Design of a Gm-C filter of very low trans-conductance & highly resistant to out-of-band Electromagnetic Interferences

Presented by

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Application: G_m-C filter of few Hz cut-off

- **Bio-Medical applications:**

- Pulse rate of a human body is 72 pulses/min on average.
- Hence, to isolate and detect our pulses from environmental noises in electronic systems, low pass filters of cut-off frequency in Hz is needed.
- This is how we can reduce the out of band noises and interferences by low pass filtering.

Cut-off frequency of Gm-C filter

- $(v_{in} - v_{out}) * G_m = I_{out}$

Again,

- $v_{out} = I_{out} * (\frac{1}{sC_{load}})$

Hence, $(v_{in} - v_{out}) * G_m = sC_{load}v_{out}$

- $v_{in} * G_m = (sC_{load} + G_m) * v_{out}$
- $\frac{v_{out}}{v_{in}} = \frac{1}{1+\frac{sC_{load}}{G_m}}$

○ Hence, DC gain= 0dB

○ Cut-off frequency of the filter is : $\frac{G_m}{2\pi C_{load}}\ Hz$

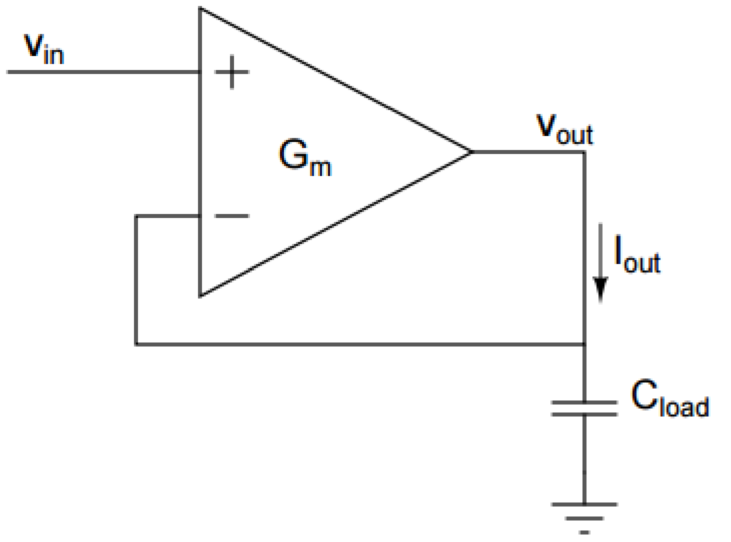


Fig1: G_m-C filter using voltage follower configuration

Block diagram of G_m-C filter

- Cut-off freq= $\frac{G_m}{2\pi C_{load}}$
- For 70Hz, C_{load}=1pF , G_m=0.47nA/V.

$$G_m = \frac{i_{out}}{v_{id}} = \frac{i_{out}}{v_{im}} * Attn.factor\left(\frac{1}{k}\right)$$

- Voltage attenuator at input stage reduces G_m

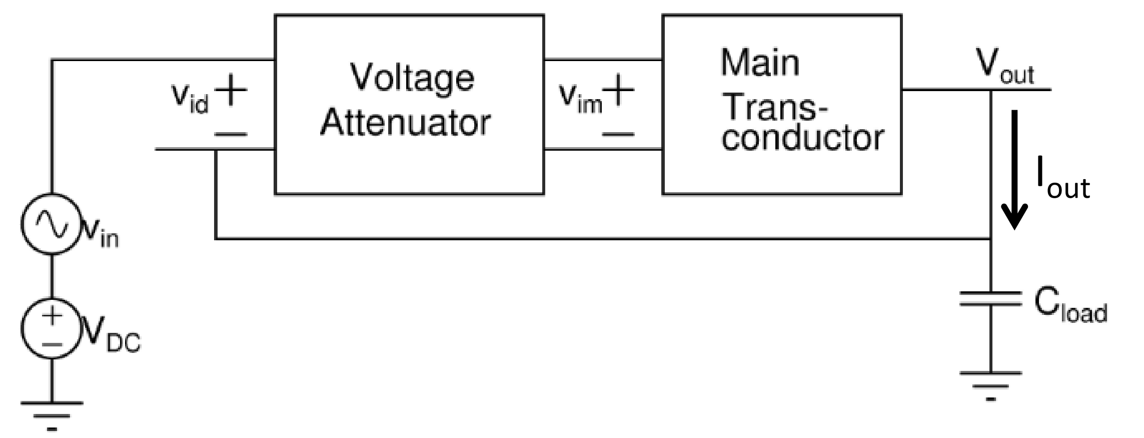


Fig2. Block diagram of G_m-C filter

- 2nd stage **Trans-conductor**: Input: v_{im} (voltage), output: I_{out} (current)
- All transistors in G_m-C filter are in **sub-threshold** region.

Cross-coupled trans-conductor design:

$$I_{o1} = I_1 + I_3 \; \& \; I_{o2} = I_2 + I_4$$

$$\frac{I_1}{I_2} = \frac{\exp\left(\frac{v_{ss1}-v_{m1}}{nV_T}\right)}{\exp\left(\frac{v_{ss1}-v_{m2}}{nV_T}\right)} = \exp\left(\frac{-v_{im}}{nV_T}\right)$$

$$\frac{I_3}{I_4} = \frac{\exp\left(\frac{v_{ss2}-v_{m2}}{nV_T}\right)}{\exp\left(\frac{v_{ss2}-v_{m1}}{nV_T}\right)} = \exp\left(\frac{v_{im}}{nV_T}\right)$$

Now,

$$\frac{I_1 - I_2}{I_1 + I_2} = \frac{\exp\left(\frac{-v_{im}}{nV_T}\right) - 1}{\exp\left(\frac{-v_{im}}{nV_T}\right) + 1} = \frac{\exp\left(\frac{-v_{im}}{2nV_T}\right) - \exp\left(\frac{v_{im}}{2nV_T}\right)}{\exp\left(\frac{-v_{im1}}{2nV_T}\right) + \exp\left(\frac{v_{im}}{2nV_T}\right)} = -\tanh\left(\frac{v_{im}}{2nV_T}\right)$$

$$\Rightarrow I_1 - I_2 = -I_{ss1} \tanh\left(\frac{v_{im}}{2nV_T}\right)$$

$$\Rightarrow I_3 - I_4 = I_{ss2} \tanh\left(\frac{v_{im}}{2nV_T}\right)$$

$$\Rightarrow I_{od} = I_{o1} - I_{o2} = (I_{ss2} - I_{ss1}) \tanh\left(\frac{v_{im}}{2nV_T}\right) \;\ldots\ldots(1)$$

Inputs of the trans-conductor: v_{m1} & v_{m2}

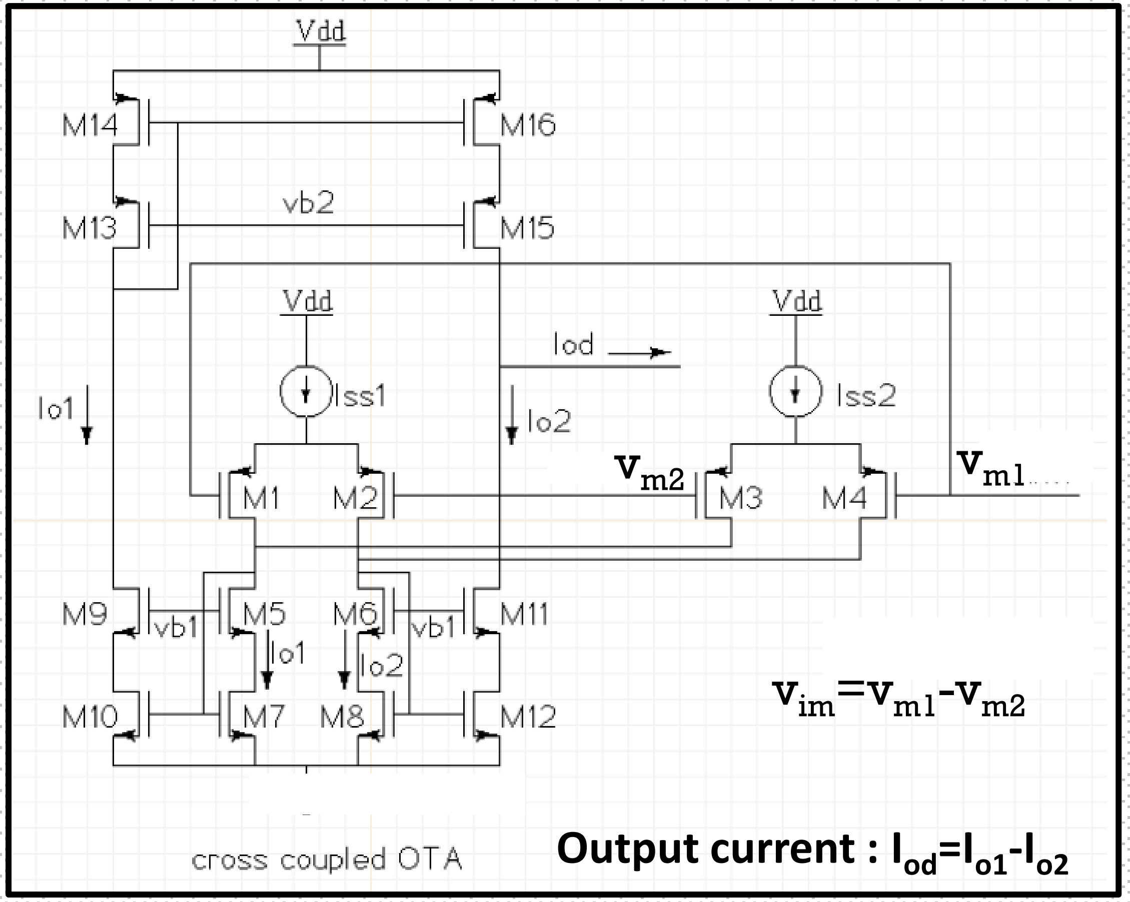


Fig3. Cross-coupled Trans-conductor

Voltage attenuator design:

❑ DC Analysis:

Assume all g_m's & I_{bias}'s are equal. Hence,

$I_1 = I_3 = I_5$ & $I_2 = I_4 = I_6$

Now, for DC analysis, $V_1 = V_2$

By symmetry,

$V_a = V_b = V_c$ &

$V_1 = V_{m1} = V_{m2} = V_2$ …………… (2)

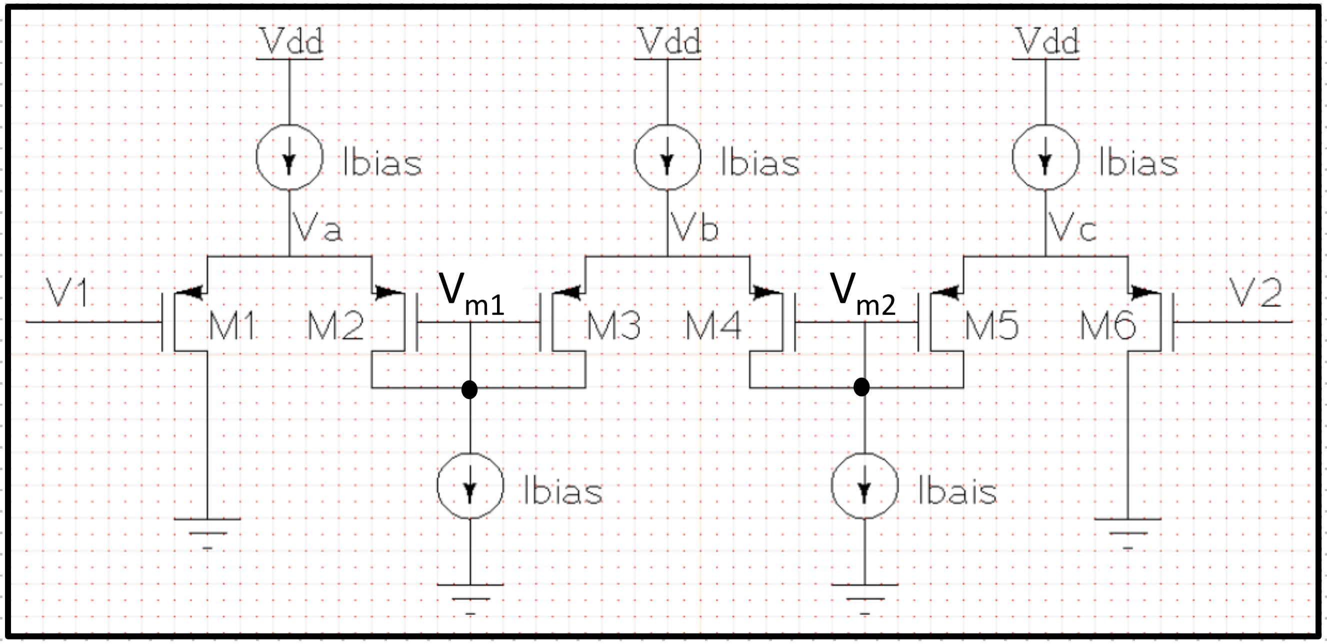


Fig4. Normal voltage attenuator

Fig ref: Sawigun, C.; Pal, D.; Demosthenous, A., "A wide-input linear range sub-threshold transconductor for sub-Hz filtering," in Circuits and Systems (ISCAS), Proceedings of 2010 IEEE International Symposium on , vol., no., pp.1567-1570,2010

Voltage attenuator design (Ctd.)

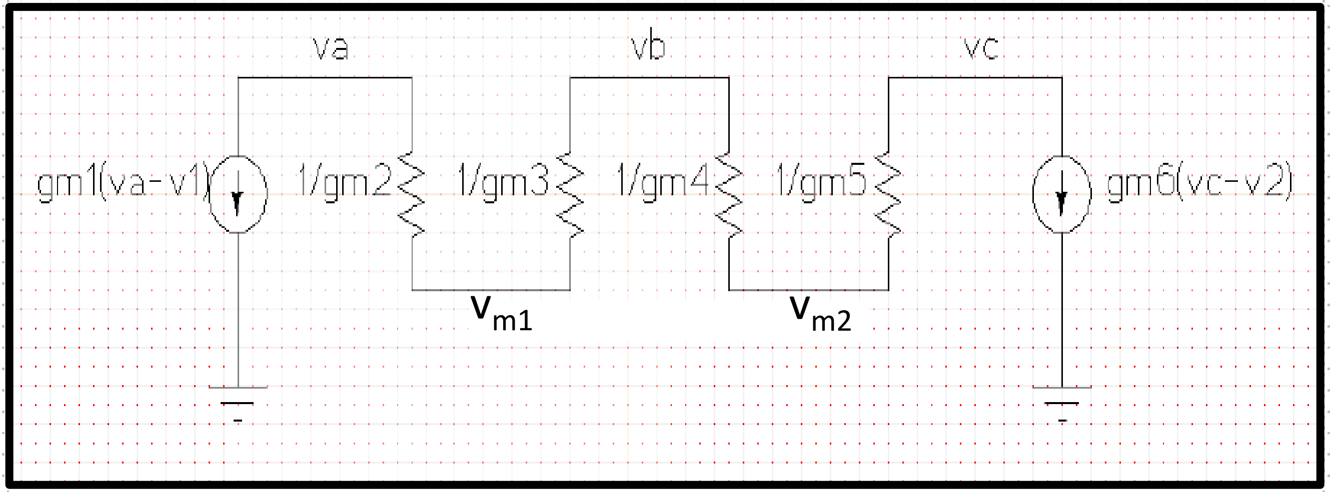


Fig5. ac equivalent of voltage attenuator

- **Small signal Analysis:**

Assumed all g_m's are equal.

