Design of a Gm-C filter of very low transconductance & highly resistant to out-of-band Electromagnetic Interferences

Presented by
Snehasish Roychowdhury

Department of Electrical Engineering.
IIT Bombay
Application: $G_m$-C filter of few Hz cut-off

- Bio-Medical applications:
  
  o Pulse rate of a human body is 72 pulses/min on average.
  o Hence, to isolate and detect our pulses from environmental noises in electronic systems, low pass filters of cut-off frequency in Hz is needed.
  o This is how we can reduce the out of band noises and interferences by low pass filtering.
Cut-off frequency of Gm-C filter

- \((v_{in} - v_{out}) \times G_m = I_{out}\)

Again,

- \(v_{out} = I_{out} \times \left(\frac{1}{sC_{load}}\right)\)

Hence,

- \((v_{in} - v_{out}) \times G_m = sC_{load} v_{out}\)

- \(v_{in} \times G_m = (sC_{load} + G_m) \times v_{out}\)

- \(\frac{v_{out}}{v_{in}} = \frac{1}{1 + \frac{sC_{load}}{G_m}}\)

- Hence, DC gain = 0dB

- Cut-off frequency of the filter is: \(\frac{G_m}{2\pi C_{load}}\) Hz
Block diagram of $G_m$-C filter

- **Cut-off freq** = $\frac{G_m}{2\pi C_{load}}$
- For 70Hz, $C_{load} = 1\,\text{pF}$, $G_m = 0.47\,\text{nA/V}$.
- $G_m = \frac{i_{out}}{v_{id}} = \frac{i_{out}}{v_{im}} \ast \text{Attenuation factor} \left(\frac{1}{k}\right)$
- Voltage attenuator at input stage reduces $G_m$
- 2nd stage **Trans-conductor**: Input: $v_{im}$ (voltage), output: $I_{out}$ (current)
- All transistors in $G_m$-C filter are in **sub-threshold** region.

---

**Fig2. Block diagram of $G_m$-C filter**
Cross-coupled trans-conductor design:

\[ I_{o1} = I_1 + I_3 \quad \text{&} \quad I_{o2} = I_2 + I_4 \]

\[ I_1 = \frac{\exp\left(\frac{v_{ss1} - v_{m1}}{nV_T}\right)}{\exp\left(\frac{v_{ss1} - v_{m2}}{nV_T}\right)} = \exp\left(-\frac{v_{im}}{nV_T}\right) \]

\[ I_2 = \frac{\exp\left(\frac{v_{ss2} - v_{m2}}{nV_T}\right)}{\exp\left(\frac{v_{ss2} - v_{m1}}{nV_T}\right)} = \exp\left(\frac{v_{im}}{nV_T}\right) \]

Now,

\[ \frac{I_1 - I_2}{I_1 + I_2} = \frac{\exp\left(-\frac{v_{im}}{nV_T}\right) - 1}{\exp\left(-\frac{v_{im}}{nV_T}\right) + 1} = \frac{\exp\left(-\frac{v_{im}}{2nV_T}\right) - \exp\left(\frac{v_{im}}{2nV_T}\right)}{\exp\left(-\frac{v_{im}}{2nV_T}\right) + \exp\left(\frac{v_{im}}{2nV_T}\right)} = -\tanh\left(\frac{v_{im}}{2nV_T}\right) \]

\[ I_1 - I_2 = -I_{ss1} \tanh\left(\frac{v_{im}}{2nV_T}\right) \]

\[ I_3 - I_4 = I_{ss2} \tanh\left(\frac{v_{im}}{2nV_T}\right) \]

\[ I_{od} = I_{o1} - I_{o2} = (I_{ss2} - I_{ss1}) \tanh\left(\frac{v_{im}}{2nV_T}\right) \quad \text{...... (1)} \]

Fig3. Cross-coupled Trans-conductor
Voltage attenuator design:

- **DC Analysis:**
  
  Assume all $g_m$'s & $I_{bias}$'s are equal. Hence,
  
  $I_1 = I_3 = I_5$ & $I_2 = I_4 = I_6$
  
  Now, for DC analysis, $V_1 = V_2$
  
  By symmetry,
  
  $V_a = V_b = V_c$ &
  
  $V_1 = V_{m1} = V_{m2} = V_2$ .................. (2)

Voltage attenuator design (Ctd.)

