition of τ such that each element of the partition is a solution to the maximum EDP problem on G, τ. Now the application to the RWA problem is follows. Given G and τ consider a solution of the maximum EDP problem that obtains a realizable set $\tau_1 = \alpha(G, \tau)$. Since the paths in τ_1 are edge disjoint they can be assigned the same wavelength, say λ_1. Now construct the set $\tau_1' = \tau - \tau_1$, i.e., τ_1' is the set of connections not contained in τ_1. An iteration of maximum EDP on the G and τ_1' will give $\tau_2 = \alpha(G, \tau_1')$ and the paths in τ_2 can be assigned the same wavelength, say λ_2. This process can be repeated till τ_i' is empty for some i. The minimum such i is denoted by $\chi(G, \tau)$. Finding $\chi(G, \tau)$ is the problem of minimizing the number of wavelengths in wavelength routed optical networks. Observe that if the solution to the maximum EDP is optimum, the optimum $\chi(G, \tau)$ can be obtained from above.

The maximum EDP is also hard to approximate and good heuristics are not well known [4]. The current best approximation guarantee for arbitrary graphs is $\mathcal{O}(\max \sqrt{m}, \mathtt{diam}(G))$, where m is the number of edges and $\mathtt{diam}(G)$ is the diameter of the graph G [4]. Srinivasan and Baveja [5] improved the approximation algorithms based on LP relaxation. Recently, Guruswami *et al* [6] have shown that simple greedy algorithms and randomized rounding algorithms can yield approximations similar to those given in [5]. This motivates us to investigate the use of greedy EDP algorithms for RWA and compare it with the results from [3].

In the following we describe the details of the solution of the RWA problem using a greedy solution to maximum EDP.

A. *RWA with Greedy EDP*

We use the bounded greedy algorithm for the maximum EDP that was first described in [4] to assign routes and a wavelength to set of lightpaths. In this algorithm, we define an upper bound d for the number of edges in a path that will be considered by the algorithm as being part of the solution. Kleinberg [4] has argued that a reasonable value for d is $\max(\mathtt{diam}(G), \sqrt{|E|})$ and we use this in our algorithm. The inputs to the algorithm are the graph G of the physical network, lightpath set τ and the bound d defined as above. The output of the algorithm is $\mathcal{P}$ which is the set of pairs (τ_i, P_i) where τ_i is a routed lightpath and P_i is the physical path assigned to this lightpath. The algorithm, BGAforEDP(G, τ, d), operates as follows. Randomly select a lightpath τ_i from the set τ, find the shortest path P_i for this connection. If path length of P_i is less than the bound d, add (τ_i, P_i) to the path set $\mathcal{P}$ and τ_i to the set of routed lightpaths $\alpha(G, \tau)$ and delete the edges in G used by P_i from G. If the path length of P_i is greater than d, lightpath τ_i is not routed. This is repeated for all the connections in τ. The algorithm is summarized in Figure 1. At the end of the algorithm, $\alpha(G, \tau)$ contains the lightpaths that will be assigned the same wavelength.

An iterative algorithm for solving RWA using maximum EDP can be developed immediately. Each iteration of the BGAforEDP will result in a set of lightpaths $\alpha(G, \tau)$ that can be assigned the same wavelength. Remove this set from τ and get the set of unassigned lightpaths. We run BGAforEDP with the physical topology graph G and the set of unassigned lightpaths to obtain the set of lightpaths that will be assigned another wavelength. This is repeated till all the lightpaths are assigned. The pseudo code for this algorithm is shown in Figure 2.

```
Algorithm BGAforEDP(G,τ,d)
Begin
    k=Number of lightpaths to be routed
    α(G,τ) = φ; P(G,τ) = φ
    Randomly permute elements of τ
    For i = 1 to k
        Find shortest path P_i for τ_i
        If ((path length of P_i) ≤ d) then
            Select path P_i for τ_i
            α(G,τ) = α(G,τ) ⋃ τ_i
            P = P ⋃ (τ_i, P_i)
            Delete edges in path P_i from G
        EndIf
    EndFor
End
```

Fig. 1. Pseudo code for algorithm BGAforEDP

```
Algorithm Greedy_EDP_RWA(G,τ)
Begin
d = max (diam(G), √|E|)
λ = 0
While ( τ ≠ φ) do
    λ = λ + 1
    BGAforEDP(G,τ,d)
    Assign λ to all paths P_i ∈ P
    τ = τ − α(G,τ)
EndWhile
End
```

Fig. 2. Pseudo code for the Algorithm Greedy_EDP_RWA.

III. Numerical Results

We now describe some numerical results with which we compare the performance of the Greedy_EDP_RWA and the ILP and graph coloring based algorithm described in [3]. We compare the number of wavelengths used to route a given lightpath set on a given physical topology. We also compare the execution time for both these algorithms. The details of our experiments are as follows. We obtain a random graph for the physical topology by specifying the number of nodes in the graph, N, and the probability of an edge, P_e between any node pair. The lightpath set τ is also generated randomly by populating the $N \times N$ matrix with 1s and 0s where the probability of 1 is specified by P_l and a 1 in position (i, j) indicates a lightpath between node i and node j. Note that Banerjee and Mukherjee provide the numerical results for their algorithm in a similar manner. For a given G and τ, we run the algorithm in [3] and Greedy_EDP_RWA. The linear program formulation is solved using *lpsolve*. The randomized rounding algorithm is run ten times (each time with a different seed) on each LP output and the best result obtained is chosen. The Greedy_EDP_RWA algorithm is also run with different permutations of τ and the assignment that uses the least number of wavelengths is chosen.