⇨ $(v_a - v_1) + (v_c - v_2) = 0$

& $\frac{g_m}{4}(v_a - v_c) = g_m(v_c - v_2)$

⇨ $(v_a - v_b) = \frac{2}{3}(v_1 - v_2)$ ….. (3)

Again, $\frac{g_m}{4}(v_a - v_c) = g_m(v_a - v_{m1}) = g_m(v_{m2} - v_c)$

By solving: $v_{m1} - v_{m2} = \frac{v_a - v_b}{2}$ ……. (4)

From (3) & (4), $v_{m1} - v_{m2} = v_{im} = \frac{v_1 - v_2}{3} = \frac{v_{id}}{\mathrm{k}}$ …………. (5)

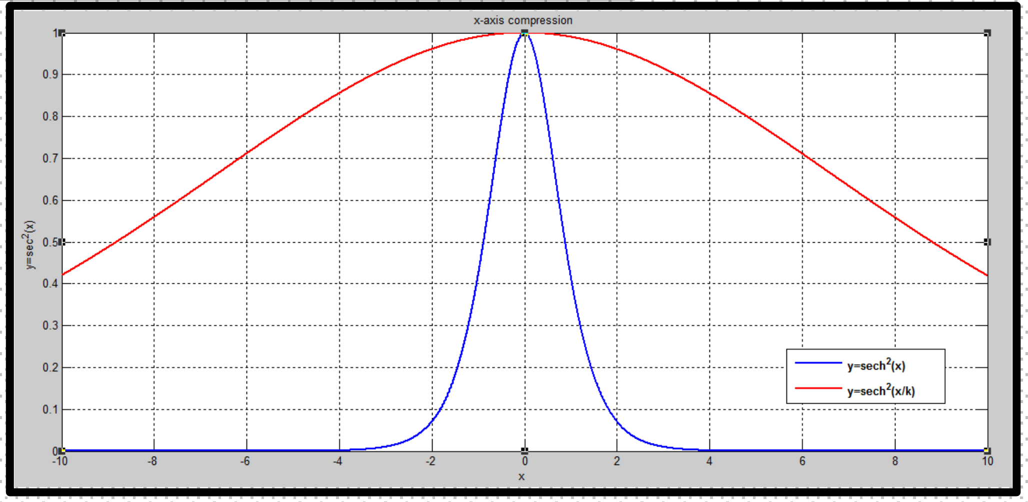
Attenuation factor for one stage attenuator = $({}^{1}/_{k})$.

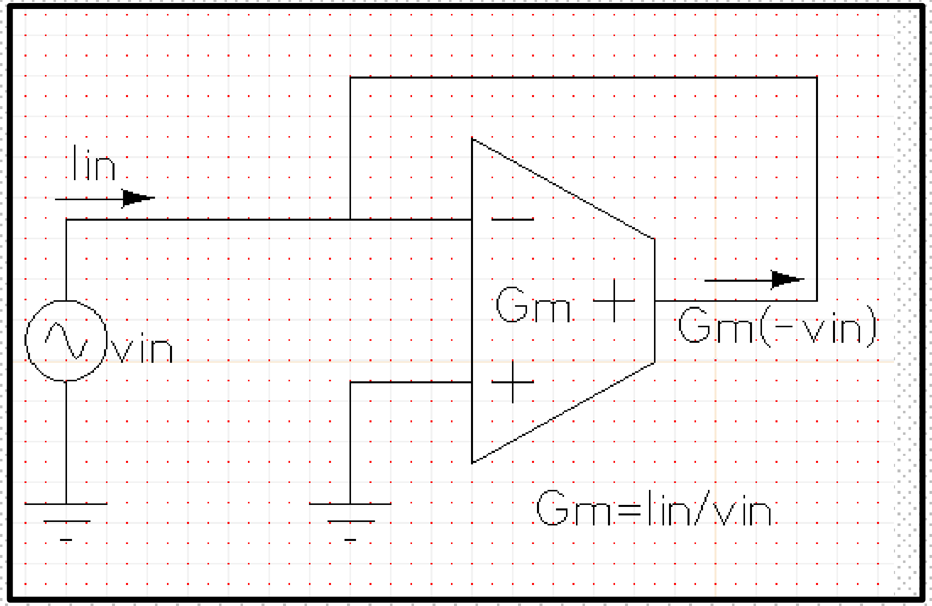
Overall trans-conductor

From (1) & (5),

$$I_{od} = (I_{ss2} - I_{ss1}) \tanh\left(\frac{v_{id}}{2knV_T}\right) \ \&$$

$$G_m = \frac{(I_{ss2} - I_{ss1})}{2knV_T} \text{sech}^2\left(\frac{v_{id}}{2knV_T}\right) \qquad \ldots(6)$$





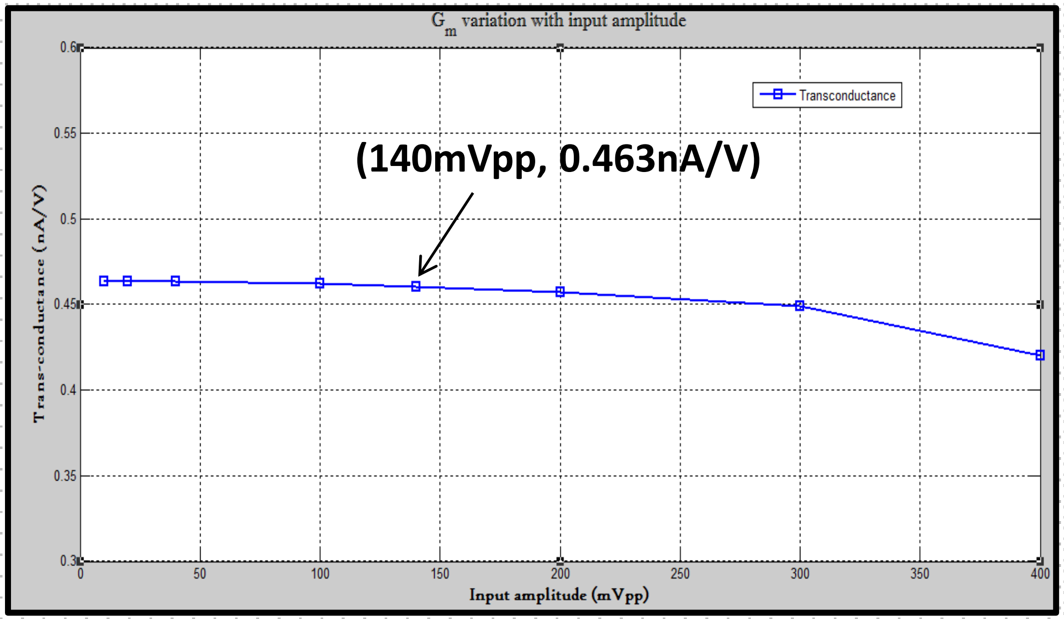


Fig6. G_m variation with input amplitude

- Voltage Attenuator takes role in x-axis compression too.
- Cross-coupled trans-conductor reduces G_m.
- From Simulation: $I_{ss1} = 4nA, I_{ss2} = 5.23nA, k = 3$
- From eqn(6): $G_m \approx 0.47$nA/V. From simulation,

G_m has almost constant value: 0.4637nS upto 140mVpp.

1st order Gm-C filter response

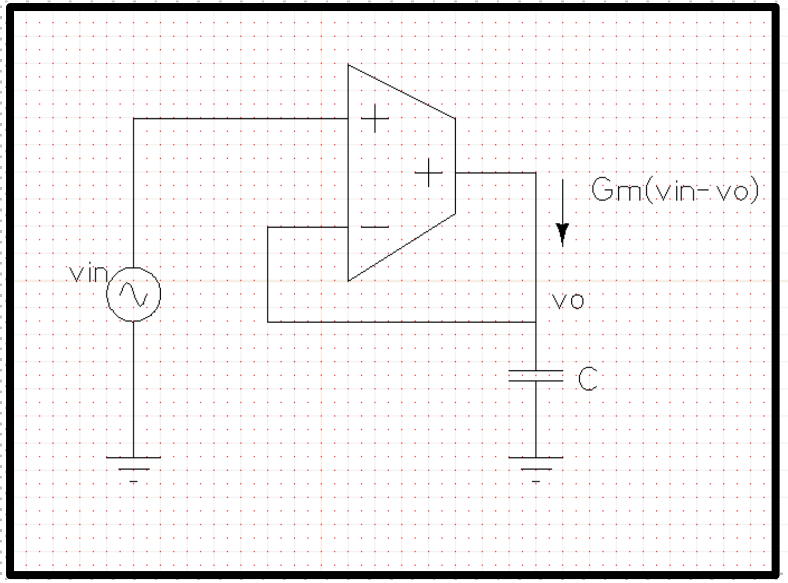


Fig7. Closed loop G_m-C filter

Cut-off frequency, $f_o = \frac{g_m}{2\pi C_{Load}}$

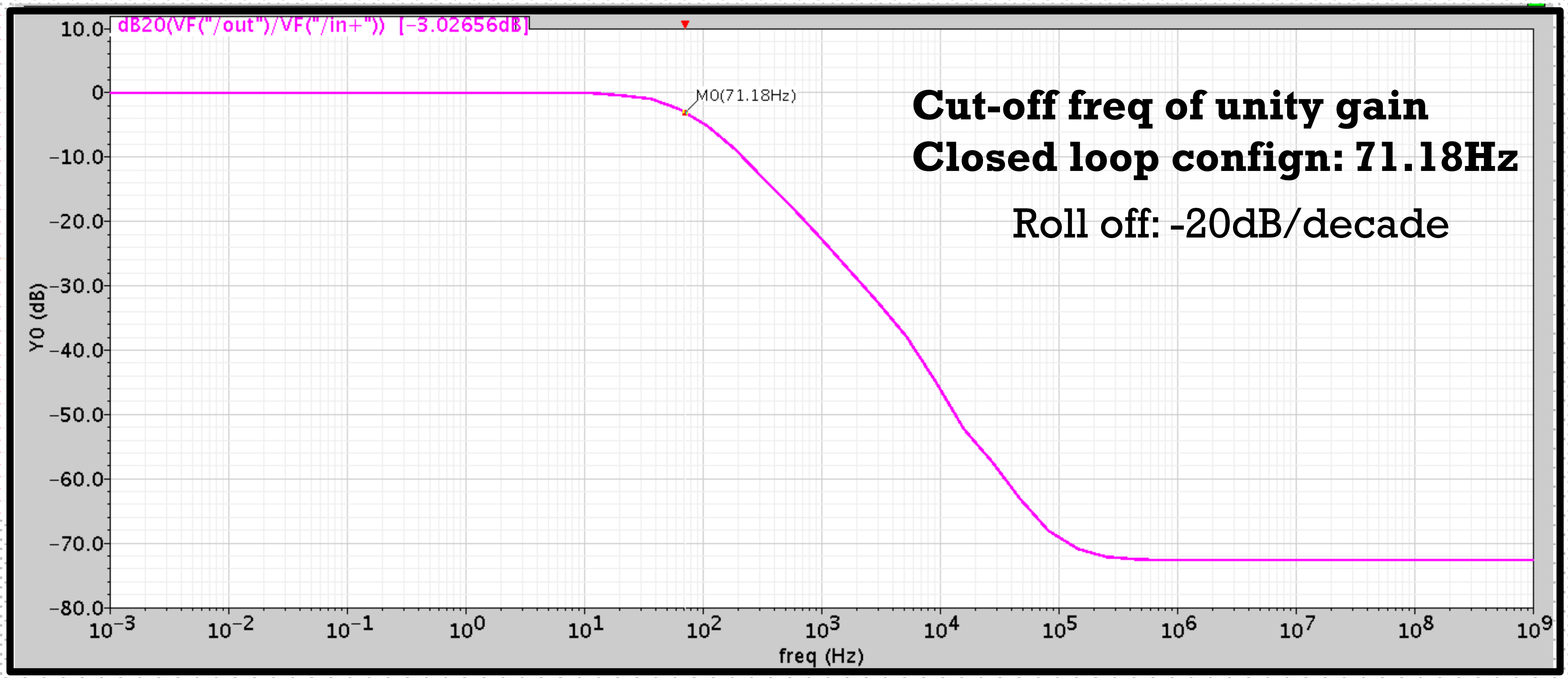
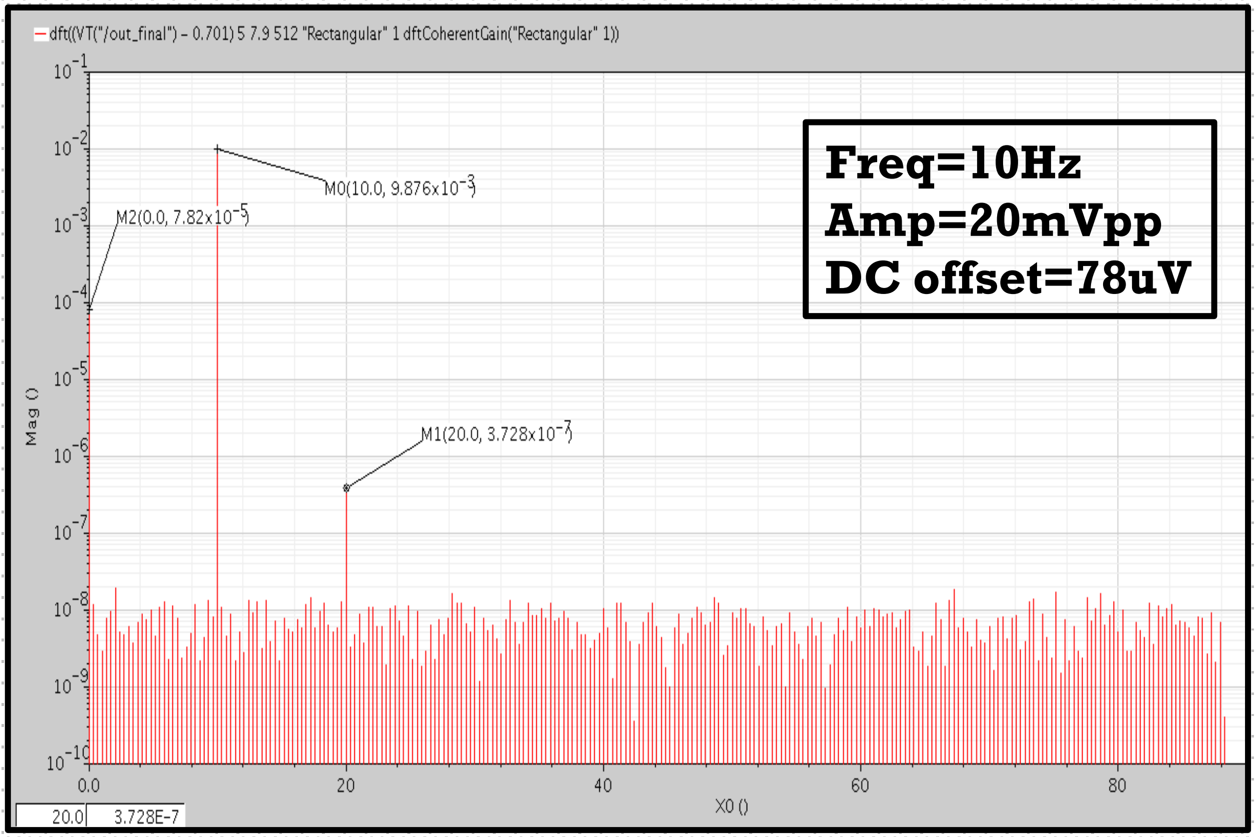


Fig8. ac response of the G_m-C filter

For G_m=0.463 nA/V , C_{load}=1pF , Theoretically cut-off frequency, f_o=70.53Hz.