Small signal Analysis:
Assumed all $g_m$’s are equal.
\[
(v_a - v_1) + (v_c - v_2) = 0
\]
& \[
\frac{g_m}{4} (v_a - v_c) = g_m (v_c - v_2)
\]
\[
(v_a - v_b) = \frac{2}{3} (v_1 - v_2) \quad \ldots (3)
\]
Again, \[
\frac{g_m}{4} (v_a - v_c) = g_m (v_a - v_{m1}) = g_m (v_{m2} - v_c)
\]
By solving: \[
v_{m1} - v_{m2} = \frac{v_a - v_b}{2} \quad \ldots (4)
\]
From (3) & (4), \[
v_{m1} - v_{m2} = v_{im} = \frac{v_1 - v_2}{3} = \frac{v_{id}}{k} \quad \ldots \ldots \ldots (5)
\]
Attenuation factor for one stage attenuator = \((1/k)\).
Overall trans-conductor

From (1) & (5),

\[ I_{od} = (I_{ss2} - I_{ss1}) \tanh \left( \frac{v_{id}}{2knV_T} \right) \quad \& \]

\[ G_m = \frac{(I_{ss2} - I_{ss1})}{2knV_T} \text{sech}^2 \left( \frac{v_{id}}{2knV_T} \right) \quad \ldots (6) \]

- Voltage Attenuator takes role in x-axis compression too.
- Cross-coupled trans-conductor reduces \( G_m \).
- From Simulation: \( I_{ss1} = 4nA, I_{ss2} = 5.23nA, k = 3 \)
- From eqn(6): \( G_m \approx 0.47nA/V \). From simulation, \( G_m \) has almost constant value: 0.4637nS upto 140mVpp.

Fig6. \( G_m \) variation with input amplitude
1st order Gm-C filter response

Cut-off freq of unity gain
Closed loop config: 71.18Hz
Roll off: -20dB/decade

For \( g_m = 0.463 \text{ nA/V} \), \( C_{load} = 1 \text{pF} \), Theoretically cut-off frequency, \( f_o = 70.53 \text{Hz} \).
HF linearity issue (coherent sampling)

Fig10 : FFT Analysis (Log Magnitude vs frequency) with coherent sampling $N_{\text{WINDOW}} = 29, N_{\text{RECOED}} = 512$

Freq=10Hz
Amp=20mVpp
DC offset=78uV

Freq=1MHz
Amp=20mVpp
DC offset=0.24mV
High frequency Interferences

- Input stage offset is critical, as it is amplified at the output stage. So, we want to reduce offset at input stage, Voltage Attenuator.

- Unity gain configuration at high frequency

\[ V_{out} = V_b = V_{DC} \]

Hence, CM interference,

\[ v_{cm} = \frac{v_a + v_b}{2} = V_{DC} + \frac{v_{emi}}{2} \]

\[ v_{dm} = v_a - v_b = v_{emi} \]

- It behaves like open loop

With both CM & DM Interferences. DM interference can’t be avoided.

- CM interference effect at output can be made zero by proper biasing technique.
**CM & DM Interferences**

- \( H_{cm} = \left. \frac{v_{o1,cm}}{v_{cm}} \right| v_{dm}=0 \)
- \( H_{dm} = \left. \frac{v_{o1,dm}}{v_{dm}/2} \right| v_{cm}=0 \)
- **Hence**, \( v_{o1} = H_{cm} v_{cm} + H_{d} v_{d} \)
  
  & \( v_{o2} = H_{cm} v_{cm} - H_{d} v_{d} \)

**Fig12: Voltage attenuator**

**DM half circuit**

**CM half circuit**
Offset issues in front end Attenuator

Let, \( x = A \sin(\omega t) \)

For 2\(^{nd}\) order harmonics at output:

\[
x^2 = A^2 \sin^2(\omega t) = \frac{A^2}{2} (1 - \cos(2\omega t))
\]

Hence, 2\(^{nd}\) order harmonics give DC offset that can’t be removed by low-pass filtering.

Offset 1 :

due to second order term : \( a_2(H_{cm}v_{cm}+H_dv_d)^2 \),

Offset 2 :

due to second order term : \( a_2(H_{cm}v_{cm}-H_dv_d)^2 \),

Hence, Offset 1 & Offset 2 are significantly different values from each other.
Offset issues in front end Attenuator

- This offset will increase further in second stage, where differential input is amplified by a high gain factor.
- Let, net output \( v_{out} = G \cdot \{(Offset1 - Offset2) + 2H_d \cdot v_d \} \), \( G \): gain of second stage = high
- Draws huge offset at output.
- Hence, elimination of offset at the 1st stage is important.

