Algorithm for obtaining a Direct Decomposition into Cyclic Invariant Subspaces

(following GANTMACHER: Theory of Matrices, Vol-I)

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CONTEXT

V is a finite-dimensional vector space over any field F, $L : V \to V$ a linear function

ALGORITHM

STEP 1

Determine a vecctor, say, w_1, whose minimal polynomial (mp), p_1, is the mp of the whole space V.

Calculate $css(w_1)$. This is V_1 of the decomposition

$$V = V_1 \oplus V_2 \oplus ... \oplus V_k$$

to be calculated.

If $V_1 = V$, **STOP**

If $V_1 \subset V$, go to Step 2

STEP 2

All mp calculations are now to be done MODULO $css(w_1)$ or V_1.

Determine a vector, say, $w_2^{'}$, not in $css(w_1)$, whose mp, RELATIVE TO or MODULO V_1, say $\overline{p_2}$, is the same as the relative mp of V.

From $w_2^{'}$, calculate w_2 as in the algorithm for determination of a vector whose mp is the same as the mp of the whole space.

Then, $css(w_2)$ will be independent of $css(w_1)$ and it will be our V_2.

(As a check on your calculations, this mp will also be the ABSOLUTE mp of w_2.)

STEP 3

If $css(w_1) \oplus css(w_2) = V$, **STOP**.

If not, choose a $w_3^{'}$, not in $css(w_1) \oplus css(w_2)$, etc doing calculations MODULO $css(w_1) \oplus css(w_2)$. A vector w_3 obtained this way generates $css(w_3) = V_3$.

(Again, the relative mp obtained here is also the absolute mp of w_3)

...and so on ....

As a check on your calculations, p_1 will be divisble by p_2, p_2 by p_3, and so on ....Further $dim(V)$ will be equal the sum of the degrees of all these mp's.