HF linearity issue (coherent sampling)



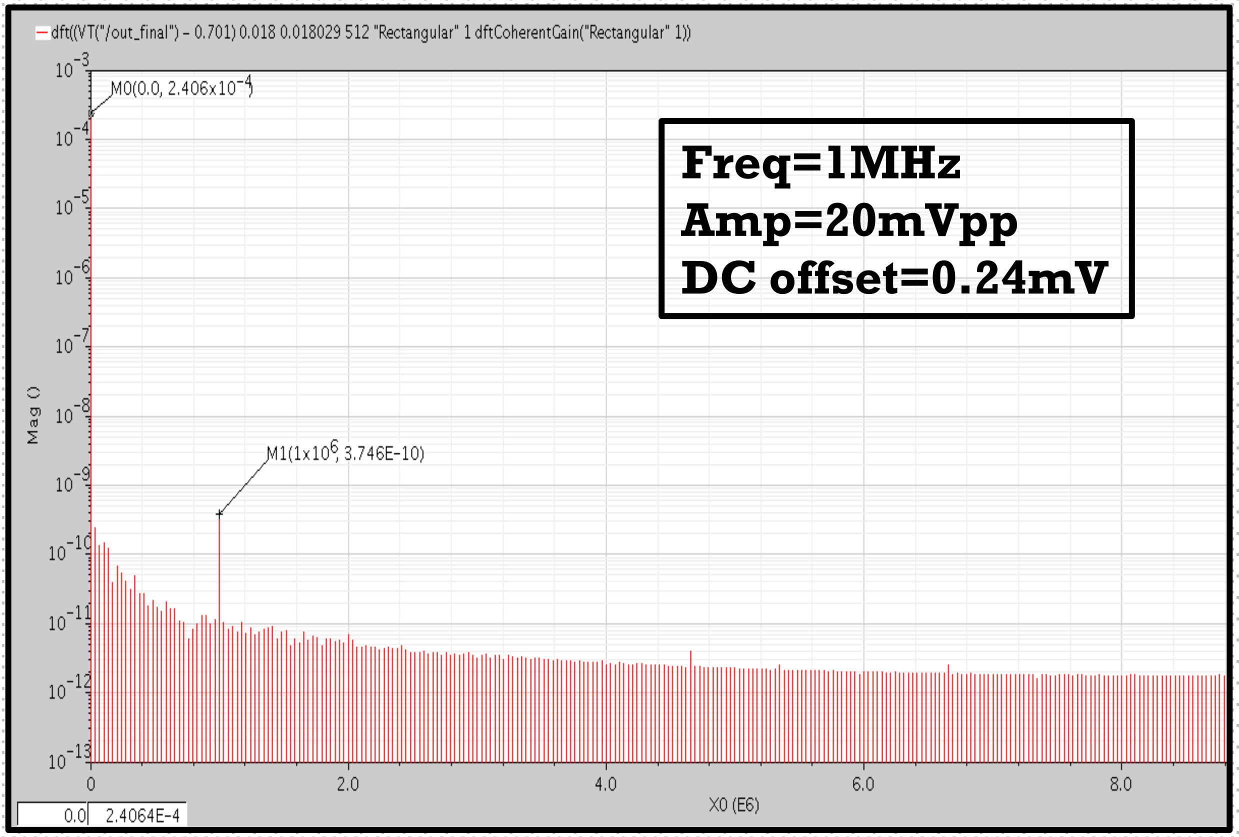


Fig10 : FFT Analysis (Log Magnitude vs frequency) with coherent sampling N_{WINDOW}=29, N_{RECOED}=512

High frequency Interferences

- Input stage offset is critical, as it is amplified at the output stage. So, we want to reduce offset at input stage, Voltage Attenuator.

- Unity gain configuration at high frequency

$V_{out} = V_b = V_{DC}$

Hence, CM interference,

$v_{cm} = \frac{v_a + v_b}{2} = V_{DC} + \frac{v_{emi}}{2}$

$v_{dm} = v_a - v_b = v_{emi}$

- It behaves like open loop With both CM & DM Interferences. DM interference can't be avoided.

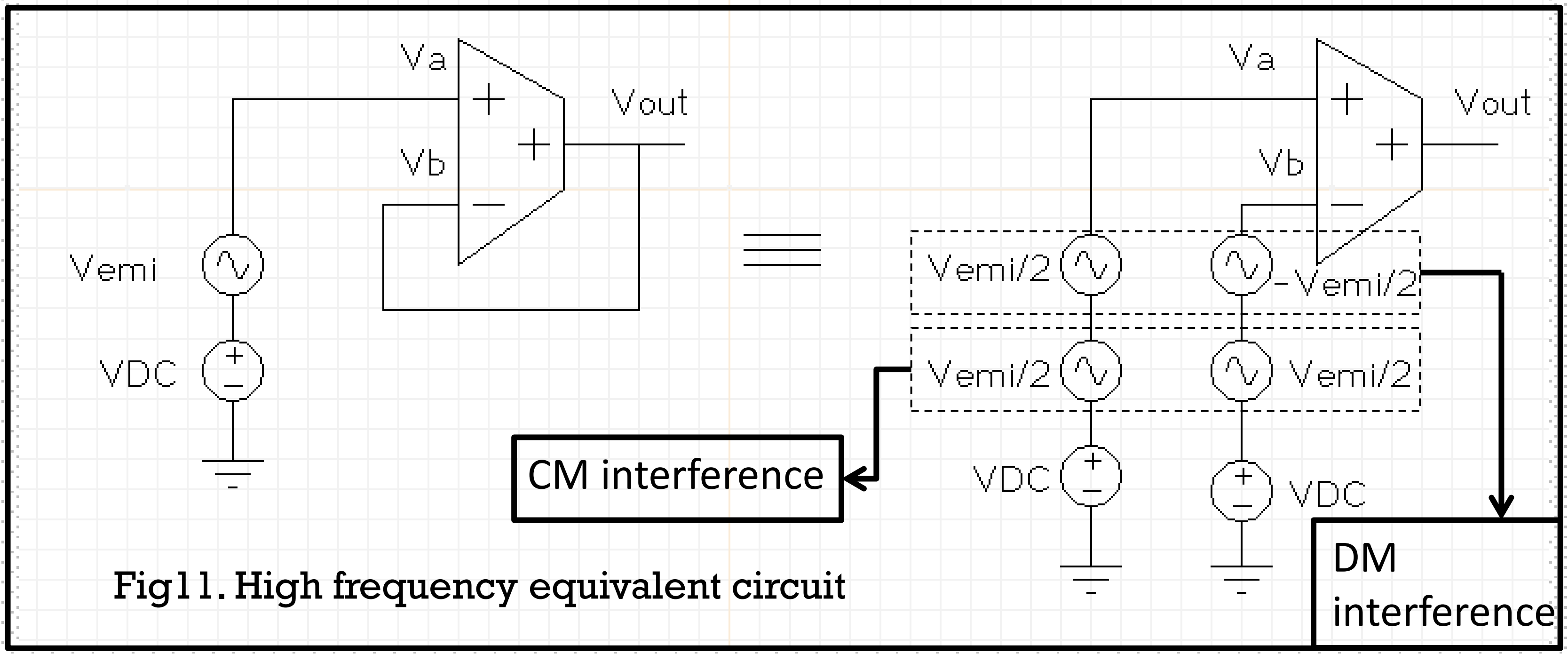


Fig11. High frequency equivalent circuit

- CM interference effect at output can be made zero by proper biasing technique.

CM & DM Interferences

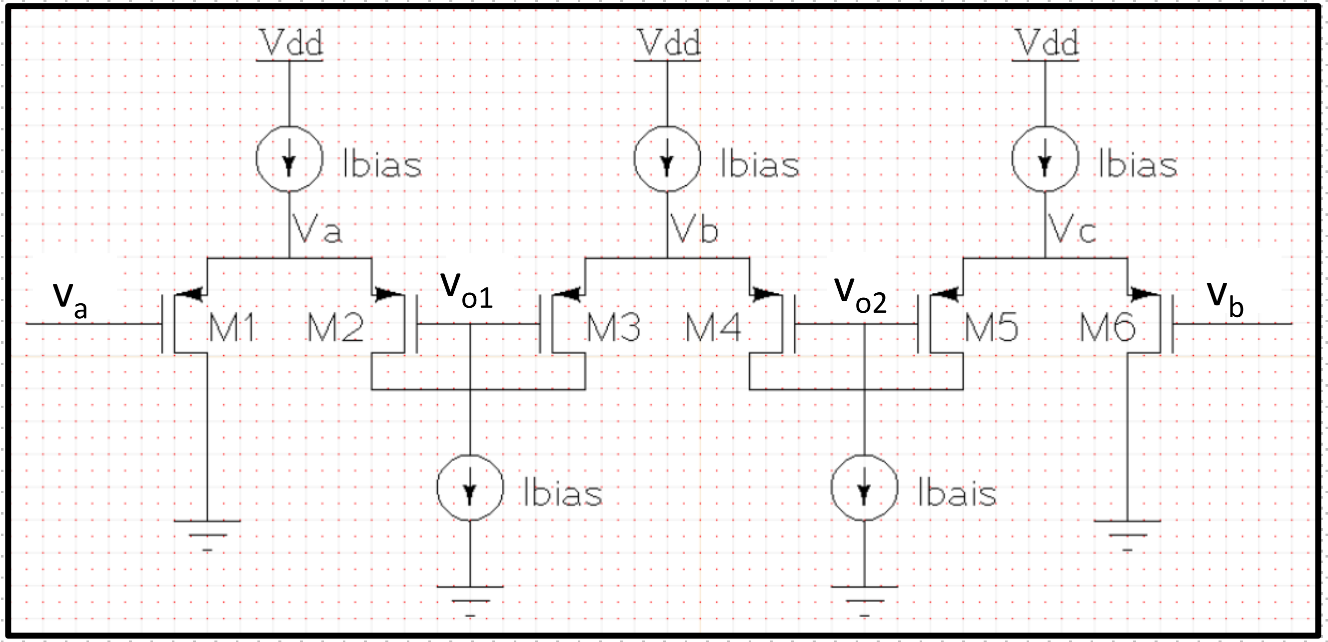
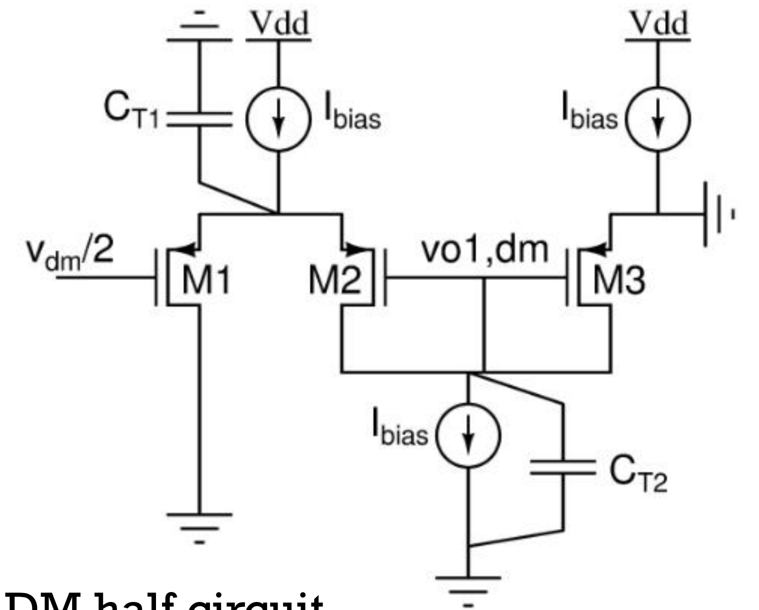
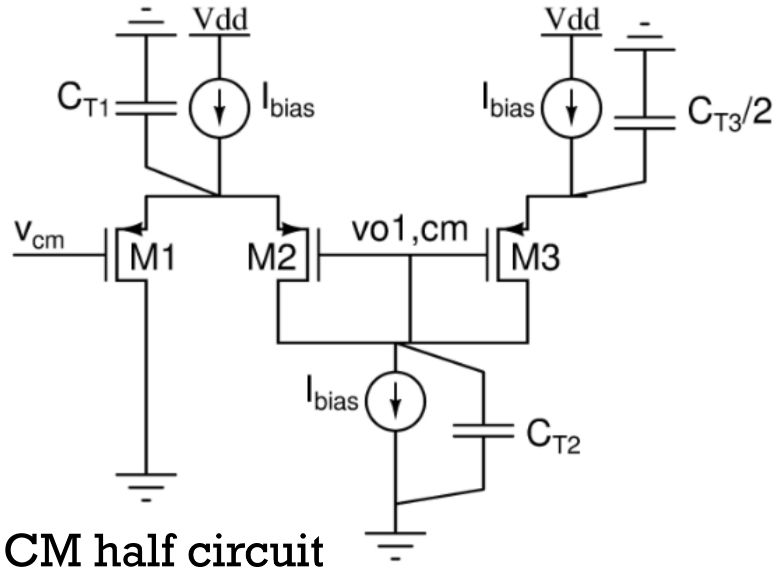