**Offset 1:**
- due to: \( a_2 (H_{cm} \cdot v_{cm} + H_d \cdot v_d)^2 \)

**Offset 2:**
- due to: \( a_2 (H_{cm} \cdot v_{cm} - H_d \cdot v_d)^2 \)

**Solution:** as \( H_d \neq 0 \), we can make \( H_{cm} = 0 \) for **Offset1 = Offset 2**.

By this, \( v_{out} \) gets rid of any DC offset.
Reduce CM interference:

Outputs of attenuator: \( v_{o1} = H_{cm} v_c = v_{o2} = v_o \) (say) \{as \( v_{dm}=0 \) here\}

KCL at node \( s_1 \) (7), \( v_{o1} \) (8), \( s_2 \) (9):

\[
(sC_{gs1} + g_m)(v_c - v_{s1}) + (g_m + sC_{gs2})(v_o - v_{s1}) = sC_{T1} v_{s1} \tag{7}
\]

\[
(g_m + sC_{gs2})(v_o - v_{s1}) + (g_m + sC_{gs3})(v_o - v_{s2}) + sC_{T2} v_o = 0 \tag{8}
\]

\[
(g_m + sC_{gs3})v_o = v_{s2}(g_m + s(C_{gs3} + C_{T3})) \tag{9}
\]

From (9), find \( v_{s2} \) & Put in (8):

\[
v_{s1} = \frac{v_{o1} \left(2g_m + s(C_{gs2} + C_{gs3} + C_{T2}) - \frac{(g_m + sC_{gs3})^2}{g_m + s(C_{gs3} + C_{T3})}\right)}{(g_m + sC_{gs2})}
\]

Put \( v_{s1} \) in (7):

\[
H_{cm} = \frac{v_{o1}}{v_{cm}} = \frac{(sC_{gs1})}{(g_m + sC_{gs2}) - \frac{2g_m + s(C_{T1} + C_{gs2})}{g_m + sC_{gs2}}} \left(\frac{2g_m + s(C_{gs2} + C_{gs3} + C_{T2}) - \frac{(g_m + sC_{gs3})^2}{g_m + s(C_{gs3} + C_{T3})}}{g_m + s(C_{gs3} + C_{T3})}\right)
\]

- **Observation:** \( H_{cm} \) decreases if \( C_{T1} \) & \( C_{T3} \) decreases. We want to minimize \( C_{T1} \) & \( C_{T3} \).

- \( C_{T1} = C_{db} + C_{sb1} + C_{sb2} \), \( C_{T3} = C_{sb3} + C_{db}/2 \)

13-Jun-16
Snehasish Roy Chowdhury
Reduction of CM interference (Ctd.)

- $C_{T1} = C_{db} + C_{sb1} + C_{sb2}$,  $C_{T3} = C_{sb3} + C_{db}/2$
- We want $C_{sb} = 0$, but in twin-tub CMOS process, shorting S to B causes high well capacitance ($C_{GND}$) at source.

- Hence, one auxiliary pair source node ($V_{s1}$) is connected to bulk of main pair. This is **Source-buffered structure**.
- $C_{sb1} + C_{sb2}$ sees almost same potential across it. Though these two potentials are not exactly equal, but voltage across $C_{sb1} + C_{sb2}$ is very small, causing it to be virtually shorted.
- In advantage, S & B of main pair are decoupled.

Common mode Transfer Function ($H_{cm}$)

Fig 17: Proposed Attenuator structure

\[ C_1 = C_{gs1} + C_{ext} \]

Fig 18: ac equivalent model of Half Circuit for Common mode inputs

\[
\begin{align*}
g_{m}v_{x1} &= g_{m}v_{gs1} + g_{mb}v_{bs1} \\
g_{m}v_{x2} &= g_{m}v_{gs2} + g_{mb}v_{bs1} \\
g_{m}v_{x3} &= g_{m}v_{gs3} + g_{mb}v_{bs3}
\end{align*}
\]
Calculation for CMTF ($H_{cm}$)

Applying KCL at A, B, C, D, E, X nodes, we get six equations. By solving these equations, we get:

$$H_{cm}(\infty) = \left(\frac{v_x}{v_1}\right) = \frac{\alpha_2 \gamma_1 - \alpha_1 \gamma_2}{\alpha_1 \beta_2 - \alpha_2 \beta_1}$$

Here, $\alpha_1$, $\alpha_2$, $\beta_1$, $\beta_2$, $\gamma_1$, $\gamma_2$ are constant values at very high Frequency, coming from parasitic capacitances & $C_1$.

