

Basis of Finite Element Method

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Introduction

Practical electromagnetic problems are complicated and require numerical techniques to obtain approximate solution. The finite element method (FEM) is an extensively used numerical method. The method was first proposed in 1940s and applied firstly by Courant, a mathematician, to solve torsion problem. The FEM was mainly used in mechanical and structural fields. A practical problem of *aircraft design* was first solved by using FEM. In 1965, Zienkiewicz and Cheung, mathematicians, reported applicability of the FEM for all the fields having same type of governing equations. Thereafter, FEM became popular also in the applications of electromagnetics. As compared to other methods in computational electromagnetics, the FEM has many advantages, viz. better applicability to complex, 3-D, anisotropic, nonlinear and coupled problems. Finite element formulation can be implemented in Ritz method (or Variational method) and Galerkin method. In the Ritz method, boundary value problem is formulated in terms of functional (function of functions, e.g., energy expression), whose minimization gives accurate solution. However, formation of functional is difficult in some complex problems and hence, it constraints scope of the Variational method. But, the method is simpler to understand the working of finite element method. It can be explained by considering an example of parallel plate capacitor in the next section.

Illustrative Example: Parallel Plate Capacitor

Problem definition:- To compute potential distribution over dielectric region Ω by using FEM.

A geometry of the parallel plate capacitor is as shown in the Fig. 1. The lower plate of the capacitor is at ground potential whereas, the upper plate is assigned a potential of 10 V. It should be noted that, when fringing fields are neglected (as done here) the problem becomes a trivial uniform field case having exact analytical solution. However, the problem is solved using FEM since the explanation of the involved procedure becomes simpler. The field distribution in this case is governed by Laplace equation:

$$\begin{aligned}\nabla^2 \phi &= 0 \\ \phi|_{y=1, 0 < x < 1} &= 10 \quad \text{on } \Omega \\ \phi|_{y=0, 0 < x < 1} &= 0\end{aligned}\tag{1}$$

The functional for the system can be taken as expression of the energy stored in the capacitor. The energy expression for a single-dielectric case can be given by:-

$$\mathcal{E} = \frac{1}{2} \int_{\Omega} |\nabla \phi|^2 d\Omega\tag{2}$$

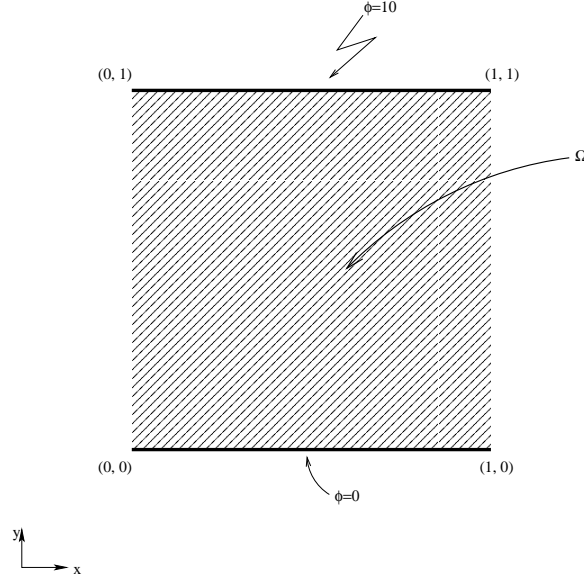


Figure 1: A Parallel Plate Capacitor

Additional ϵ term will appear inside integral for multiple dielectric system. For single dielectric case, considered here, the term is of no consequence as it will anyway get cancelled when the total energy expression is minimized later (eq. 36). The procedure of the FEM can be divided into the following four steps.

1. Discretization of the domain
2. Approximation of the solution
3. Assembly of the system
4. Boundary conditions and solution of the final system

The problem domain is discretized into finite number of elements (or sub-domains). Selection of the type of elements is based on geometry of the problem, shape of the element (i.e., triangular or quadrilateral for 2-D analysis and cubic, tetrahedral or prismatic for 3-D analysis) and nodal or vector formulation. Size of the element depends on the specific geometry requirements. In case of eddy current problems, selection of the element size depends upon skin depth. To compute force or torque in the electromagnetic systems (such as permanent or electro magnets and motors), size of elements is decided by the air-gap in the system. Irregular boundaries in the problem domain should be divided with more number of elements in order to obtain accurate solution (refer Fig. 2). The problem domain of the parallel plate capacitor can be discretized as shown in Fig. 3.