Fig12: Voltage attenuator



DM half circuit



CM half circuit

- $H_{cm} = \frac{v_{o1,cm}}{v_{cm}}\Big| v_{dm=0}$
- $H_{dm} = \frac{v_{o1,dm}}{v_{dm}/2}\Big| v_{cm=0}$
- Hence, $v_{o1} = H_{cm}v_{cm} + H_d v_d$

  & $v_{o2} = H_{cm}v_{cm} - H_d v_d$

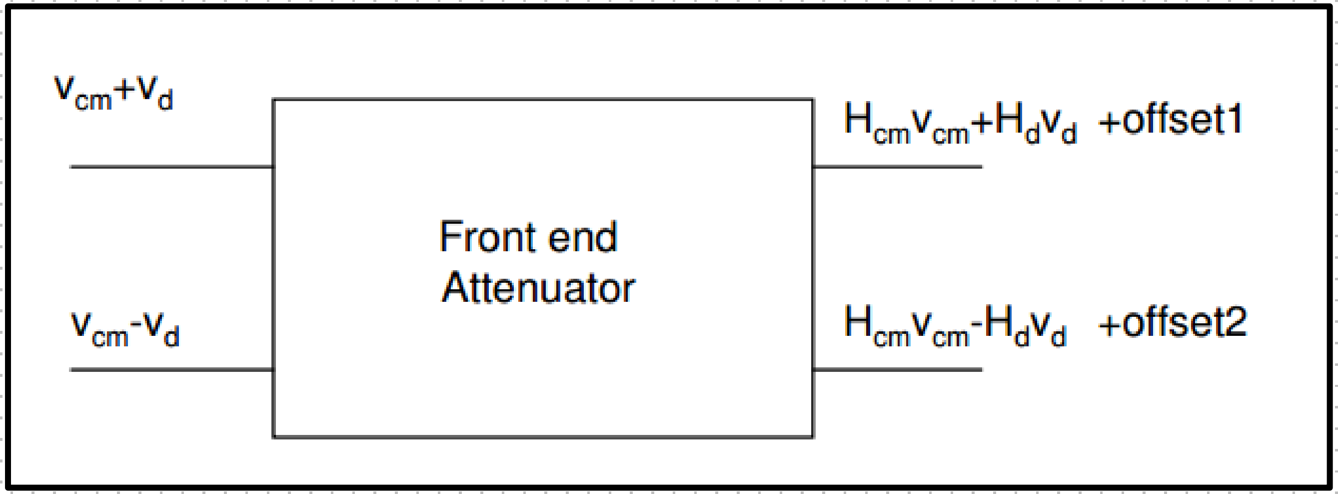
Offset issues in front end Attenuator

Let, $x = Asin(\omega t)$

For 2nd order harmonics at output:

$$x^2 = A^2 sin^2(\omega t) = \frac{A^2}{2}(1 - cos(2\omega t))$$

Hence, 2nd order harmonics give DC offset that can't be removed by low-pass filtering.



Offset 1 :

due to second order term : $a_2(H_{cm}v_{cm}+H_dv_d)^2$,

Offset 2 :

due to second order term : $a_2(H_{cm}v_{cm}-H_dv_d)^2$,

Hence, Offset 1 & Offset 2 are significantly different values from each other.

Offset issues in front end Attenuator

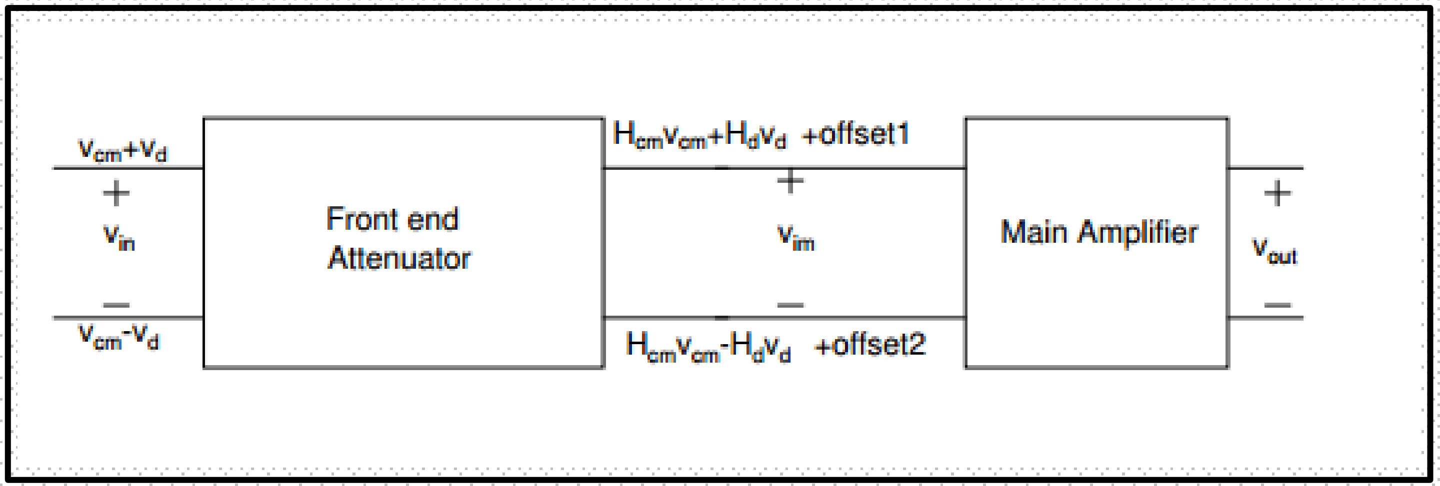
- This offset will increase further in second stage, where differential input is amplified by a high gain factor.
- Let, net output v_{out}=G*{(Offset1-Offset2)+2$H_d v_d$} , **G: gain of second stage= high**
- Draws huge offset at output.
- Hence, elimination of offset at the 1st stage is important.

Offset 1 :

due to : $a_2(H_{cm}v_{cm}+H_d v_d)^2$,

Offset 2 :

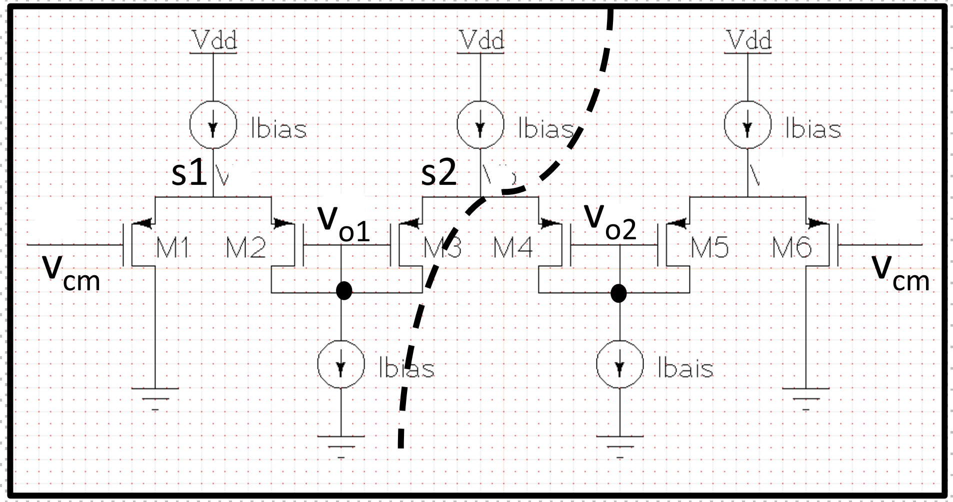
due to : $a_2(H_{cm}v_{cm}-H_d v_d)^2$,

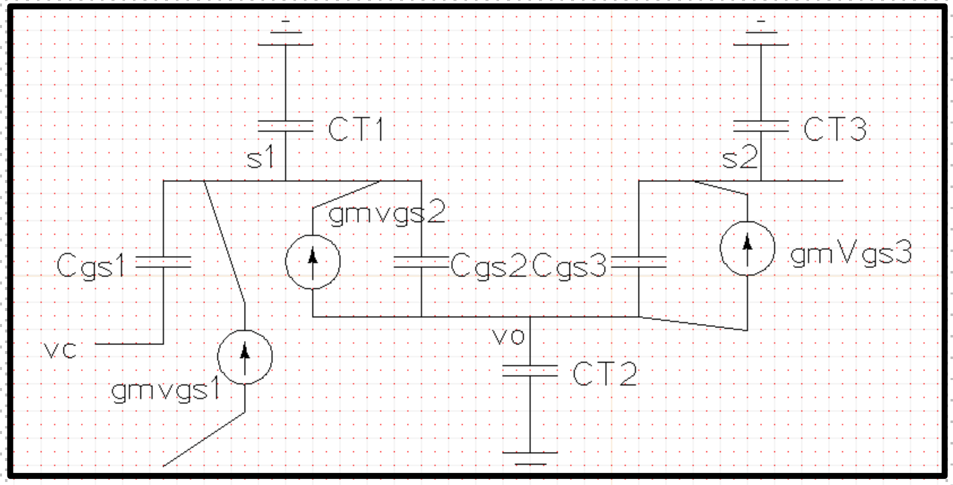


Solution : as $H_d \neq 0$, we can make $\mathbf{H_{cm}=0}$ for **Offset1 = Offset 2.**

By this, v_{out} gets rid of any DC offset.

Reduce CM interference:





Outputs of attenuator: $v_{o1} = H_{cm}v_c = v_{o2} = v_o$ (say) {as v_{dm}=0 here}

KCL at node s_1 (7), v_{o1} (8), s_2 (9):

$$(sC_{gs1} + g_m)(v_c - v_{s1}) + (g_m + sC_{gs2})(v_o - v_{s1}) = sC_{T1}v_{s1} \qquad (7)$$

$$(g_m + sC_{gs2})(v_o - v_{s1}) + (g_m + sC_{gs3})(v_o - v_{s2}) + sC_{T2}v_o = 0 \qquad (8)$$

$$(g_m + sC_{gs3})v_o = v_{s2}(g_m + s(C_{gs3} + C_{T3})) \qquad (9)$$

From (9), find v_{s2} & Put in (8):

$$v_{s1} = \frac{v_{o1}\left(2g_m + s(C_{gs2} + C_{gs3} + C_{T2}) - \frac{(g_m + sC_{gs3})^2}{g_m + s(C_{gs3} + C_{T3})}\right)}{(g_m+sC_{gs2})}$$

Put v_{s1} in (7): $H_{cm} = \frac{v_{o1}}{v_{cm}} = \frac{(sC_{gs1})}{(g_m+sC_{gs2}) - \frac{2g_{m1}+s(C_{T1}+C_{gs1}+C_{gs2})}{(g_m+sC_{gs2})}\left\{2g_m+s(C_{gs2}+C_{gs3}+C_{T2}) - \frac{(g_m+sC_{gs3})^2}{g_m+s(C_{gs3}+C_{T3})}\right\}}$

- **Observation**: H_{cm} decreases if C_{T1} & C_{T3} decreases. We want to minimize C_{T1} & C_{T3}.
- $C_{T1}=C_{db}+C_{sb1}+C_{sb2}$, $C_{T3}=C_{sb3}+ C_{db}/2$

Reduction of CM interference(Ctd.)

- $C_{T1}=C_{db}+C_{sb1}+C_{sb2}$, $C_{T3}=C_{sb3}+C_{db}/2$
- We want $C_{sb}=0$, but in twin-tub CMOS process, shorting S to B causes high **well capacitance (C_{GND})** at source.

- Hence, one auxiliary pair source node(V_{s1}) is connected to to bulk of main pair. This is **Source-buffered structure**.
- $C_{sb1}+C_{sb2}$ sees almost same potential across it. Though these two potentials are not exactly equal, but voltage across $C_{sb1}+C_{sb2}$ is very small, causing it to be virtually shorted.
- In advantage, S & B of main pair are decoupled.