Table I shows the number of wavelengths used for each of combination of G and τ and also the execution time for both the algorithms. Note that Greedy_EDP_RWA consistently takes less time than that the LP based algorithm and the number of wavelengths is always the same, except in the case of the 20

N	P_e	P_l	Algorithm of [3]		Greedy_EDP_RWA	
			λs	time (s)	min,max λs	time (s)
6	0.4	0.4	5	0.04	*5*,5	0.03
6	0.8	0.4	1	0.04	*1*,2	0.01
10	0.7	0.2	3	0.06	*3*,3	0.01
10	0.7	0.4	4	0.22	*4*,4	0.12
20	0.2	0.1	3	1.02	*3*,4	0.95
20	0.2	0.2	8	2.71	*8*,9	2.54
20	0.2	0.4	11	9.99	*11*,14	3.63
20	0.2	0.6	12	13.9	*13*,15	4.03
20	0.3	0.1	3	0.98	*3*,4	0.95
20	0.4	0.2	2	2.72	*2*,2	0.69
20	0.6	0.1	1	1.11	*1*,1	0.39
30	0.2	0.1	4	10.8	*4*,5	7.02
30	0.2	0.2	7	22.5	*7*,8	10.57
40	0.2	0.1	3	38.5	*3*,3	13.44

TABLE I

Numerical results to illustrate the comparative performance of Greedy_EDP_RWA and the algorithm of Banerjee and Mukherjee [3]

node network with $P_e = 0.2$ and $P_l = 0.6$. We mention here that the choices of P_e and P_l were governed by the size of the problem that could be given to `lpsolve`.

IV. Limited Wavelengths

As we said earlier, the RWA can also take the form maximizing the number of lightpaths that are routed using a fixed number of wavelengths, say L. This is similar to the objective function of Ramaswami and Sivarajan [2]. The Greedy_EDP_RWA algorithm defined above may be used in this form of RWA as follows. Observe that there can only be a maximum of L partitions of the lightpath set τ with each partition being assigned the same wavelength. It is easy to see that we will need to perform L iterations of the main loop of Greedy_EDP_RWA. The RWA problem constrained in this manner cannot be solved using the method of Banerjee and Mukherjee [3] or any of the other LP based formulations is not straightforward because in these, the wavelength assignment is done separately through a graph coloring algorithm.

To estimate the performance of Greedy_EDP_RWA for RWA constrained in this manner we performed the following experiment. We constructed a random graph for the physical network with N nodes and imposed a fully connected virtual topology on this network leading to $N(N-1)$ lightpaths in the network. We obtained the carried traffic as a function of L for this network. The experiment was performed with 100 randomly generated physical networks, and the mean and variance of the carried traffic for different L was obtained. In Figure 3 we plot the mean and variance for a network with $N = 15$ nodes. The experiment was conducted for random physical networks with edge probabilities $P_e = 0.3$ (average of 63 directed edges) and $P_e = 0.4$ (average of 84 directed edges). Observe that the denser physical network requires about 10 wavelengths to construct a fully connected virtual topology with probability very nearly 1 while the sparser network requires considerably more than 20, i.e., the reduction in the required number of wavelengths decreases faster than linearly with increasing edges. Also observe that the incremental carried traffic with increasing number of wavelength diminishes very rapidly with increasing L.

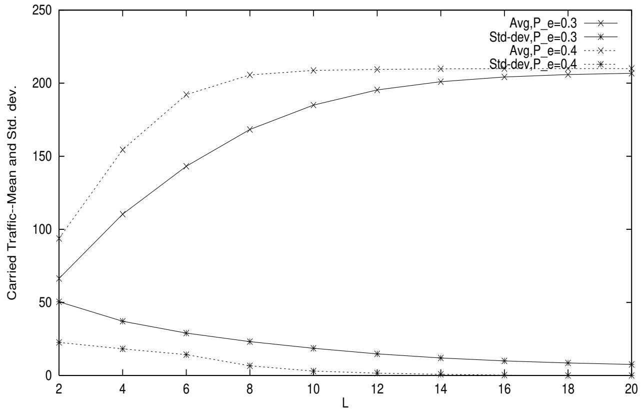

Fig. 3. Average carried traffic in 15-node physical networks generated as random graphs with edge probability 0.3 and 0.4. The average is obtained from 100 samples.

V. Conclusion

In this paper we have presented an edge disjoint path based solution for the routing of lightpaths of the virtual topology of an optical network and the assignment of wavelengths to them. This method is appealing in its simplicity and yet provides a performance in terms of the number of wavelengths required to deploy a given virtual topology comparable to that of more complex solution methods based on integer linear programming and graph coloring. Further, this algorithm can also be used to solve the RWA in which the total number of available wavelengths is upper bounded and the carried traffic needs to be maximized.

Acknowledgments

The authors would like to thank an anonymous referees for helping improve the presentation in the paper.

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