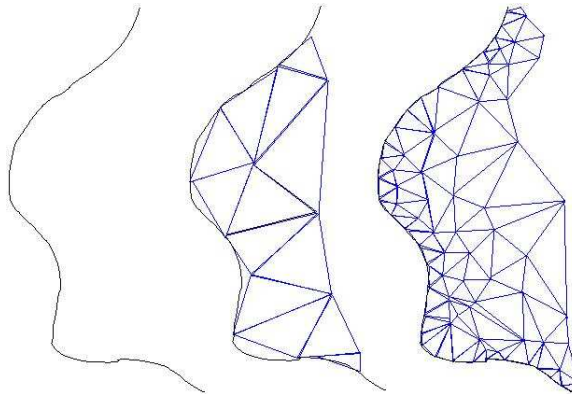


Figure 2: Irregular boundary divided with more number of elements.

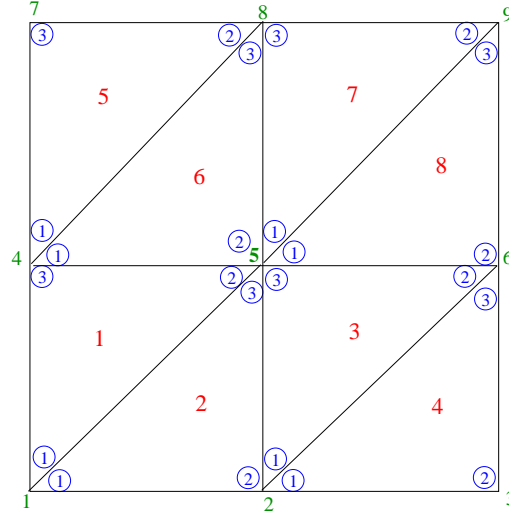


Figure 3: Discretization of the domain for parallel plate capacitor

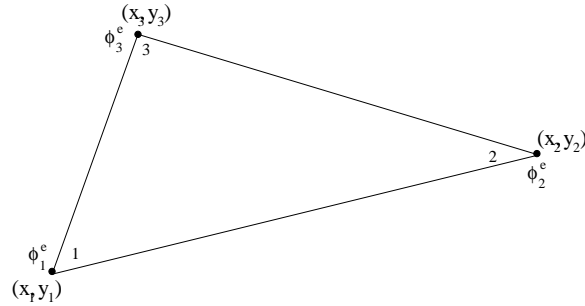


Figure 4: Triangular element for 2-D analysis

After discretization of the domain into large (finite) number of elements, approximate solution is assumed over each element. The best choice for approximating the desired solution is *piece-wise linear polynomial*. The polynomial approximation for 2-D and 3-D analysis can be given by:-

$$\widetilde{\phi}^e = a + bx + cy + dxy \Rightarrow \text{for 2-D quadrilateral element size} \quad (3)$$

$$\widetilde{\phi}^e = a + bx + cy \Rightarrow \text{for 2-D triangular element size} \quad (4)$$

$$\widetilde{\phi}^e = a + bx + cy + dz \Rightarrow \text{for 3-D tetrahedral element size} \quad (5)$$

$$\widetilde{\phi}^e = a + bx + cy + dz + exy + fyz + gzx + hxyz \Rightarrow \text{for 3-D cubic element size} \quad (6)$$

It should be noted that the number of constants in the above expressions is equal to the number of nodes of the chosen element type. The global solution over the entire problem domain can be found by augmenting individual elemental approximate solutions. The system assembly for simple 2-D triangular elemental domain (refer Fig. 4) can be shown as:-

$$\widetilde{\phi}_1^e = a + bx_1 + cy_1 \quad (7)$$

$$\widetilde{\phi}_2^e = a + bx_2 + cy_2 \quad (8)$$

$$\widetilde{\phi}_3^e = a + bx_3 + cy_3 \quad (9)$$

$$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}^{-1} \begin{bmatrix} \widetilde{\phi}_1^e \\ \widetilde{\phi}_2^e \\ \widetilde{\phi}_3^e \end{bmatrix} \quad (10)$$

We shall, henceforth, drop the \sim on $\widetilde{\phi}^e$ s for simplicity and understand that ϕ refers to the approximate solution.