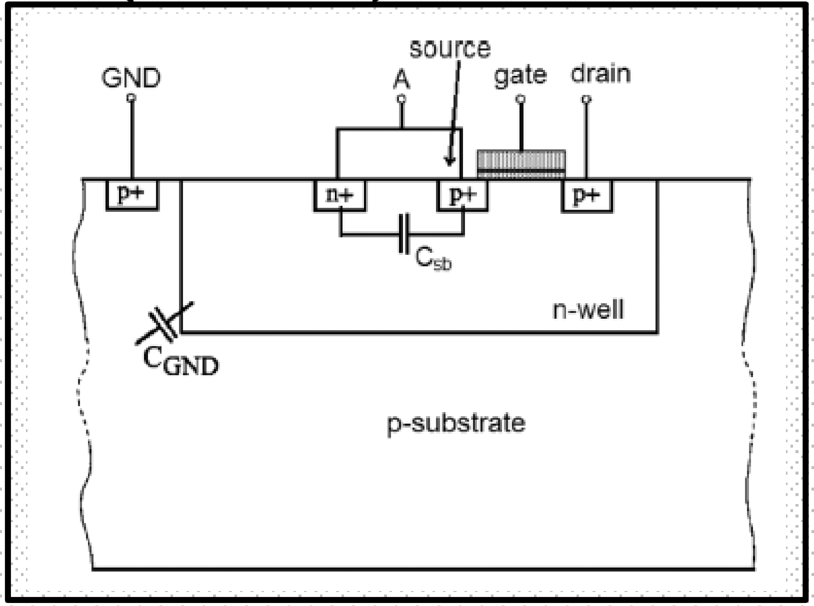


Fig14: vertical p-MOS, Fig15: Source-buffering

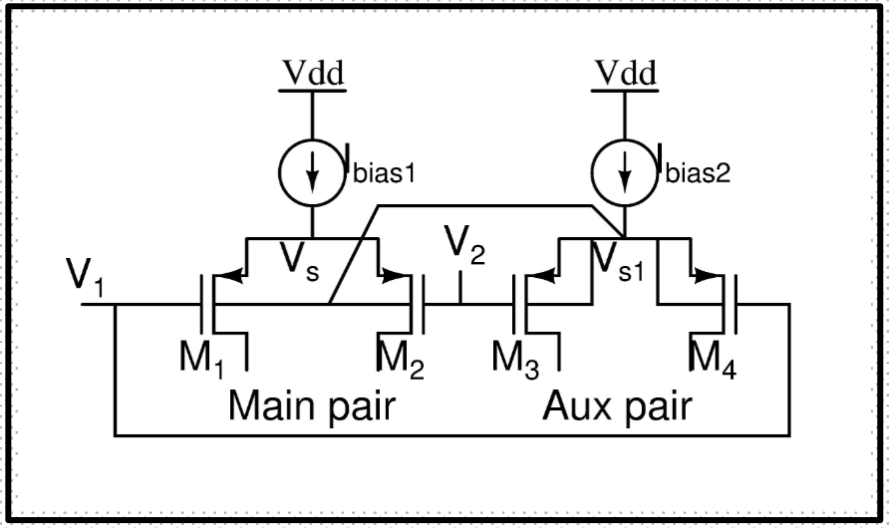


Fig14 Ref: Jingjing Yu; Amer, A.; Sanchez-Sinencio, E., "Electromagnetic Interference Resisting Operational Amplifier," in *Circuits and Systems I :Regular Papers, IEEE Transactions on* , vol.61, no.7, pp.1917-1927, July 2014

Common mode Transfer Function (H_{cm})

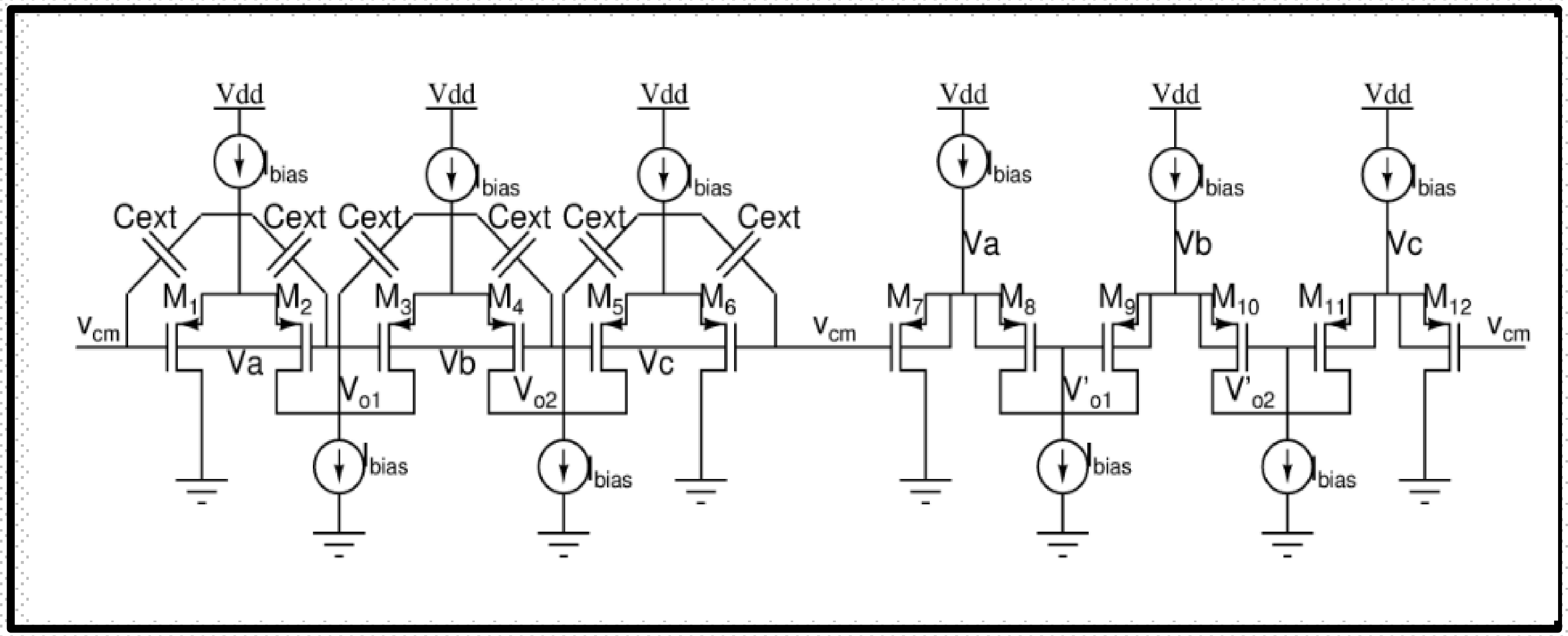


Fig17: Proposed Attenuator structure

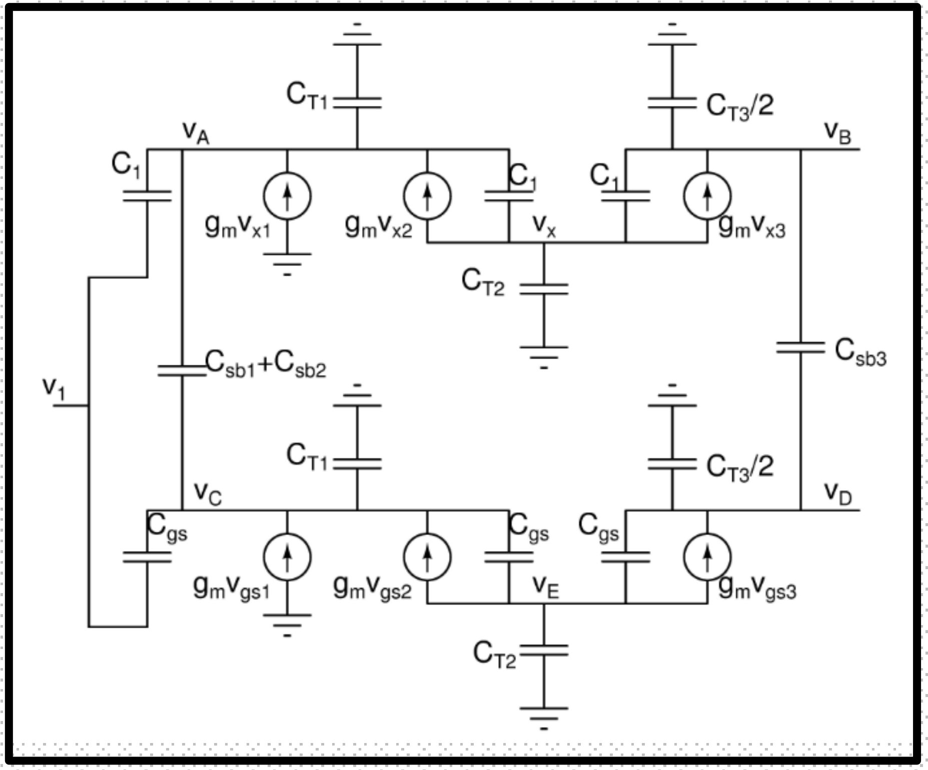


Fig18: ac equivalent model of Half Circuit for Common mode inputs

$$C_1 = C_{gs1} + C_{ext}$$

$$\boldsymbol{g_m v_{x1} = g_m v_{gs1} + g_{mb} v_{bs1}}$$
$$\boldsymbol{g_m v_{x2} = g_m v_{gs2} + g_{mb} v_{bs1}}$$
$$\boldsymbol{g_m v_{x1} = g_m v_{gs3} + g_{mb} v_{bs3}}$$

Calculation for CMTF (H_{cm})

Applying KCL at A,B,C,D,E,X nodes, we get six equations.

By solving these equations, we get:

$$H_{cm}(\infty) = \left(\frac{v_x}{v_1}\right) = \frac{\alpha_2\gamma_1 - \alpha_1\gamma_2}{\alpha_1\beta_2 - \alpha_2\beta_1}$$

Here, α_1, α_2 ,β_1, β_2 ,γ_1, γ_2 are constant values at very high Frequency, coming from parasitic capacitances & C_1.

By making , $H_{cm}(\infty)=0$, we find a quadratic equation of C_1.

By solving the equation , ($C_1=C_{gs1}+C_{ext}$) we find:

$$C_{ext} = 95fF$$

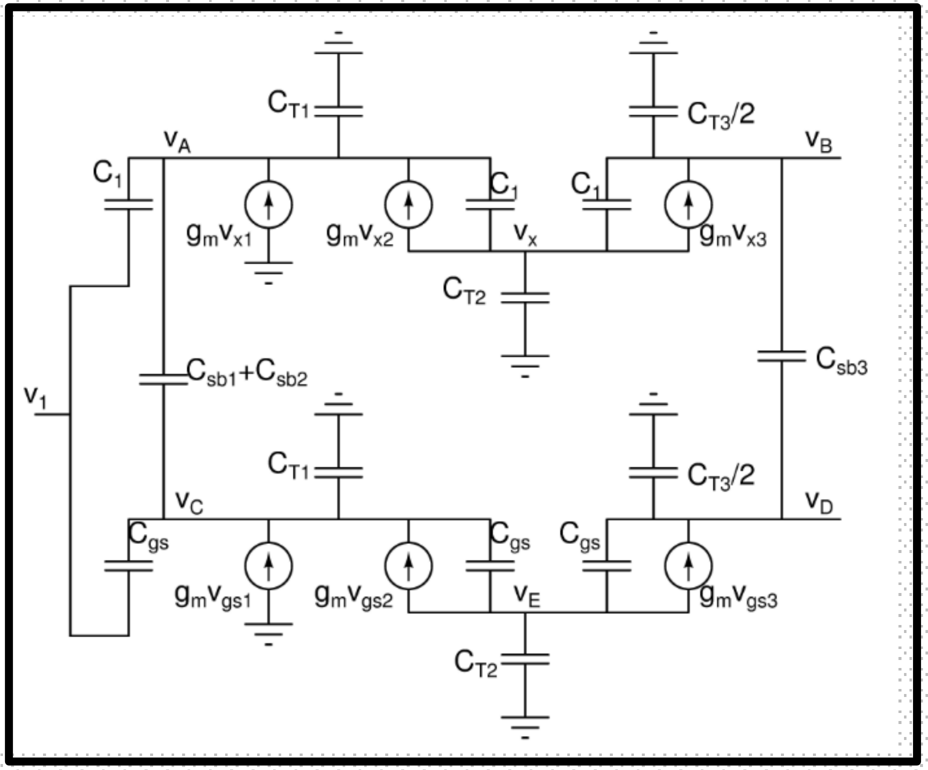


Fig18: ac equivalent model of Half Circuit for Common mode inputs

Adding this amount of external capacitance will make H_{cm}= 0 at very high frequency.

Comparison I (100mVpp)

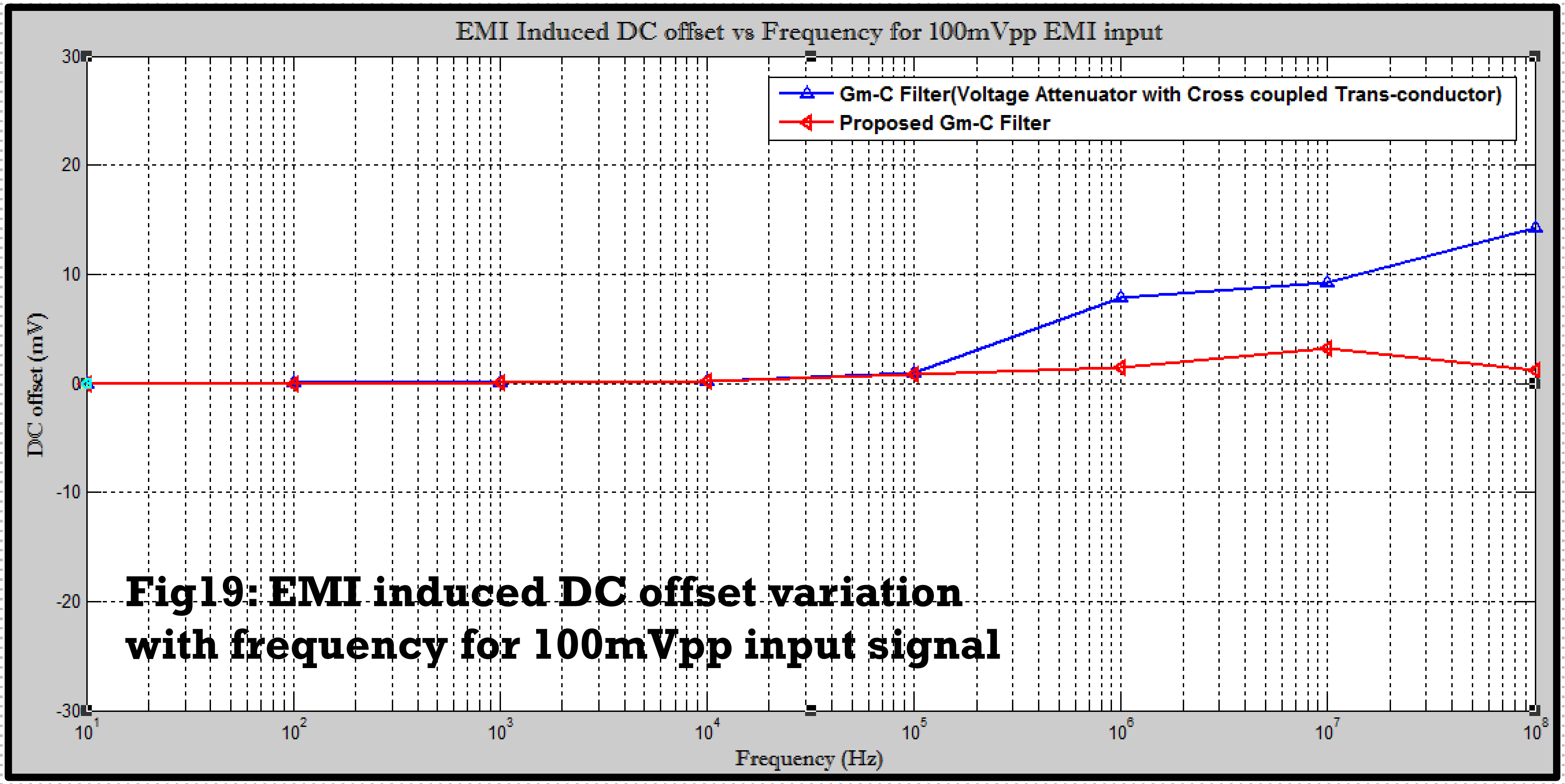
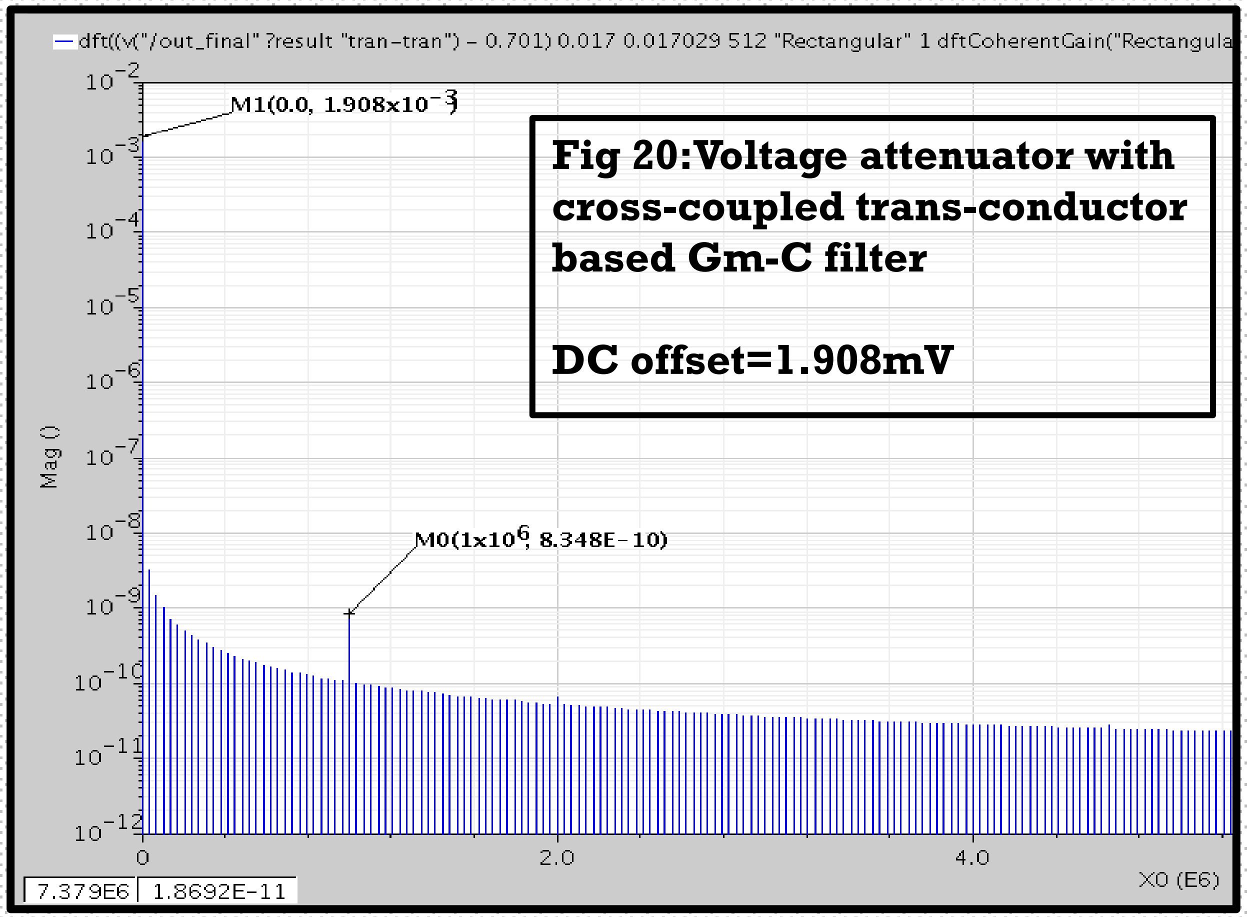