By making $H_{cm}(\infty)=0$, we find a quadratic equation of $C_1$.

By solving the equation, $(C_1 = C_{gs1} + C_{ext})$ we find:

$$C_{ext} = 95 fF$$

Adding this amount of external capacitance will make $H_{cm} = 0$ at very high frequency.
Comparison I (100mVpp)

- In frequency range [10Hz-100MHz], Maximum EMI induced offset (Magnitude): (i) Voltage attenuator with cross-coupled transconductor based \( G_m - C \) filter:: 1.996% (ii) Proposed \( G_m - C \) filter:: 0.459%

- Upto 100KHz, offsets are almost same as parasitic effect comes at high frequencies.
- Upto 100KHz, EMI induced Offset is very low (~uV). This proves good linearity of the filter at low frequency.
- Proposed \( G_m - C \) filter shows small offset even at high freq due to its immunity to CM interference.
- Acc to calculation, offset will reach zero at infinite frequency

Fig19: EMI induced DC offset variation with frequency for 100mVpp input signal
Comparison II (FFT: 50mVpp, 1MHz)

Fig 20: Voltage attenuator with cross-coupled trans-conductor based Gm-C filter
DC offset=1.908mV

Fig 21: Proposed EMI Resisting Gm-C Filter
DC offset=0.46mV

Coherent sampling: $f_{in}=1$MHz, $N_{WINDOW}=29$, $N_{RECORD}=512$ : irreducible. DC Offset is reduced in EMI resisting filter.
Comparison III (1MHz)

With input amplitude, EMI induced DC offset increases. Rate of increment is less in proposed Filter.

- For non-linearity of the filter at high frequency, DC offset is high. Let, $v_{in} = A\sin(\omega t)$. i.e. $v_{in}^2 = \frac{A^2}{2} (1 - \cos(\omega t))$. This DC Offset is high when amplitude is high as seen in figure.
- But, by making $H_{cm}(\infty) = 0$, we are reducing non-linearity of the filter at high frequency.
- This is why at $f \gg GBW$ ($f=1\text{MHz}$ here), this proposed filter is much immune to Electromagnetic Interference. (CM interference precisely)
Conclusion

- **Advantage:**
  - For out-of-band EMI frequencies, when parasitic capacitances come in picture, *EMI induced DC offset* becomes a challenging issue. Hence, This approach for EMI resisting highly linear sub-kilohertz $G_m$-$C$ filter is noble in this field.

- **Disadvantage:**
  - Added power dissipation
  - Transistor area increment
THANK YOU
Source buffering

- We want to make $H_{cm} = 0$ & accordingly we’ll choose $C_1$
- KCL at node C(10),E(11),D(12):

  \[(sC_{gs} + g_m)(v_1 + v_E - 2v_A - 2v_{bs1}) = sC_{T1}v_A + s(C_{bs1} + C_{T1})v_{bs1} \]  \(\text{.} (10)\)

  \[(2v_E - v_D - v_C)(sC_{gs} + g_m) + sC_{T2}v_E = 0 \]  \(\text{.} (11)\)

  \[(v_E - v_B - v_{bs2})(sC_{gs} + g_m) = s\frac{C_{bs2} + C_{T3}}{2}v_{bs2} + s\frac{C_{T3}}{2}v_B \]  \(\text{.} (12)\)

Solving (11) & (12) to omit $v_E$:

\[
\left[\left[g_m + s\left(C_{gs} + \frac{C_{bs2} + C_{T3}}{2}\right)\right]\right] \left[2g_m + s(2C_{gs2} + C_{T2})\right] - \left(g_m + sC_{gs}\right)^2 \times \left(v_{bs2} + v_B\right) = (v_A + v_{bs1})(g_m + sC_{gs})^2 \]

\(\text{(13)}\)

Put $v_E$ value from (12) in (10):

\[(sC_{gs} + g_m)v_1 + v_{bs2} \left[g_m + s\left\{C_{gs} + \frac{C_{bs2} + C_{T3}}{2}\right\}\right] + v_B \left[g_m + s\left\{C_{gs} + \frac{C_{T3}}{2}\right\}\right] = v_A\left[2g_m + s(2C_{gs} + C_{T1})\right] + v_{bs1}\left[2g_m + s(2C_{gs} + C_{T1} + C_{sb1})\right] \]