$$\phi^e = \begin{bmatrix} 1 & x & y \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & x & y \end{bmatrix} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}^{-1} \begin{bmatrix} \phi_1^e \\ \phi_2^e \\ \phi_3^e \end{bmatrix} \quad (11)$$

$$\Rightarrow \phi^e = \sum_{i=1}^3 N_i(x, y) \phi_i^e \quad (12)$$

where,

$$\begin{aligned} N_1(x, y) &= \frac{1}{2\Delta} [(x_2 y_3 - x_3 y_2) + (y_2 - y_3)x + (x_3 - x_2)y] \\ N_2(x, y) &= \frac{1}{2\Delta} [(x_3 y_1 - x_1 y_3) + (y_3 - y_1)x + (x_1 - x_3)y] \\ N_3(x, y) &= \frac{1}{2\Delta} [(x_1 y_2 - x_2 y_1) + (y_1 - y_2)x + (x_2 - x_1)y] \end{aligned} \quad (13)$$

and,

$$\Delta = \text{Area of the elemental triangular element} = \frac{1}{2} \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} \quad (14)$$

$N_i(x, y)$ is the expansion function and has the property that,

$$N_i(x_j, y_j) = \delta_{ij} \quad (15)$$

where,

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad (16)$$

Thus, $N_i(x, y)$ defined over the particular element vanishes outside the element. Eqn. 12 can be substituted in Eqn. 2 to obtain expression for the functional (i.e., energy \mathcal{E}). Thus, the functional expression for one element e can be given by:-

$$\mathcal{E} = \sum_e \frac{1}{2} \int_{\Omega^e} \left| \nabla \left(\sum_{i=1}^3 N_i(x, y) \phi_i^e \right) \right|^2 d\Omega^e \quad (17)$$

$$\Rightarrow \mathcal{E} = \frac{1}{2} \sum_e \int_{\Omega^e} \left| \left(\sum_{i=1}^3 \{ \nabla N_i(x, y) \} \phi_i^e \right) \right|^2 d\Omega^e \quad (18)$$

$$\Rightarrow \mathcal{E} = \frac{1}{2} \sum_e \int_{\Omega^e} |(\nabla N_1(x, y) \phi_1^e + \nabla N_2(x, y) \phi_2^e + \nabla N_3(x, y) \phi_3^e)|^2 d\Omega^e \quad (19)$$

where, the operator ∇ is del operator which can be defined in the Cartesian co-ordinate systems as:-

$$\nabla = \frac{\partial}{\partial x} \mathbf{a}_x + \frac{\partial}{\partial y} \mathbf{a}_y + \frac{\partial}{\partial z} \mathbf{a}_z \quad (20)$$

For any vector \bar{a} , $\bar{a} \cdot \bar{a} = |\bar{a}|^2$

$$\begin{aligned} \Rightarrow \mathcal{E} &= \frac{1}{2} \sum_e \int_{\Omega^e} [(\nabla N_1(x, y) \phi_1^e + \nabla N_2(x, y) \phi_2^e + \nabla N_3(x, y) \phi_3^e) \cdot \\ &\quad (\nabla N_1(x, y) \phi_1^e + \nabla N_2(x, y) \phi_2^e + \nabla N_3(x, y) \phi_3^e)] d\Omega^e \end{aligned} \quad (21)$$

$$\Rightarrow \mathcal{E} = \frac{1}{2} \sum_e \sum_{i=1}^3 \sum_{j=1}^3 \int_{\Omega^e} \phi_i^e \nabla N_i(x, y) \cdot \nabla N_j(x, y) \phi_j^e d\Omega^e \quad (22)$$

where Ω^e is the elemental domain.

$$\Rightarrow \mathcal{E} = \frac{1}{2} \sum_e \sum_{i=1}^3 \sum_{j=1}^3 \phi_i^e \left(\int_{\Omega^e} \nabla N_i(x, y) \cdot \nabla N_j(x, y) d\Omega^e \right) \phi_j^e \quad (23)$$

$$a_{ij}^e \equiv \int_{\Omega^e} \nabla N_i \cdot \nabla N_j d\Omega^e \quad (24)$$

$$\mathbf{A}^e = \begin{bmatrix} a_{11}^e & a_{12}^e & a_{13}^e \\ a_{21}^e & a_{22}^e & a_{23}^e \\ a_{31}^e & a_{32}^e & a_{33}^e \end{bmatrix} \quad (25)$$

\mathbf{A}^e is referred to as the elemental stiffness matrix. The elemental energy can, thus, be represented as,

$$\mathcal{E}^e = \phi^{eT} \mathbf{A}^e \phi^e \quad (26)$$

where, $\phi^e = \{ \phi_1^e \quad \phi_2^e \quad \phi_3^e \}^T$ and $\mathcal{E} = \sum_e \mathcal{E}^e$.