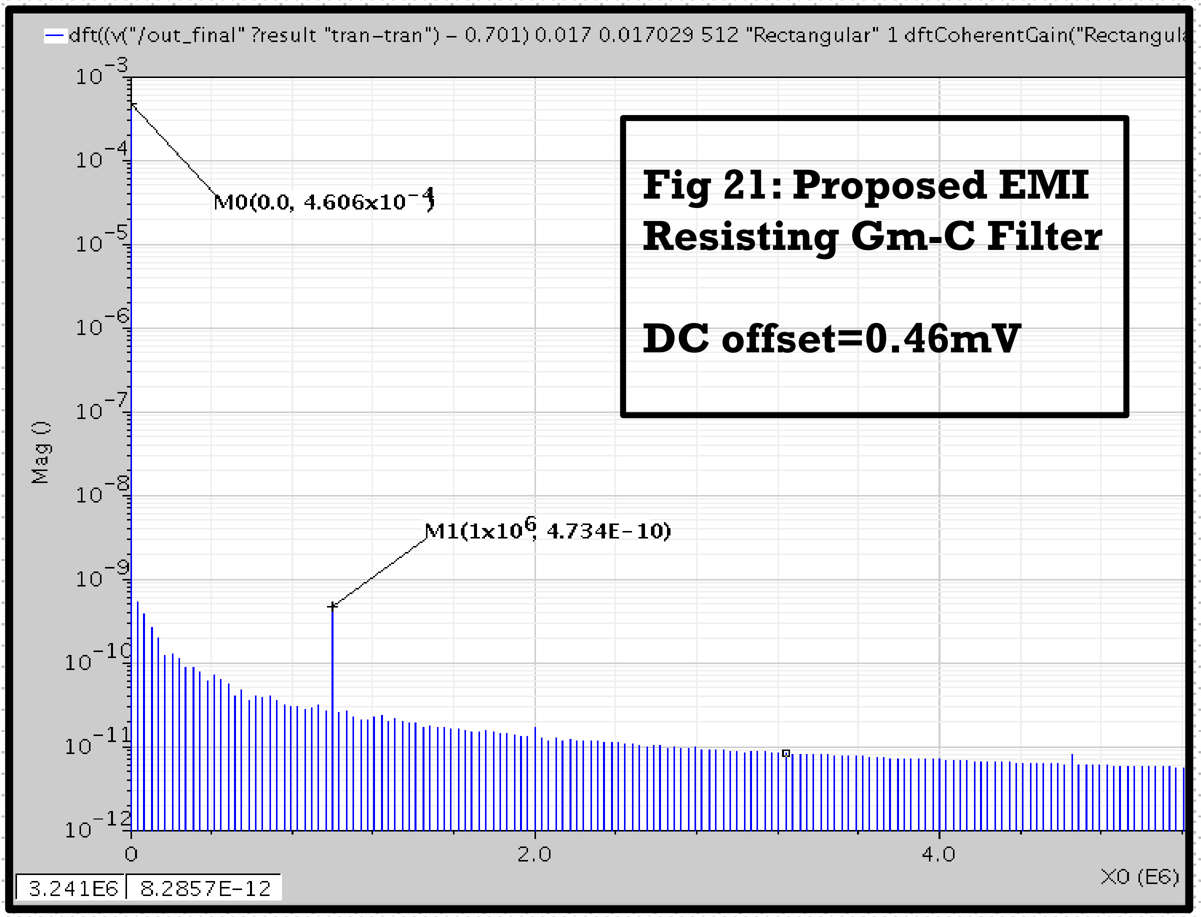
Fig19: EMI induced DC offset variation with frequency for 100mVpp input signal

- Upto 100KHz, offsets are almost same as parasitic effect comes at high frequencies.
- Upto 100KHz, EMI induced Offset is very low(~uV). This proves good linearity of the filter at low frequency.
- Proposed Gm-C filter shows small offset even at high freq due to its immunity to CM interference.
- Acc to calculation, offset will reach zero at infinite frequency

- In frequency range **[10Hz-100MHz]**, Maximum EMI induced offset (Magnitude): (i) Voltage attenuator with cross-coupled transconductor based G_m-C filter:: **1.996%** (ii) Proposed G_m-C filter:: **0.459%**

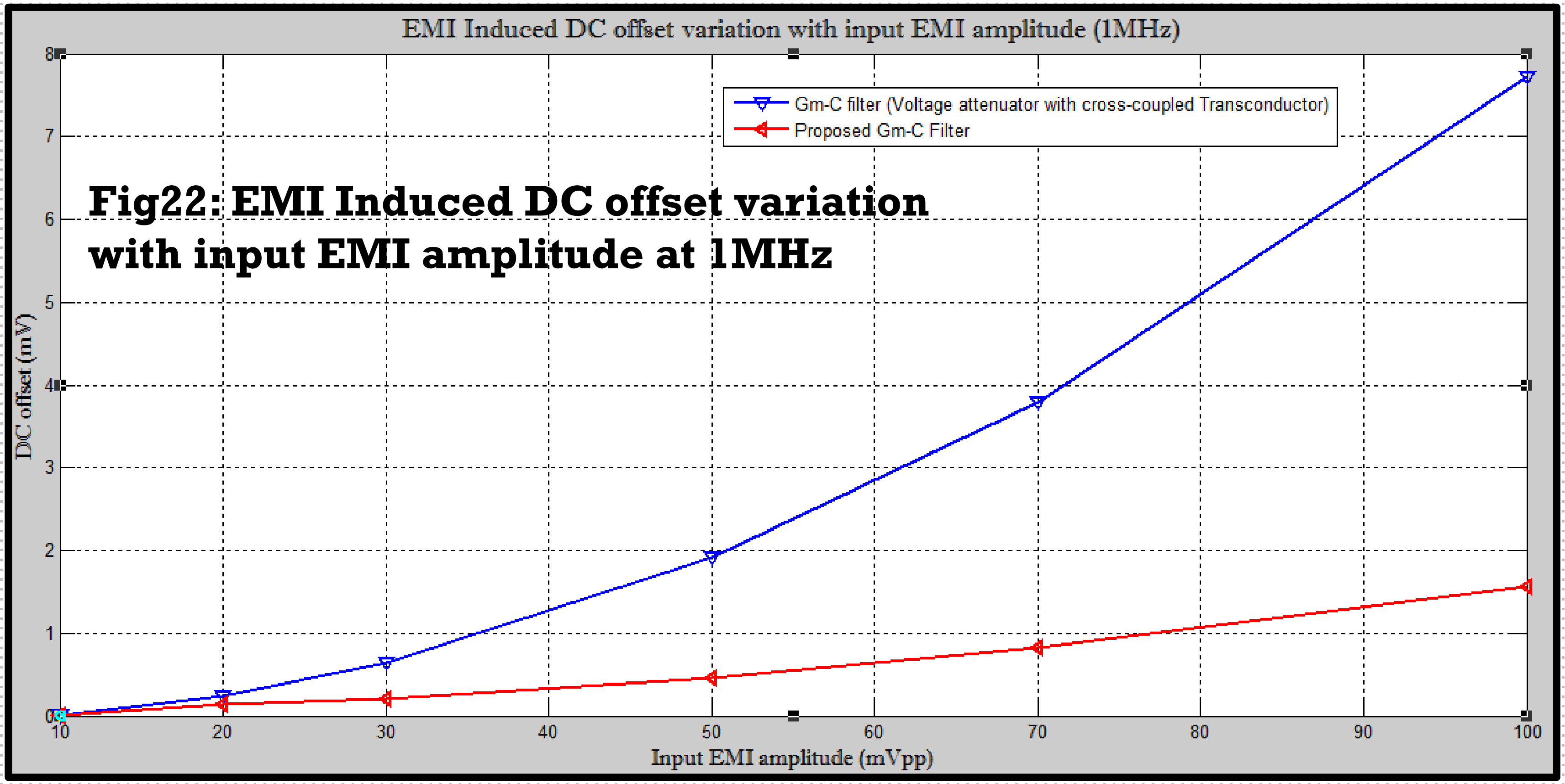
Comparison II (FFT: 50mVpp, 1MHz)





Coherent sampling: f_{in}=1MHz, N_{WINDOW}=29, N_{RECORD}=512 : irreducible. DC Offset is reduced in EMI resisting filter.

Comparison III (1MHz)



- For non-lineatity of the filter at high frequency, DC offset is high. Let, v_{in}=Asin(ωt). ie. $v_{in}^2 = \frac{A^2}{2}(1-\cos(\omega t))$ .This DC Offset is high when amplitude is high as seen in figure.
- But, by making $H_{cm}(\infty)=0$, we are reducing non-linearity of the filter at high frequency.
- This is why at $f \gg GBW$ (f=1MHz here), this proposed filter is much immune to Electromagnetic Interference. (CM interference precisely)

With input amplitude,EMI induced DC offset increases. Rate of increment is less in proposed Filter.

Conclusion

- **Advantage:**

  - For out-of band EMI frequencies, when parasitic capacitances come in picture, **EMI induced DC offset** becomes a challenging issue. Hence, This approach for EMI resisting highly linear sub-kilohertz G_m-C filter is noble in this field.