\(\text{.} (14)\)
Now, KCL at A(15), B(16), X(17):

\[(sC_1 + g_m)(v_1 + v_x) + (2g_{mb} + sC_{bs1})v_{bs1} = [2g_m + s(2C_1 + C_{T1})]v_A\]  \hspace{1cm} \ldots \ldots (15)

\[(sC_1 + g_m)v_x + \left( g_{mb1} + \frac{sC_{sb2}}{2} \right) v_{sb2} = \left( g_m + s \left( C_1 + \frac{C_{T2}}{2} \right) \right) v_B\]  \hspace{1cm} \ldots \ldots (16)

\[[2g_m + s(2C_1 + C_{T2})]v_x + g_{mb}(v_{bs1} + v_{bs2}) = (v_A + v_B)(g_m + sC_1)\]  \hspace{1cm} \ldots \ldots (17)

For minimizing, we assume:

- \(M(v_{bs2} + v_B) = N(v_A + v_{bs1})\) \hspace{1cm} \ldots \ldots (13.a)

- \(Pv_1 + Qv_{bs2} + Rv_B = Sv_A + Tv_{bs1}\) \hspace{1cm} \ldots \ldots (14.a)

- \(A(v_1 + v_X) + Bv_{bs1} = Cv_A\) \hspace{1cm} \ldots \ldots (15.a)

- \(Dv_X + Ev_{bs2} = Fv_B\) \hspace{1cm} \ldots \ldots (16.a)

- \(Gv_X + H(v_{bs1} + v_{bs2}) = I(v_A + v_B)\) \hspace{1cm} \ldots \ldots (17.a)

- Solving (13.a) & (14.a):

\[v_{bs2} = X_2v_1 + Y_2v_B + Z_2v_A\] where, \(X_2 = \frac{NP}{(TM-NQ)}\), \(Y_2 = \frac{-(S-T)}{TM-NQ}\), \(Z_2 = \frac{-(TM-NR)}{TM-NQ}\) \hspace{1cm} \ldots \ldots (18)

\[v_{bs1} = X_1v_1 + Y_1v_B + Z_1v_A\] where, \(X_1 = \frac{MP}{(TM-NQ)}\), \(Y_1 = \frac{M(R-Q)}{TM-NQ}\), \(Z_1 = \left[ 1 + \frac{M(S-T)}{TM-NQ} \right] \hspace{1cm} \ldots \ldots (19)\]
At high frequency \((s \to \infty)\), we find the following:

\[
\begin{align*}
X_2 &= \frac{NP}{(TM-NQ)} = \frac{C_{gs}^3}{(2C_{gs} + C_{T1} + C_{sb1})
\left([C_{gs} + \frac{C_{sb2} + C_{T3}}{2}] (2C_{gs} + C_{T2}) - C_{gs}^2\right) - C_{gs}^2 (C_{gs} + \frac{C_{sb2} + C_{T3}}{2})} \\
Y_2 &= -\frac{N(S-T)}{(TM-NQ)} = \frac{C_{sb1}^2}{(2C_{gs} + C_{T1} + C_{sb1})
\left([C_{gs} + \frac{C_{sb2} + C_{T3}}{2}] (2C_{gs} + C_{T2}) - C_{gs}^2\right) - C_{gs}^2 (C_{gs} + \frac{C_{sb2} + C_{T3}}{2})} \\
Z_2 &= \frac{- (TM-NR)}{(TM-NQ)} = \frac{C_{gs}^2 (C_{gs} + \frac{C_{T3}}{2}) - (2C_{gs} + C_{T1} + C_{sb1})
\left([C_{gs} + \frac{C_{sb2} + C_{T3}}{2}] (2C_{gs} + C_{T2}) - C_{gs}^2\right) - C_{gs}^2 (C_{gs} + \frac{C_{sb2} + C_{T3}}{2})} \\
X_1 &= \frac{MP}{TM-NQ} = \frac{\left([C_{gs} + \frac{C_{sb2} + C_{T3}}{2}] (2C_{gs} + C_{T2}) - C_{gs}^2\right) C_{gs}^3}{(2C_{gs} + C_{T1} + C_{sb1})
\left([C_{gs} + \frac{C_{sb2} + C_{T3}}{2}] (2C_{gs} + C_{T2}) - C_{gs}^2\right) - C_{gs}^2 (C_{gs} + \frac{C_{sb2} + C_{T3}}{2})} \\
Y_1 &= \frac{M(R-Q)}{TM-NQ} = \frac{\left([C_{gs} + \frac{C_{sb2} + C_{T3}}{2}] (2C_{gs} + C_{T2}) - C_{gs}^2\right) \frac{CT_3}{2}}{(2C_{gs} + C_{T1} + C_{sb1})
\left([C_{gs} + \frac{C_{sb2} + C_{T3}}{2}] (2C_{gs} + C_{T2}) - C_{gs}^2\right) - C_{gs}^2 (C_{gs} + \frac{C_{sb2} + C_{T3}}{2})} \\
Z_1 &= \frac{M(S-T)}{TM-NQ} = \frac{\left([C_{gs} + \frac{C_{sb2} + C_{T3}}{2}] (2C_{gs} + C_{T2}) - C_{gs}^2\right) C_{sb1}^3}{(2C_{gs} + C_{T1} + C_{sb1})
\left([C_{gs} + \frac{C_{sb2} + C_{T3}}{2}] (2C_{gs} + C_{T2}) - C_{gs}^2\right) - C_{gs}^2 (C_{gs} + \frac{C_{sb2} + C_{T3}}{2})} - 1
\end{align*}
\]