For example,

$$a_{11}^e = \int_{\Omega^e} \nabla N_1 \cdot \nabla N_1 d\Omega^e \quad (27)$$

where, from equation 13,

$$a_{11}^e = \frac{1}{4\Delta^2} [(y_2 - y_3)^2 + (x_3 - x_2)^2] \int_{\Omega^e} d\Omega^e = \frac{1}{4\Delta} [(y_2 - y_3)^2 + (x_3 - x_2)^2] \quad (28)$$

Similarly, we have,

$$a_{12}^e = \int_{\Omega^e} \nabla N_1 \cdot \nabla N_2 d\Omega^e = \frac{1}{4\Delta} [(y_2 - y_3)(y_3 - y_1) + (x_3 - x_2)(x_1 - x_3)] \quad (29)$$

$$a_{13}^e = \int_{\Omega^e} \nabla N_1 \cdot \nabla N_3 d\Omega^e = \frac{1}{4\Delta} [(y_2 - y_3)(y_1 - y_2) + (x_3 - x_2)(x_2 - x_1)] \quad (30)$$

The global connectivity matrix can be given as:-

$$\begin{array}{c} \mathbf{e} \end{array} \begin{array}{c} 1^e \quad 2^e \quad 3^e \\ \begin{pmatrix} 1 & 5 & 4 \\ 1 & 2 & 5 \\ 2 & 6 & 5 \\ 2 & 3 & 6 \\ 4 & 8 & 7 \\ 4 & 5 & 8 \\ 5 & 9 & 8 \\ 5 & 6 & 9 \end{pmatrix} \end{array}$$

Figure 5: Connectivity matrix

The global stiffness matrix is of the form:

$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{19} \\ A_{21} & A_{22} & A_{23} & \cdots & A_{29} \\ A_{31} & A_{32} & A_{33} & \cdots & A_{39} \\ \vdots & & & \ddots & \vdots \\ A_{91} & A_{92} & A_{93} & \cdots & A_{99} \end{pmatrix} \quad (31)$$

Elements in the global stiffness matrix can be understood by referring Fig. 6.

$$\begin{aligned}
A_{11} &= a_{11}^{e1} + a_{11}^{e2} \\
A_{12} &= a_{12}^{e2} = A_{21} \\
A_{1i} &= A_{i1} = 0 \quad i = 3, 6, 7, 8, 9 \\
A_{22} &= a_{22}^{e2} + a_{11}^{e3} + a_{11}^{e4} \\
A_{23} &= a_{12}^{e4} = A_{32} \\
&\vdots \\
A_{55} &= a_{22}^{e1} + a_{33}^{e2} + a_{33}^{e3} \\
&\quad + a_{22}^{e6} + a_{11}^{e7} + a_{11}^{e8} \\
A_{51} &= a_{21}^{e1} + a_{31}^{e2} = A_{15}
\end{aligned}$$

and so on.

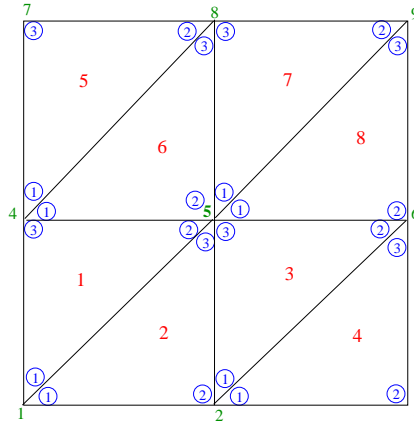


Figure 6: Discretisation of the problem domain by triangular elements

Thus, the global stiffness matrix is:

$$\mathbf{A} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & \cdots & 9 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ \vdots \\ 9 \end{matrix} & \begin{pmatrix} a_{11}^{e1} + a_{11}^{e2} & a_{12}^{e2} & 0 & \cdots & 0 \\ a_{21}^{e2} & a_{22}^{e2} + a_{11}^{e3} + a_{11}^{e4} & a_{12}^{e4} & \cdots & 0 \\ 0 & a_{21}^{e4} & a_{22}^{e4} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_{22}^{e7} + a_{33}^{e8} \end{pmatrix} \end{matrix} \quad (32)$$