- **Disadvantage:**

  - Added power dissipation
  - Transistor area increment

THANK YOU

Source buffering

- We want to make H_{cm}=0 & accordingly we'll choose C_1
- KCL at node C(10),E(11),D(12):

$$(sC_{gs}+g_{m1})(v_1+v_E-2v_A-2v_{bs1})=sC_{T1}v_A+s(C_{bs1}+C_{T1})v_{bs1} \quad .(10)$$

$$(2v_E-v_D-v_C)(sC_{gs}+g_{m1})+sC_{T2}v_E=0 \quad .(11)$$

$$(v_E-v_B-v_{bs2})(sC_{gs}+g_{m1})=s\frac{C_{bs2}+C_{T3}}{2}v_{bs2}+s\frac{C_{T3}}{2}v_B \quad .(12)$$

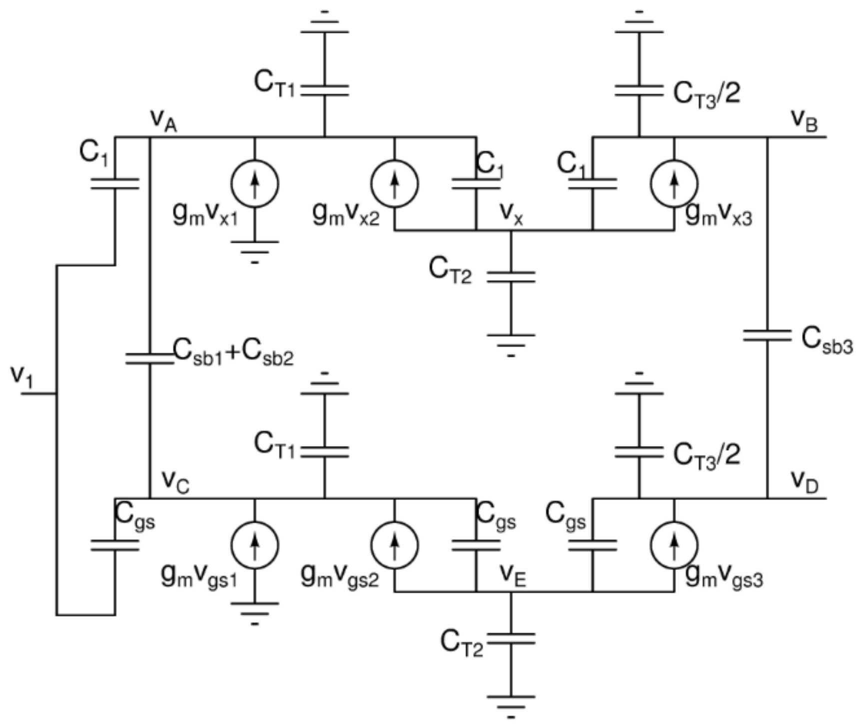


Fig12: ac equivalent of the CM Half circuit

Solving (11) & (12) to omit v_E:

$$\left[\left[g_m+s\left\{C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right\}\right]\left[2g_m+s(2C_{gs2}+C_{T2})\right]-(g_m+sC_{gs})^2\right]*(v_{bs2}+v_B)=(v_A+v_{bs1})(g_m+sC_{gs})^2 \quad (13)$$

Put v_E value from(12) in (10):

$$(sC_{gs}+g_{m1})v_1+v_{bs2}\left[g_m+s\left\{C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right\}\right]+v_B\left[g_m+s\left\{C_{gs}+\frac{C_{T3}}{2}\right\}\right]=$$
$$v_A[2g_m+s(2C_{gs}+C_{T1})]+v_{bs1}[2g_m+s(2C_{gs}+C_{T1}+C_{sb1})] \quad (14)$$

Now, KCL at A(15), B(16), X(17):

$$(sC_1 + g_m)(v_1 + v_x) + (2g_{mb} + sC_{bs1})v_{bs1} = [2g_m + s(2C_1 + C_{T1})]v_A \quad \text{........ (15)}$$

$$(sC_1 + g_m)v_x + \left(g_{mb1} + \frac{sC_{sb2}}{2}\right) v_{sb2} = \left(g_m + s\left(C_1 + \frac{C_{T3}}{2}\right)\right) v_B \quad \text{....... (16)}$$

$$[2g_m + s(2C_1 + C_{T2})]v_x + g_{mb}(v_{bs1} + v_{bs2}) = (v_A + v_B)(g_m + sC_1) \quad \text{....... (17)}$$

For minimizing,we assume:

- $M(v_{bs2} + v_B) = N(v_A + v_{bs1})$ .... (13.a) ,
- $Pv_1 + Qv_{bs2} + Rv_B = Sv_A + Tv_{bs1}$ .... (14.a)
- $A(v_1 + v_X) + Bv_{bs1} = Cv_A$.. (15.a)
- $Dv_X + Ev_{bs2} = Fv_B$ .... (16.a)
- $Gv_X + H(v_{bs1} + v_{bs2}) = I(v_A + v_B)$ .... (17.a)

$M = \left[\left[g_m + s\left\{C_{gs} + \frac{C_{sb2}+C_{T3}}{2}\right\}\right][2g_m + s(2C_{gs2} + C_{T2})] - (g_m + sC_{gs})^2\right]$,
$N = (g_m + sC_{gs})^2$

$P = (sC_{gs} + g_{m1})$, $Q = \left[g_m + s\left\{C_{gs} + \frac{C_{sb2}+C_{T3}}{2}\right\}\right]$, $R = \left[g_m + s\left\{C_{gs} + \frac{C_{T3}}{2}\right\}\right]$

$S = [2g_m + s(2C_{gs} + C_{T1})]$, $T = [2g_m + s(2C_{gs} + C_{T1} + C_{sb1})]$,
$A = (sC_1 + g_m), B = (2g_{mb} + sC_{bs1})$

$C = [2g_m + s(2C_1 + C_{T1})]$, $D = (sC_1 + g_m)$, $E = \left(g_{mb1} + \frac{sC_{sb2}}{2}\right)$,

$F = \left(g_m + s\left(C_1 + \frac{C_{T3}}{2}\right)\right)$

$G = [2g_m + s(2C_1 + C_{T2})], H = g_{mb}, I = (g_m + sC_1)$

- Solving (13.a) & (14.a):

$$v_{bs2} = X_2v_1 + Y_2v_B + Z_2v_A \quad \text{where, } X_2 = \frac{NP}{(TM-NQ)}, Y_2 = \frac{-(S-T)}{(TM-NQ)}, Z_2 = \frac{-(TM-NR)}{(TM-NQ)} \quad \text{.........(18)}$$

$$v_{bs1} = X_1v_1 + Y_1v_B + Z_1v_A \quad \text{where , } X_1 = \frac{MP}{(TM-NQ)}, Y_1 = \frac{M(R-Q)}{(TM-NQ)}, Z_1 = -\left[1 + \frac{M(S-T)}{(TM-NQ)}\right] \quad \text{......... (19)}$$

At high frequency (s→∞), we find the following:

- $X_2 = \frac{NP}{(TM-NQ)} = \frac{C_{gs}^3}{(2C_{gs}+C_{T1}+C_{sb1})\left[\left(C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right)(2C_{gs}+C_{T2})-C_{gs}^2\right]-C_{gs}^2(C_{gs}+\frac{C_{sb2}+C_{T3}}{2})}$
- $Y_2 = \frac{-N(S-T)}{(TM-NQ)} = \frac{C_{sb1}*C_{gs}^2}{(2C_{gs}+C_{T1}+C_{sb1})\left[\left(C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right)(2C_{gs}+C_{T2})-C_{gs}^2\right]-C_{gs}^2(C_{gs}+\frac{C_{sb2}+C_{T3}}{2})}$
- $Z_2 = \frac{-(TM-NR)}{(TM-NQ)} = \frac{C_{gs}^2(C_{gs}+\frac{C_{T3}}{2})-(2C_{gs}+C_{T1}+C_{sb1})\left[\left(C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right)(2C_{gs}+C_{T2})-C_{gs}^2\right]}{(2C_{gs}+C_{T1}+C_{sb1})\left[\left(C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right)(2C_{gs}+C_{T2})-C_{gs}^2\right]-C_{gs}^2(C_{gs}+\frac{C_{sb2}+C_{T3}}{2})}$
- $X_1 = \frac{MP}{TM-NQ} = \frac{\left[\left(C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right)(2C_{gs}+C_{T2})-C_{gs}^2\right]C_{gs}}{(2C_{gs}+C_{T1}+C_{sb1})\left[\left(C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right)(2C_{gs}+C_{T2})-C_{gs}^2\right]-C_{gs}^2(C_{gs}+\frac{C_{sb2}+C_{T3}}{2})}$
- $Y_1 = \frac{M(R-Q)}{TM-NQ} = \frac{-\left[\left(C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right)(2C_{gs}+C_{T2})-C_{gs}^2\right]*\frac{C_{T3}}{2}}{(2C_{gs}+C_{T1}+C_{sb1})\left[\left(C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right)(2C_{gs}+C_{T2})-C_{gs}^2\right]-C_{gs}^2(C_{gs}+\frac{C_{sb2}+C_{T3}}{2})}$
- $Z_1 = \frac{M(S-T)}{TM-NQ} = \frac{\left[\left(C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right)(2C_{gs}+C_{T2})-C_{gs}^2\right]*C_{sb1}}{(2C_{gs}+C_{T1}+C_{sb1})\left[\left(C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right)(2C_{gs}+C_{T2})-C_{gs}^2\right]-C_{gs}^2\left(C_{gs}+\frac{C_{sb2}+C_{T3}}{2}\right)} - 1$

All terms are independent of C_1

Put equation(19) in (15.a): $v_1(A+BX_1)+BY_1v_B+Av_X=(C-BZ_1)v_A$

Put equation(18) in (16.a): $Dv_X+EX_2v_1+EZ_2v_A=(F-EY_2)v_B$

- Put value of v_A from first equation in second equation: $\alpha_1 v_B=\beta_1 v_X+\gamma_1 v_1$ (20)

  where, $\alpha_1=[(F-EY_2)(C-BZ_1)-EZ_2BY_1]$, $\beta_1=(D(C-BZ_1)+EZ_2A)$, $\gamma_1=\{EX_2(C-BZ_1)+EZ_2(A+BX_1)\}$

From (18) & (19), put values v_{bs1} & v_{bs2} in (17.a): $\alpha_2 v_B=\beta_2 v_X+\gamma_2 v_1$ (21)

where, $\alpha_2=\left[I+\frac{IBY_1}{C-BZ_1}-H(Y_1+Y_2)-\frac{H(Z_1+Z_2)BY_1}{C-BZ_1}\right]$, $\beta_2=\left[G+\frac{AH(Z_1+Z_2)}{C-BZ_1}-\frac{AI}{C-BZ_1}\right]$, $\gamma_2=\left[H(X_1+X_2)+\frac{H(Z_1+Z_2)(A+BX_1)}{C-BZ_1}-\frac{(A+BX_1)I}{C-BZ_1}\right]$

From (20) & (21): $\left(\frac{\alpha_2}{\alpha_1}\right)(\beta_1 v_X+\gamma_1 v_1)=\beta_2 v_X+\gamma_2 v_1$

$$\boldsymbol{H_{cm}=\frac{\alpha_2\gamma_1-\alpha_1\gamma_2}{\alpha_1\beta_2-\alpha_2\beta_1}}$$

For $H_{cm}=0$, we find: $\boldsymbol{\alpha_2\gamma_1-\alpha_1\gamma_2=0}$

⇨ $[I(C-BZ_1)+IBY_1-H(Y_1+Y_2)(C-BZ_1)-H(Z_1+Z_2)BY_1]*[EX_2(C-BZ_1)+EZ_2(A+BX_1)]=$
$[(F-EY_2)(C-BZ_1)-EZ_2BY_1]*[H(X_1+X_2)(C-BZ_1)+H(Z_1+Z_2)(A+BX_1)-(A+BX_1)I]$

1st term: $[I(C-BZ_1)+IBY_1-H(Y_1+Y_2)(C-BZ_1)-H(Z_1+Z_2)BY_1]$ divide numerator and denominator by [order of max(num, den) = 2] , we'll find the term is simplified to: $\left[(C_1)\left((2C_1+C_{T1})-C_{bs1}Z_1\right)+C_1C_{bs1}Y_1\right]$

2nd term: $[EX_2(C-BZ_1)+EZ_2(A+BX_1)]$ divide numerator and denominator by s^2 gives:

$\left[\frac{C_{sb2}}{2}X_2\left((2C_1+C_{T1})-C_{bs1}Z_1\right)+\frac{C_{sb2}}{2}Z_2(\mathrm{C}_1+\mathrm{C}_{\mathrm{bs1}}X_1)\right]$

3rd term: $[(F - EY_2)(C - BZ_1) - EZ_2BY_1]$ is simplified by dividing num & den by s² and put s→∞

$$:\left[\left(\left(C_1 + \frac{C_{T3}}{2}\right) - \left(\frac{C_{sb2}}{2} * Y_2\right)\right)\left((2C_1 + C_{T1}) - (C_{bs1}Z_1)\right)\right] - \left(\frac{C_{sb2}}{2}\right)(C_{bs1})Z_2Y_1$$

4th term: $[H(X_1 + X_2)(C - BZ_1) + H(Z_1 + Z_2)(A + BX_1) - (A + BX_1)I]$ is simplifed by dividing num & den by s² and put s→∞

$$:[-(C_1 + C_{bs1}X_1)(C_1)]$$

$$\boldsymbol{C_1^2(4X_2 + 2Z_2 - 2) + C_1\{((2C_{T1}X_2 - 2C_{bs1}Z_1X_2 + 2C_{bs1}X_1Z_2) + (C_{T1} - C_{bs1}Z_1 + C_{bs1}Y_1)(2X_2 + Z_2)) - (C_{T1} - C_{bs1}Z_1 - C_{bs1}Z_2Y_1 + 2C_{bs1}X_1)\} + C_{T1} - C_{bs1}Z_1 + C_{bs1}Y_1\ C_{T1}X_2 - C_{bs1}Z_1X_2 + C_{bs1}X_1Z_2 - C_{bs1}X_1(C_{T1} - C_{bs1}Z_1 - C_{bs1}Z_2Y_1)} = 0$$

This follows the form: $aC_1{}^2 + bC_1 + c = 0$

From DC simulation: C_{gs}=7.05fF , C_{sb1}=17.14fF , C_{sb2}=17.02fF, C_{T1}=10.46fF , C_{T2}=10.47fF , C_{T3}=10.47fF , g_m=42.4nA/V

Hence, putting the values in all equations, $a = -4.03, b = -2.53 * 10^{-15}$ & $c = 4.12 * 10^{-26}$

$$\boldsymbol{C_1 = -\frac{2.53 * 10^{-15} - \sqrt{(-2.53 * 10^{-15})^2 + 4 * 4.03 * 4.12 * 10^{-26}}}{2 * 4.03} = 100.79\ fF}$$

- $C_1 = C_{gs} + C_{ext}$, Here, C_{gs}=7.05fF , ⇨ $C_{ext} = 93.74fF$
- We consider, $\boldsymbol{C_{ext} = 95fF}$

RETURN

Infinite frequency for 1st term:

1st term:

$$[I(C - BZ_1) + IBY_1 - H(Y_1 + Y_2)(C - BZ_1) - H(Z_1 + Z_2)BY_1]$$

$$= (g_m + sC_1)[2g_m + s(2C_1 + C_{T1}) - (2g_{mb} + sC_{bs1})Z_1 + (2g_{mb} + sC_{bs1})Y_1] - [g_{mb}(Y_1 + Y_2)(2g_m + s(2C_1 + C_{T1}))] - [g_{mb}(Z_1 + Z_2)(2g_{mb} + sC_{bs1})Y_1]$$

The critical frequencies above which (at least 10 times) we are considering the approximation of s→∞ is valid:

$$(i)\ \frac{g_m}{C_1} = \frac{42.2n}{100.79f} = 67.48\ KHz$$

$$(ii)\ \frac{2g_m}{2C_1 + C_{T1}} = 64.27\ KHz$$

$$(iii)\ \frac{2g_{mb}}{C_{bs1}} = 74.9\ KHz$$

Infinite frequency for 2nd term:

2nd term:

$[EX_2(C - BZ_1) + EZ_2(A + BX_1)]$

$=\left(g_{mb1} + \frac{sC_{bs2}}{2}\right)[X_2\{2g_m + s(2C_1 + C_{T1}) - (2g_{mb} + sC_{bs1})Z_1\} + Z_2\{(g_m + sC_1) + (2g_{mb} + sC_{bs1})X_1\}]$