All terms are independent of \(C_1\)
Put equation (19) in (15.a): $v_1 (A + BX_1) + BY_1 v_B + Av_X = (C - BZ_1)v_A$

Put equation (18) in (16.a): $Dv_X + EX_2 v_1 + EZ_2 v_A = (F - EY_2)v_B$

- Put value of $v_A$ from first equation in second equation: $\alpha_1 v_B = \beta_1 v_X + \gamma_1 v_1$

  where, $\alpha_1 = [(F - EY_2)(C - BZ_1) - EZ_2 BY_1]$, $\beta_1 = (D(C - BZ_1) + EZ_2 A)$, $\gamma_1 = \{EX_2(C - BZ_1) + EZ_2(A + BX_1)\}$

From (18) & (19), put values $v_{bs1}$ & $v_{bs2}$ in (17.a): $\alpha_2 v_B = \beta_2 v_X + \gamma_2 v_1$

where, $\alpha_2 = \left[1 + \frac{IBY_1}{C - BZ_1} - H(Y_1 + Y_2) - \frac{H(Z_1 + Z_2)BY_1}{C - BZ_1}\right]$, $\beta_2 = \left[G + \frac{AH(Z_1 + Z_2)}{C - BZ_1} - \frac{AI}{C - BZ_1}\right]$, $\gamma_2 = \left[H(X_1 + X_2) + \frac{H(Z_1 + Z_2)(A + BX_1)}{C - BZ_1} - \frac{(A + BX_1)I}{C - BZ_1}\right]$

From (20) & (21): $(\frac{\alpha_2}{\alpha_1}) (\beta_1 v_X + \gamma_1 v_1) = \beta_2 v_X + \gamma_2 v_1$

$$H_{cm} = \frac{\alpha_2 Y_1 - \alpha_1 Y_2}{\alpha_1 \beta_2 - \alpha_2 \beta_1}$$

For $H_{cm} = 0$, we find: $\alpha_2 Y_1 - \alpha_1 Y_2 = 0$

$\Rightarrow [(C - BZ_1) + IBY_1 - H(Y_1 + Y_2)(C - BZ_1) - H(Z_1 + Z_2)BY_1] * [EX_2(C - BZ_1) + EZ_2(A + BX_1)] = [(F - EY_2)(C - BZ_1) - EZ_2 BY_1] * [H(X_1 + X_2)(C - BZ_1) + H(Z_1 + Z_2)(A + BX_1) - (A + BX_1)I]$

1st term: $[(C - BZ_1) + IBY_1 - H(Y_1 + Y_2)(C - BZ_1) - H(Z_1 + Z_2)BY_1]$ divide numerator and denominator by $s^2$, we’ll find the term is simplified to: $[(C_1)((2C_1 + C_{T1}) - C_{bs1} Z_1) + C_1 C_{bs1} Y_1]$

2nd term: $[EX_2(C - BZ_1) + EZ_2(A + BX_1)]$ divide numerator and denominator by $s^2$ gives:

$$\left[\frac{c_{sh2}}{2} X_2 ((2C_1 + C_{T1}) - C_{bs1} Z_1) + \frac{c_{sh2}}{2} Z_2 (C_1 + C_{bs1} X_1)\right]$$
\[ (F - EY_2)(C - BZ_1) - EZ_2BY_1 \] is simplified by dividing num & den by \( s^2 \) and put \( s \to \infty \)

\[
\left[ \left( \left( C_1 + \frac{C_{T3}}{2} \right) - \left( \frac{C_{sb2}}{2} \right) Y_2 \right) \left( 2C_1 + C_{T1} \right) - \left( C_{bs1}Z_1 \right) \right] - \left( \frac{C_{sb2}}{2} \right) \left( C_{bs1} \right) Z_2Y_1
\]