Thus, equation 23 for the functional can now be expressed as,

$$\mathcal{E} = \frac{1}{2} \phi^T \mathbf{A} \phi \quad (33)$$

where, \mathbf{A} is the matrix in equation 32 and ϕ is a vector of all nodal potential values,

$$\phi = \{\phi_1 \quad \phi_2 \quad \phi_3 \quad \phi_4 \quad \phi_5 \quad \phi_6 \quad \phi_7 \quad \phi_8 \quad \phi_9\}^T \quad (34)$$

The total energy of the system can be minimized by differentiating it with respect to nodal potential values.

$$\nabla \mathcal{E} = 0 \quad (35)$$

$$\Rightarrow \begin{pmatrix} \frac{\partial \mathcal{E}}{\partial \phi_1} \\ \frac{\partial \mathcal{E}}{\partial \phi_2} \\ \vdots \\ \frac{\partial \mathcal{E}}{\partial \phi_9} \end{pmatrix} = 0 \quad (36)$$

A set of n -number of simultaneous equations obtained from the above equation can be written as below:-

$$\begin{aligned} A_{11}\phi_1 + A_{12}\phi_2 + A_{13}\phi_3 + \dots + A_{19}\phi_9 &= 0 \\ A_{21}\phi_1 + A_{22}\phi_2 + A_{23}\phi_3 + \dots + A_{29}\phi_9 &= 0 \\ \vdots & \\ A_{91}\phi_1 + A_{92}\phi_2 + A_{93}\phi_3 + \dots + A_{99}\phi_9 &= 0 \end{aligned} \quad (37)$$

The above set of equations can be written in terms of matrix form.

$$\Rightarrow \mathbf{A}\phi = 0 \quad (38)$$

The boundary conditions for the problem of parallel plate capacitor are:-

$$\begin{aligned} \phi_1 = \phi_2 = \phi_3 &= 0 \\ \phi_7 = \phi_8 = \phi_9 &= 10 \end{aligned} \quad (39)$$

This specifies six of the nine unknowns in Eqn. 37.

Equation 37 now becomes the reduced order system as shown below:

$$\begin{aligned} A_{44}\phi_4 + A_{45}\phi_5 + A_{46}\phi_6 &= -10(A_{47} + A_{48} + A_{49}) \\ A_{54}\phi_4 + A_{55}\phi_5 + A_{56}\phi_6 &= -10(A_{57} + A_{58} + A_{59}) \\ A_{64}\phi_4 + A_{65}\phi_5 + A_{66}\phi_6 &= -10(A_{67} + A_{68} + A_{69}) \end{aligned} \quad (40)$$

$$\Rightarrow \underbrace{\mathbf{A}_{3 \times 3}}_{\text{Symmetric}} \phi_{3 \times 1} = b_{3 \times 1} \quad (41)$$

The unknown values of potentials can thus be determined from Eqn. 40, which in this case will be very close to 5. The values will deviate slightly from the exact solution value on account of inaccuracies introduced due to the numerical procedure.

References

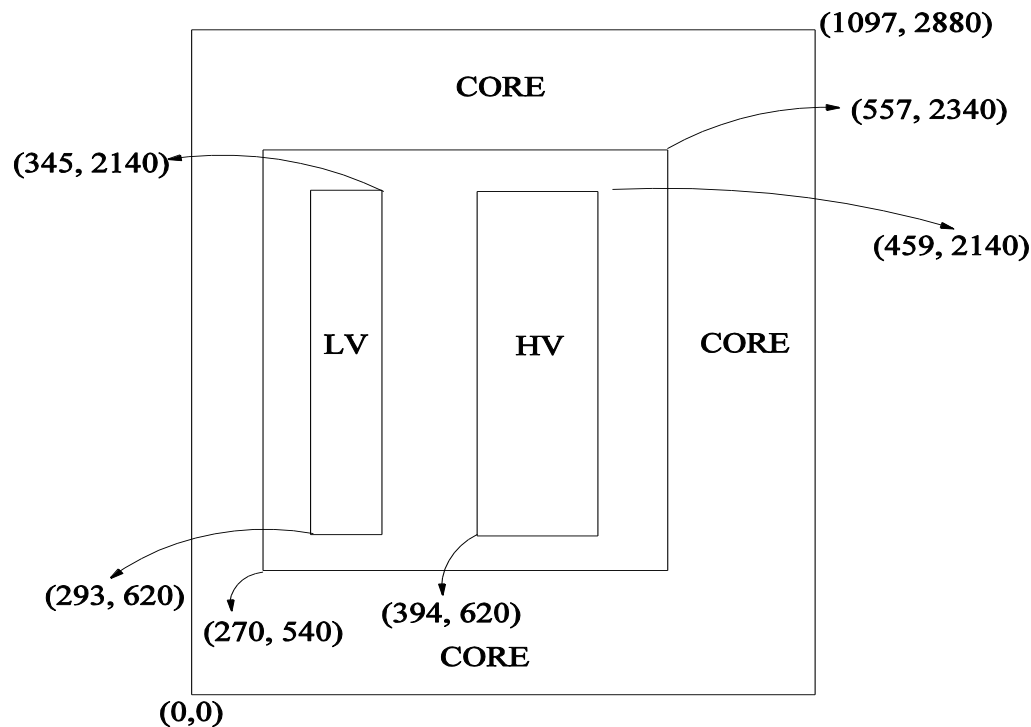
1. Matthew N. O. Sadiku, Numerical Techniques in Electromagnetics with MATLAB, Third Edition, CRC press, 2009, chapter 6.
2. S. J. Salon, Finite element analysis of electrical machines, Springer India.

