TRAINS ON TIME

Optimizing and Scheduling of railway timetables

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Students' Reading Group

July 27, 2016

Outline

- Introduction to Optimization
- Examples
- Types of Optimization problems
- Periodic Constraints
- The Cyclic Railway Timetabling Problem (CRTP)
- Assignment Constraints
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- Future Work
- Conclusion
- References

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- It will not be an understatement to claim that optimization has been done by all of us from a quite young age.

Introduction to Optimization(Contd..)

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Let us consider a very simple example.

You have to spend ₹ 200		
* 	Each slice costs ₹100	☺ ☺ ☺
** 	Each burger cost ₹50	☺ ☺

* Source: http://www.istockphoto.com/vector/pizza-slice-gm165646066-9184739

**Source: https://www.123rf.com/stock-photo/beef_cartoon.html

Let P be the number of pizzas and B the number of burgers you will have.

$$\begin{aligned} Max \quad & 3P + 2B && (1) \\ s.t. \quad & 0 \leqslant P \leqslant 2 \quad , P \in \mathbb{Z} && (2) \\ & 0 \leqslant B \leqslant 2 \quad , B \in \mathbb{Z} && (3) \\ & 100P + 50B = 200 && (4) \end{aligned}$$

Decision variables: P and B

(1): Objective function of the optimization problem

(2) - (4): The constraints of the problem.

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This is an example of a linear quadratic regulator problem

Types of Optimization problems

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Type of problem	Objective function	Type of constraints	Decision variables	Method of solving
Linear Program	Linear	Linear Equalities and Inequalities	Real	Simplex Method
Linear Integer Program	Linear	Linear Equalities and Inequalities	Integers	Heuristic Method
Quadratic Program	Quadratic	Linear Equalities and Inequalities	Real	Dynamic Programming

Periodic Constraints

Cyclic timetables: Arrival/departure times of trains repeat every hour.

- Periodically recurring events demand periodic constraints
- All events have times lying in [0,60)
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An example of a periodic constraint is as follows:-

d: any departure event

a: any arrival event

$$d - a + Tp \in [3, 5]$$

$$p \in \mathbb{Z}$$

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- **Headway Time constraints**:- leads to periodic constraints between departures from a single station
- **Dwell Time constraints**:- leads to periodic constraints between arrival and departure of trains at any particular station
- **Traversal constraints**:- leads to periodic constraints between arrival and departure of trains at adjacent stations

CRTP(Contd..)

- **Synchronization constraints**:- Depending on the requirement of number of services between a pair of stations, the services should be appropriately spread out over an hour. For example if there are 5 services from stations A to B, the trains should be spread out by approximately 10 to 14 minutes in an hour

Depending on these constraints the CRTP formulation aims at scheduling trains matching all the constraints.

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These problems leads to another important type of constraints called Assignment Constraints.

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- Every arrival event has to be linked with a departure event

$$\sum_{j \in \mathbb{D}} \mathbb{X}_{ij} = 1 \quad \forall i \in \mathbb{A}$$

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Assignment constraints(Contd..)

- Using these variables $\mathbb{X}_{ij}$, we define two constraints to specify turnaround constraints at terminal stations.

$$d_j - a_i + Tp \geqslant -57 + 60\mathbb{X}_{ij}$$

$$d_j - a_i + Tp \leqslant 65 - 60\mathbb{X}_{ij}$$

- The way Mixed Integer Linear Programs are solved, the search space for "unlinked" arrival-departure events gets quite huge as the bounds are between -57 and 65

- Solving the MILP using the above constraints the solver is unable to solve CRTP
- We thus reduce the search space by slightly modifying the constraints as below:-

$$d_j - a_i + Tp \geqslant 3\mathbb{X}_{ij}$$

$$d_j - a_i + Tp \leqslant 65 - 60\mathbb{X}_{ij}$$

- With search space reduced the solver is now able to solve the problem satisfactorily

Modeling using AMPL

- Such an optimization problem requires to be modeled quite carefully. The modeling language that has been used is AMPL(A Mathematical Programming Language)
- Modeling any problem consists of first denoting the decision variables, defining objective functions and then constraints
- For solving the AMPL model we need to call a solver (in this case Gurobi), which returns the optimal value of the decision variables and minimum value of the objective function

Modeling using AMPL(Contd..)

An example of modeling in AMPL is shown:-

- Our old problem was:-

$$
\begin{aligned}
\text{Max} \quad & 3P + 2B \\
s.t. \quad & 0 \leqslant P \leqslant 2 \quad , P \in \mathbb{Z} \\
& 0 \leqslant B \leqslant 2 \quad , B \in \mathbb{Z} \\
& 100P + 50B = 200
\end{aligned}
$$

- An equivalent AMPL model is shown:-

```
#Decision variables
var p integer >=0,<=2;
var b integer >=0,<=2;

#Objective function
maximize happiness: 3*p+2*b;

#Constraint
subject to con1:100*p+50*b=200;
~
```

The solution of the above model looks like this:-

```
Gurobi 6.5.0: optimal solution; objective 7
happiness = 7

p = 1

b = 2

soumyadutta@kanjur:~$
```

We thus confirm our previous solution.

Future work

As of now we have no objective function in our CRTP formulation. We will add the following to our formulation:-

- Increase robustness of the timetable
- Reduce traveling time between source-destination pairs

Conclusion

- Optimization provides a flexible framework for creating railway time-tables
- Often when stuck with an optimization problem, tweaking the model slightly can help us. For this however, knowledge of the problem at hand is essential.

References

- Peeters, L.W.P. (2003). *Cyclic Railway Timetable Optimization*, Erasmus Research Institute of Management (ERIM), Erasmus University Rotterdam.
- Serafini, P., & Ukovich, W. (1989). A mathematical model for periodic scheduling problems. *SIAM Journal on Discrete Mathematics*, 2(4), 550-581.
- Wolsey, Laurence.A. (1998) *Integer Programming*, John Wiley and Sons,INC.
- http://www.istockphoto.com/vector/pizza-slice-gm165646066-9184739
- http://www.123rf.com/stock-photo/beef-cartoon.html