**3rd term:**

\[ H(X_1 + X_2)(C - BZ_1) + H(Z_1 + Z_2)(A + BX_1) - (A + BX_1)I \] is simplified by dividing num & den by \( s^2 \) and put \( s \to \infty \)

\[
\left[ -(C_1 + C_{bs1}X_1)(C_1) \right]
\]

**4th term:**

\[ C_1^2(4X_2 + 2Z_2 - 2) + C_1\left( \left( 2C_{T1}X_2 - 2C_{bs1}Z_1X_2 + 2C_{bs1}X_1Z_2 \right) - \left( C_{T1} - C_{bs1}Z_1 + C_{bs1}Y_1 \right)(2X_2 + Z_2) - \left( C_{T1} - C_{bs1}Z_1 - C_{bs1}Z_2Y_1 + 2C_{bs1}X_1 \right) \right) + C_{T1} - C_{bs1}Z_1 + C_{bs1}Y_1 C_{T1}X_2 - C_{bs1}Z_1X_2 + C_{bs1}X_1Z_2 - C_{bs1}X_1(C_{T1} - C_{bs1}Z_1 - C_{bs1}Z_2Y_1) = 0
\]

This follows the form: \( aC_1^2 + bC_1 + c = 0 \)

From DC simulation: \( C_{gs} = 7.05 \text{fF} \), \( C_{sb1} = 17.14 \text{fF} \), \( C_{sb2} = 17.02 \text{fF} \), \( C_{T1} = 10.46 \text{fF} \), \( C_{T2} = 10.47 \text{fF} \), \( C_{T3} = 10.47 \text{fF} \), \( g_m = 42.4 \text{nA/V} \)

Hence, putting the values in all equations, \( a = -4.03, b = -2.53 \times 10^{-15} \) & \( c = 4.12 \times 10^{-26} \)

\[
C_1 = -\frac{2.53 \times 10^{-15} - \sqrt{(2.53 \times 10^{-15})^2 + 4 \times 4.03 \times 4.12 \times 10^{-26}}}{2 \times 4.03} = 100.79 \text{fF}
\]

- \( C_1 = C_{gs} + C_{ext} \), Here, \( C_{gs} = 7.05 \text{fF} \), \( \Rightarrow \ C_{ext} = 93.74 \text{fF} \)

- We consider,

\[
C_{ext} = 95 \text{fF}
\]

13-Jun-16
Snehasish Roychowdhury
Infinite frequency for 1\(^{st}\) term:

1\(^{st}\) term:

\[ I(C - BZ_1) + IBY_1 - H(Y_1 + Y_2)(C - BZ_1) - H(Z_1 + Z_2)BY_1 \]

\[ = (g_m + sC_1)[2g_m + s(2C_1 + C_{T1}) - (2g_{mb} + sC_{bs1})Z_1 + (2g_{mb} + sC_{bs1})Y_1] \]

\[ - [g_{mb}(Y_1 + Y_2)(2g_m + s(2C_1 + C_{T1})) - [g_{mb}(Z_1 + Z_2)(2g_{mb} + sC_{bs1})Y_1] \]

The critical frequencies above which (at least 10 times) we are considering the approximation of \( s \rightarrow \infty \) is valid:

(i) \( \frac{g_m}{C_1} = \frac{42.2n}{100.79f} = 67.48 \text{ KHz} \)

(ii) \( \frac{2g_m}{2C_1 + C_{T1}} = 64.27 \text{ KHz} \)

(iii) \( \frac{2g_{mb}}{C_{bs1}} = 74.9 \text{ KHz} \)
Infinite frequency for 2\textsuperscript{nd} term:

\textbf{2\textsuperscript{nd} term:}

\[ [EX_2(C - BZ_1) + EZ_2(A + BX_1)] \]

\[ = \left(g_{mb1} + \frac{sC_{bs2}}{2}\right)[X_2\{(2g_m + s(2C_1 + C_{T1}) - (2g_{mb} + sC_{bs1})Z_1\} + Z_2\{(g_m + sC_1) + (2g_{mb} + sC_{bs1})X_1\}] \]

The critical frequencies above which (at least 10 times) we are considering the approximation of \(s \rightarrow \infty\) is valid:

(i) \[\frac{2g_mX_2 - 2g_{mb}Z_1}{(2C_1 + C_{T1})X_2 - C_{bs1}Z_1} = 71.33 \text{ KHz} \]