Tutorial One: Calculation of leakage inductance of transformer using FEM

Consider a transformer with the following rating:

31.5 MVA, 132 kV/33kV, Y/ Δ , Ampere-turns: 135024, No. of HV turns = 980

Although it is a three-phase transformer, for calculating its leakage impedance on per-phase basis, only a single-phase is modeled as shown in the following figure. LV is inner winding surrounded by outer HV winding. For simplicity, tap winding is not considered. HV to core distance is an equivalent distance calculated by considering $2/3^{\text{rd}}$ window width. This distance has insignificant effect on the leakage inductance.



Dimensions:

Core diameter	:	540 mm	
Core-LV gap	:	23 mm	
LV radial depth	:	52 mm	LV mean diameter: 638 mm
LV-HV gap	:	49 mm	
HV radial depth	:	65 mm	HV mean diameter: 853 mm

Heights of LV and HV windings : 1520 mm

The steps to simulate the above problem in any commercial FEM software are as follows. Specific commands to be used will vary for different softwares.

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1. Draw the given transformer diagram as described above using preprocessor menu. The problem is solved in Cartesian system. The performance figures computed will be for a meter depth in z-direction; this approximation is valid for winding having large diameters. For example, the energy calculated in winding area will then have to be multiplied by its mean turn length to get the total energy stored in it). For smaller diameter windings, axisymmetric model should be used to get more accurate results.
2. Use appropriate finite element type from the menu.
3. Define material properties. It should be remembered that relative permeability of winding zones is 1 (since they consist of copper and insulation). Core should be assigned high permeability (being made of magnetic steel).
4. Choose small enough mesh size to get accurate results.
5. Define sources: enter current and turns information for each winding. Define exactly equal number of ampere-turns for both windings (magnetizing ampere-turns are neglected since we are interested in calculating leakage inductance; shunt branch in the equivalent circuit of the transformer, consisting of parallel combination of R_c and X_m , is neglected).
6. Define flux-parallel boundary condition (magnetic vector potential = 0) on the outermost boundaries.
7. Solve the problem as a magnetostatic one.
8. Plot the equipotential lines (magnetic flux lines in this case). Since the ampere-turns defined for both the windings are equal but with opposite signs, the net ampere-turns enclosed by the magnetic circuit is zero. Thus, there should not be a single line in the core part enclosing both the windings. You will observe a bunch of lines enclosing LV winding and the remaining ones enclosing HV winding, as shown in the figure 1 below.