The critical frequencies above which (at least 10 times) we are considering the approximation of $s \rightarrow \infty$ is valid:

$(i)\ \frac{2g_m X_2 - 2g_{mb} Z_1}{(2C_1 + C_{T1})X_2 - C_{bs1} Z_1} = 71.33\ KHz$

$(ii)\ \frac{(g_m + 2g_{mb} X_1)}{C_1 + C_{bs1} X_1} = 67.16\ KHz$

Infinite frequency for 3rd term:

3rd term:

$$[(F - EY_2)(C - BZ_1) - EZ_2BY_1]$$

$$= \left[\left\{(g_m + g_{mb}Y_2) + s\left(C_1 + \frac{C_{T3}}{2} - \frac{C_{bs2}}{2}Y_2\right)\right\}\{(2g_m - 2g_{mb}) + s(2C_1 + C_{T1} - C_{bs1}Z_1)\} - Z_2Y_1(g_{mb} + \frac{sC_{bs2}}{2})(2g_{mb} + sC_{bs1})\right]$$

The critical frequencies above which (at least 10 times) we are considering the approximation of s→∞ is valid:

$$(i)\ \frac{g_m + g_{mb}Y_2}{C_1 + \frac{C_{T3}}{2} - \frac{C_{bs2}}{2}Y_2} = 64.18\ KHz$$

$$(ii)\ \frac{2(g_m - g_{mb})}{2C_1 + C_{T1} - C_{bs1}Z_1} = 54.6\ KHz$$

Infinite frequency for 4th term:

4th term:

$$[H(X_1 + X_2)(C - BZ_1) + H(Z_1 + Z_2)(A + BX_1) - (A + BX_1)I]$$

$$= g_{mb}(X_1 + X_2)[2g_m + s(2C_1 + C_{T1}) - (2g_{mb} + sC_{bs1})Z_1] + g_{mb}(Z_1 + Z_2)[g_m + sC_1 + (2g_{mb} + sC_{bs1})X_1] - (g_m + sC_1)[g_m + sC_1 + (2g_{mb} + sC_{bs1})X_1]$$

The critical frequencies above which (at least 10 times) we are considering the approximation of s→∞ is valid:

$$(i)\ \frac{g_m + 2g_{mb}X_1}{C_1 + C_{bs1}X_1} = 67.16\ KHz$$

$$(ii)\ \frac{2(g_m - g_{mb}Z_1)}{2C_1 + C_{T1} - C_{bs1}Z_1} = 64.37\ KHz$$

- $M = \left[\left[g_m + s\left\{C_{gs} + \frac{C_{sb2}+C_{T3}}{2}\right\}\right]\left[2g_m + s(2C_{gs2} + C_{T2})\right] - (g_m + sC_{gs})^2\right]$,
- $N = (g_m + sC_{gs})^2, P = (sC_{gs} + g_{m1})$,
- $Q = \left[g_m + s\left\{C_{gs} + \frac{C_{sb2}+C_{T3}}{2}\right\}\right],\ R = \left[g_m + s\left\{C_{gs} + \frac{C_{T3}}{2}\right\}\right]$
- $S = [2g_m + s(2C_{gs} + C_{T1})], T = [2g_m + s(2C_{gs} + C_{T1} + C_{sb1})]$,
- $A = (sC_1 + g_m),\ B = (2g_{mb} + sC_{bs1})$
- $C = [2g_m + s(2C_1 + C_{T1})],\ D = (sC_1 + g_m)$,
- $E = \left(g_{mb1} + \frac{sC_{sb2}}{2}\right), F = \left(g_m + s\left(C_1 + \frac{C_{T3}}{2}\right)\right)$
- $G = [2g_m + s(2C_1 + C_{T2})],\ H = g_{mb},\ I = (g_m + sC_1)$

$$X_2 = \frac{NP}{(TM-NQ)},\quad Y_2 = \frac{-N(S-T)}{(TM-NQ)},\quad Z_2 = \frac{-(TM-NR)}{(TM-NQ)}$$

$$X_1 = \frac{MP}{(TM-NQ)},\quad Y_1 = \frac{M(R-Q)}{(TM-NQ)},\quad Z_1 = -\left[1 + \frac{M(S-T)}{(TM-NQ)}\right]$$

For $X_{1,2}$, $Y_{1,2}$, $Z_{1,2}$, the frequencies above which we can considers $s \to \infty$, are given as following:

- M,Q: (i) $\frac{g_m}{C_{gs}+\frac{C_{bs2}+C_{T3}}{2}} = 324.5KHz$

  (ii) $\frac{2g_m}{2C_{gs}+C_{T2}} = 549.3KHz$

- N,P : $\frac{g_m}{C_{gs}} = 957.2KHz$
- R: $\frac{2g_m}{2C_{gs}+C_{T3}} = 549.3KHz$
- T: $\frac{g_m}{C_{gs}+\frac{C_{T1}+C_{bs1}}{2}} < 549KHz$
- S: $\frac{2g_m}{2C_{gs}+C_{T1}} = 549.3KHz$

Conclusion:

All the terms are considered individually to find the frequency above which we can consider our assumption is valid. Ie H_{cm} is zero.

From all the terms above, we find highest frequency =957.2 KHz. Hence, above almost 10 times, of it ie 9-10 MHz, we can Assume frequency to be infinite. All our assumptions are valid above this frequency. This is valid as 10MHz is considered typically in EMI frequency range.

Comparison (100mVpp)

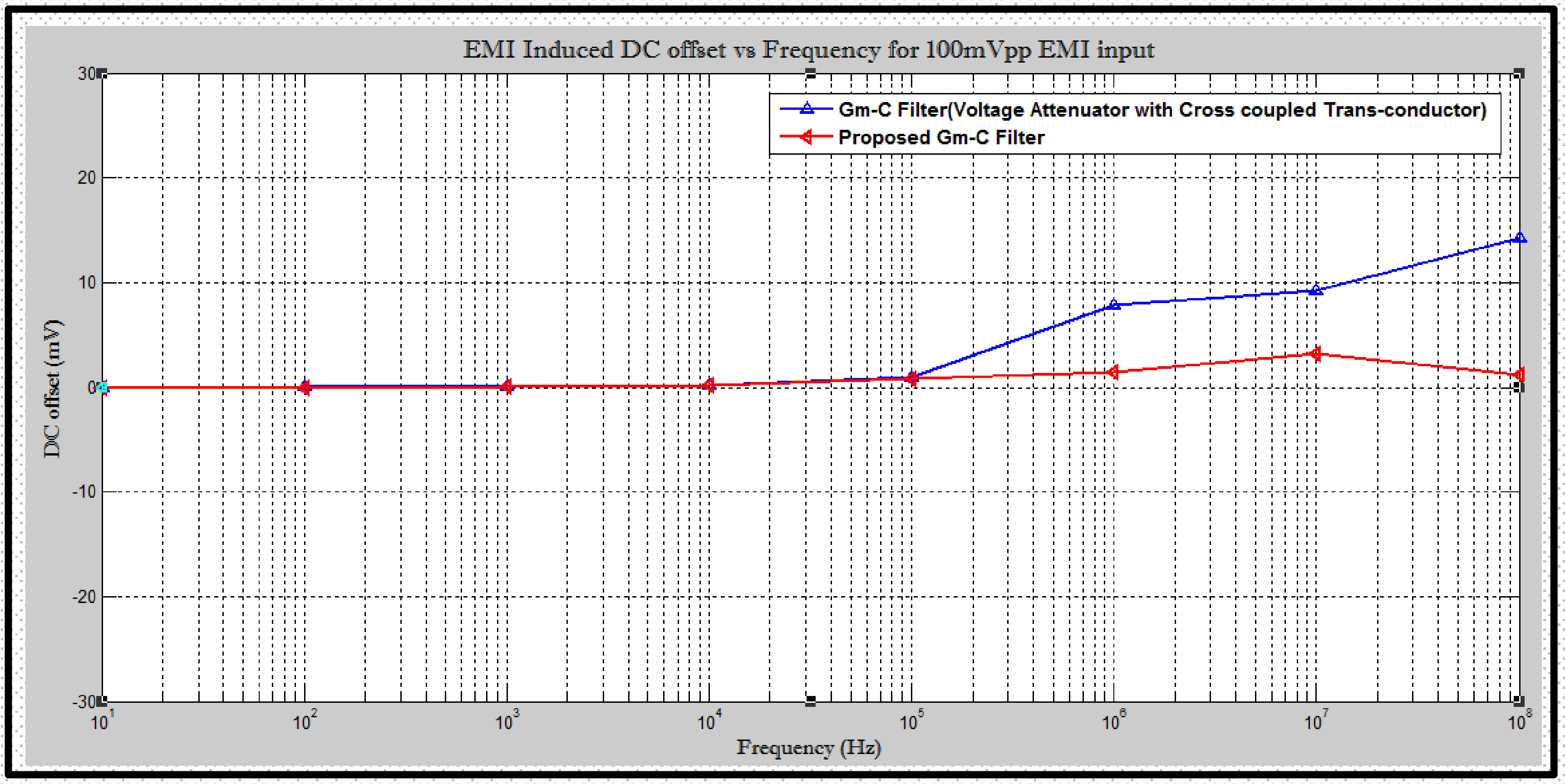
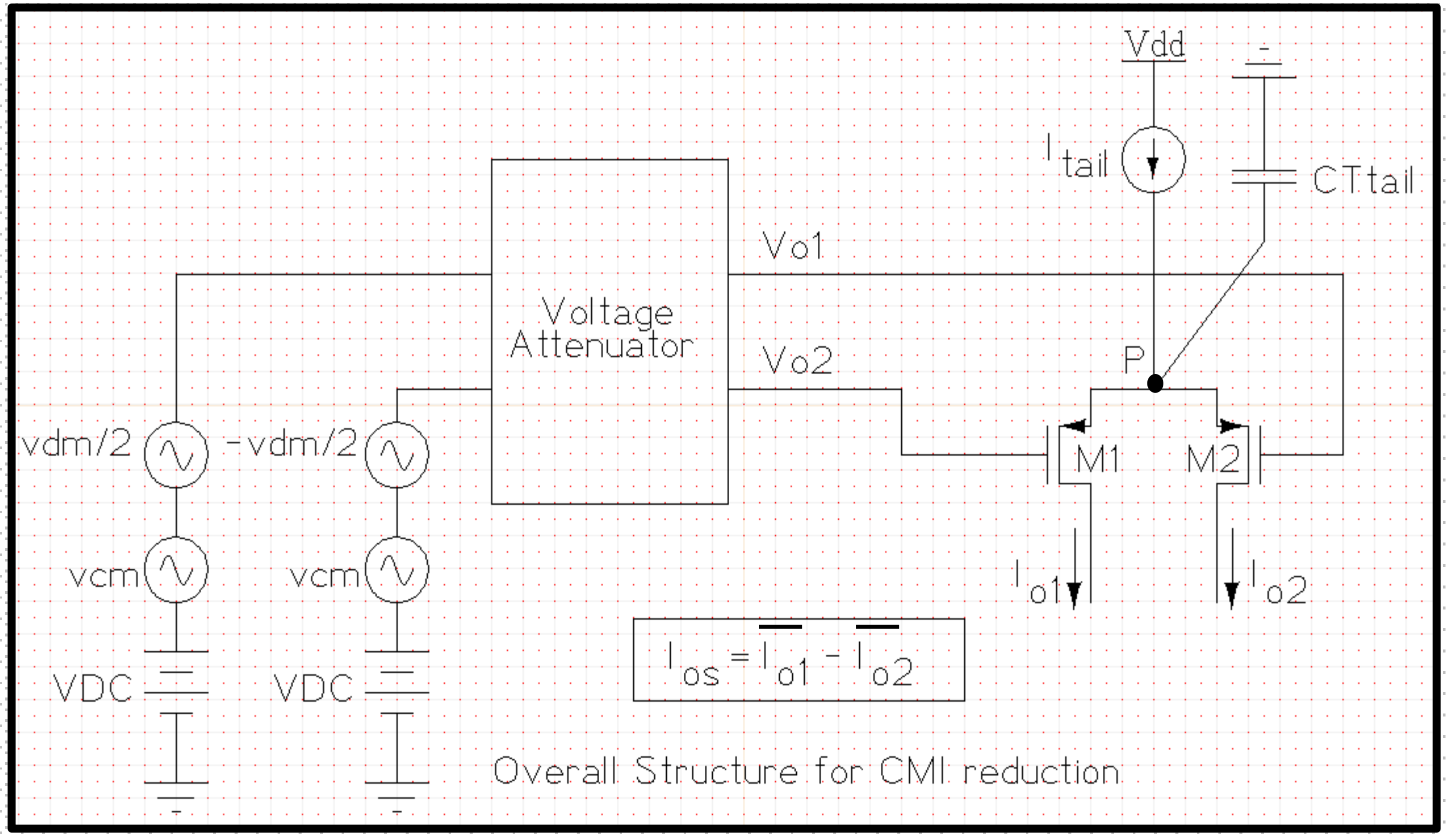


Fig19: EMI induced DC offset variation with frequency for 100mVpp input signal

- Here, we see the EMI Induced DC offset is reduced significantly for this proposed G_m-C filter compared to Uncompesated G_m-C filter From the frequency near 10MHz, which is considered to Be the minimum value of Infinty while calculating CMTF $H_{cm}(\infty)=0$.

- In frequency range **[10Hz-100MHz]**, Maximum EMI induced offset (Magnitude): (i) Voltage attenuator with cross-coupled transconductor based G_m-C filter:: **1.996%** (ii) Proposed G_m-C filter:: **0.459%**

DC offset reduction at high frequency



$v_{o1} = H_{cm}v_{cm} + H_{dm}\frac{v_{dm}}{2}$ &

$v_{o2} = H_{cm}v_{cm} - H_{dm}\frac{v_{dm}}{2}$

And, $V_{o1} = v_{o1} + V_{DC}$ & $V_{o2} = v_{o2} + V_{DC}$

- **CASE1**: If Common mode interference comes at attenuator output, then from equations above: $|v_{o1}| \neq |v_{o2}|$ Unequal capacitive division occurs across gate-source of M1 & M2 due to $C_{T,tail}$ and as a result, $|v_{gs,M1}| \neq |v_{gs,M2}|$ ie finite offset current I_{OS} at output.
- **CASE2:** Instead, if we make H_{cm}=0 somehow, ie no CM Interference at attenuator output, $|v_{o1}| = |v_{o2}|$

In this case, P acts as virtual ground as second stage doesn't see any common mode input. ie I_{os}=0.
This is why we need to minimize H_{cm} of the first stage to reduce DC offset at output.