(ii) \[\frac{(g_m + 2g_{mb}X_1)}{C_1 + C_{bs1}X_1} = 67.16 \text{ KHz} \]
Infinite frequency for 3rd term:

\[ [(F - EY_2)(C - BZ_1) - EZ_2BY_1] \]

\[ = \left( (g_m + g_{mb}Y_2) + s \left( C_1 + \frac{C_{T3}}{2} - \frac{C_{bs2}}{2}Y_2 \right) \right) \left( (2g_m - 2g_{mb}) + s(2C_1 + C_{T1} - C_{bs1}Z_1) \right) - Z_2Y_1(g_{mb} + \frac{sC_{bs2}}{2})(2g_{mb} + sC_{bs1}) \]

The critical frequencies above which (at least 10 times) we are considering the approximation of \( s \to \infty \) is valid:

(i) \( \frac{g_m + g_{mb}Y_2}{C_1 + \frac{C_{T3}}{2} - \frac{C_{bs2}}{2}Y_2} = 64.18 \, KHz \)

(ii) \( \frac{2(g_m - g_{mb})}{2C_1 + C_{T1} - C_{bs1}Z_1} = 54.6 \, KHz \)
Infinite frequency for 4th term:

4th term:

\[ H(X_1 + X_2)(C - BZ_1) + H(Z_1 + Z_2)(A + BX_1) - (A + BX_1)I \]

\[ = g_mb (X_1 + X_2)[2g_m + s(2C_1 + C_{T1}) - (2g_mb + sC_{bs1})Z_1] + g_mb (Z_1 + Z_2)[g_m + sC_1 + (2g_mb + sC_{bs1})X_1] \]

\[ - (g_m + sC_1)[g_m + sC_1 + (2g_mb + sC_{bs1})X_1] \]

The critical frequencies above which (at least 10 times) we are considering the approximation of \( s \rightarrow \infty \) is valid:

(i) \( \frac{g_m + 2g_mbX_1}{C_1 + C_{bs1}X_1} = 67.16 \text{ KHz} \)

(ii) \( \frac{2(g_m - g_mbZ_1)}{2C_1 + C_{T1} - C_{bs1}Z_1} = 64.37 \text{ KHz} \)
Conclusion:
All the terms are considered individually to find the frequency above which we can consider our assumption is valid. i.e. $H_{cm}$ is zero.
From all the terms above, we find highest frequency = 957.2 KHz. Hence, above almost 10 times, of it i.e. 9-10 MHz, we can assume frequency to be infinite. All our assumptions are valid above this frequency. This is valid as 10MHz is considered typically in EMI frequency range.
Comparison (100mVpp)

- In frequency range [10Hz-100MHz], Maximum EMI induced offset (Magnitude): (i) Voltage attenuator with cross-coupled transconductor based $G_m$-C filter: $1.996\%$ (ii) Proposed $G_m$-C filter: $0.459\%$

- Here, we see the EMI induced DC offset is reduced significantly for this proposed $G_m$-C filter compared to Uncompesated $G_m$-C filter. From the frequency near 10MHz, which is considered to be the minimum value of Infinty while calculating CMTF $H_{cm}(\infty)=0$.

Fig19: EMI induced DC offset variation with frequency for 100mVpp input signal
DC offset reduction at high frequency

\[ v_{o1} = H_{cm}v_{cm} + H_{dm} \frac{v_{dm}}{2} \quad \text{&} \quad v_{o2} = H_{cm}v_{cm} - H_{dm} \frac{v_{dm}}{2} \]

And, \( V_{o1} = v_{o1} + V_{DC} \) & \( V_{o2} = v_{o2} + V_{DC} \)

- **CASE1**: If Common mode interference comes at attenuator output, then from equations above: \( |v_{o1}| \neq |v_{o2}| \)

Unequal capacitive division occurs across gate-source of M1 & M2 due to \( C_{T,\text{tail}} \) and as a result, \( |v_{gs,M1}| \neq |v_{gs,M2}| \) ie finite offset current \( I_{\text{os}} \) at output.

- **CASE2**: Instead, if we make \( H_{cm} = 0 \) somehow, ie no CM Interference at attenuator output, \( |v_{o1}| = |v_{o2}| \)

In this case, \( P \) acts as virtual ground as second stage doesn’t see any common mode input. ie \( I_{\text{os}} = 0 \).

This is why we need to minimize \( H_{cm} \) of the first stage to reduce DC offset at output.