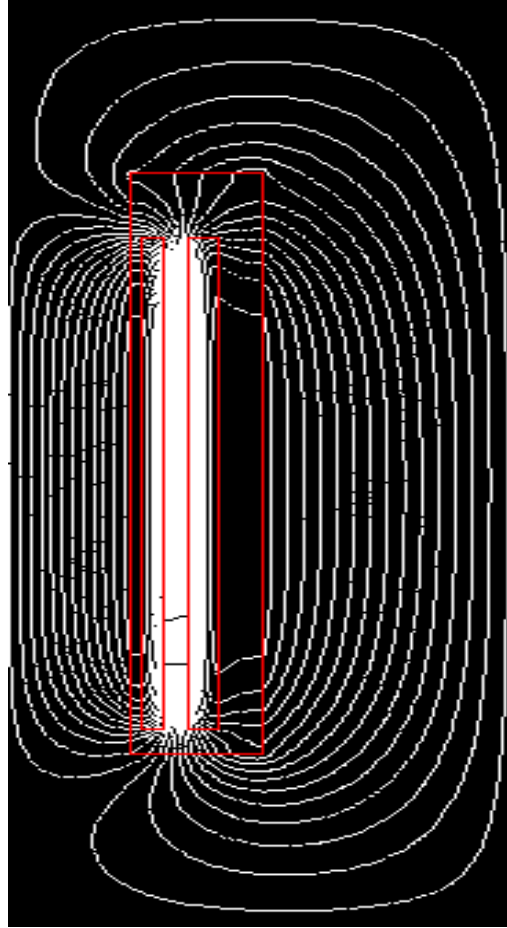


Figure 1. Magnetic vector potential plot

9. Calculate energy in all parts of the problem. The energy values obtained should be close to that given in the following table:

Sr. No.	Part	Energy in Joules per meter depth
1.	CORE	0.0391982
2.	Air	360.805
3.	LV	120.653
4.	HV	147.144

It can be seen from the table that the energy stored in the core is negligible as the flux density in it is insignificant.

It should be noted that if we want to analyze transformer in no-load state, excited winding (LV or HV) should be fed by a rated voltage source with other winding open-circuited along with definition of non-linear B-H curve for the core. In this case, rated flux density (corresponding to mutual flux) would be set up in the core, giving substantial energy in it.

10. Calculate energies in all the parts by multiplying them with corresponding mean turn lengths ($\pi \times$ mean diameters) as shown in the following table (core energy is neglected).

Sr. No.	Part	Energy in Joules
1.	Air	$\pi \times 0.739 \times 360.805 = 837.7 \text{ J}$
2.	LV	$\pi \times 0.638 \times 120.653 = 241.8 \text{ J}$
3.	HV	$\pi \times 0.853 \times 147.144 = 394.3 \text{ J}$
TOTAL ENERGY = 1473.8 J		

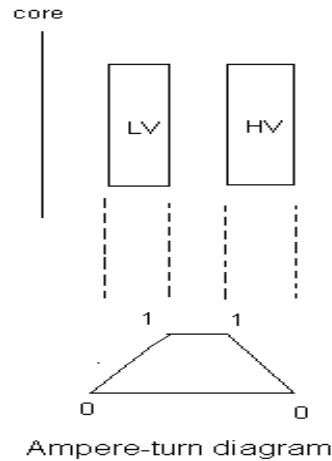
11. Finally, the inductance is calculated by the following formula.

$$\frac{1}{2} L i^2 = 1473.8$$

$$L \text{ (referred to HV side)} = \frac{1473.8 \times 2}{i_{\text{HV}}^2 = (137.78)^2} = 0.155 \text{ Henry}$$

Analytical solution:-

(Ref: S. V. Kulkarni and S. A. Khaparde, Transformer Engineering: Design and Practice, Marcel Dekker, Taylor & Francis Group, New York, 2004, chapter 3).



Leakage inductance calculations

$$\text{Effective area} = \left[\frac{1}{3} T_1 \times D_1 + T_g \times D_g + \frac{1}{3} T_2 \times D_2 \right] \times \pi$$

where, D_1 , D_g and D_2 are the mean diameters and T_1 , T_g and T_2 are the radial depths of LV, gap and HV respectively

$$\begin{aligned} \text{Effective area} &= [(5.2/3) \times 63.8 + 4.9 \times 73.9 + (6.5/3) \times 85.3] \times \pi \times 10^{-4} \text{ m}^2 \\ &= 657.5 \times \pi \times 10^{-4} \text{ m}^2 = 0.2066 \text{ m}^2 \end{aligned}$$

$$\text{Effective height (HT}_{\text{eff}}) = \text{actual height} + \{(\text{HVOD} - \text{LVID}) / (2\pi)\} \text{ cm}$$

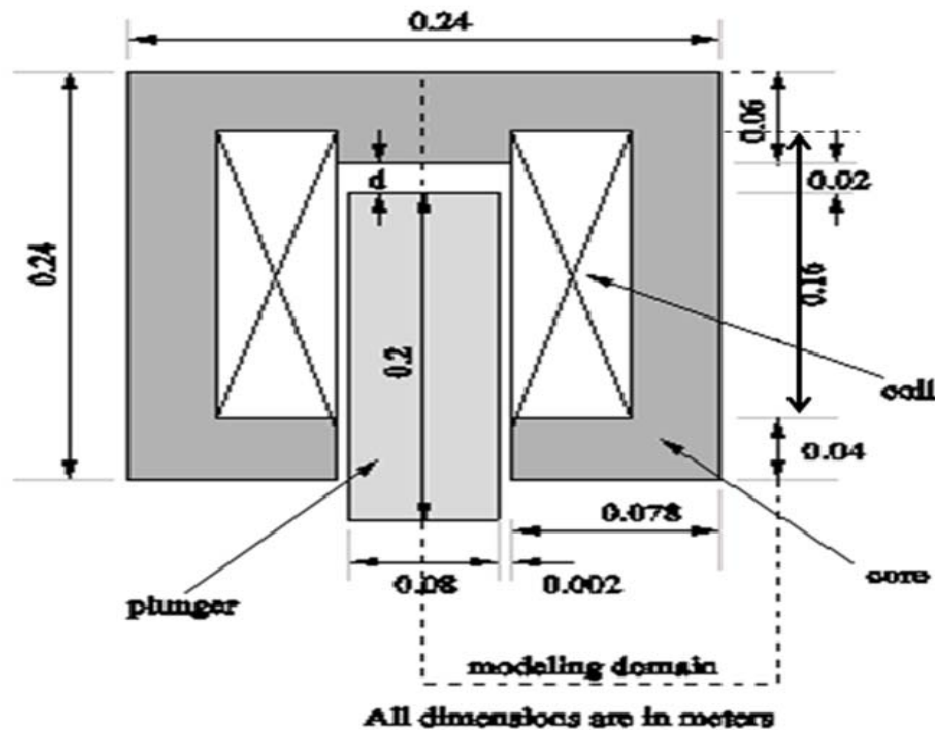
$$\begin{aligned} \text{where, HVOD is HV winding outer diameters and LVID is LV winding inner diameter} \\ &= 152 + (91.8 - 58.6) / (2\pi) = 157.3 \text{ cm} \\ &= 1.573 \text{ m} \end{aligned}$$

$$L (\text{referred to HV side}) = (\mu_0 N^2 A) / \text{HT}_{\text{eff}} = (4\pi \times 10^{-7} \times 980^2 \times 0.2066) / 1.573 = 0.158 \text{ H}$$

Tutorial Two: Force on Plunger due to Magnetic Field in a Nonlinear Solenoid

Description:

A solenoid actuator consists of a coil enclosed in a ferromagnetic core with a plunger.



Given:

Relative permeability of air and coil	1
Current density in coil	1e6 Amp/m ²

The B-H curve for the core and plunger

H (A/m)	460	640	720	890	1280	1900	3400	6000
B (T)	0.8	0.95	1	1.10	1.25	1.40	1.55	1.65

1. Draw the given electromagnet-plunger diagram as described above using preprocessor menu. The problem is solved in axisymmetric system. Since the problem is symmetric about the central vertical axis, only half of the geometry needs to be modeled. This symmetric half problem domain needs to be enclosed by a fictitious outer boundary on which boundary conditions can be imposed.

2. Use appropriate finite element type from the menu.
 3. Define material properties. It should be remembered that relative permeability of coil and air zones is 1. Core and plunger should be assigned B-H characteristics as defined in the table given above.
 4. Choose small enough mesh size to get accurate results.
 5. Define sources: enter current or current density for the coil.
 6. Define flux-parallel boundary condition (magnetic vector potential = 0) on the outermost boundaries.
 7. Solve the problem as a nonlinear magnetostatic one.
 8. Plot the equipotential lines (magnetic flux lines in this case).
- Calculate force on the plunger using facility available in commercial FEM software. The force value obtained should be about 357 N.

Analytical solution:-

The core is assumed to have infinite permeability requiring no magnetizing mmf

Cross sectional area of coil = Height \times Width

$$= 0.16 \text{ meter} \times (0.078 - 0.04) \text{ meter}$$

$$= 6.08 \times 10^{-3} \text{ m}^2$$

Let,

Ampere turn density (ATD) = Current density (J)

$$\text{Current density} = 1\text{e}6 \text{ A/m}^2$$

Ampere turn (AT) = Ampere turn density \times Area of coil

$$= \mathbf{1e6} \times 6.08 \times 10^{-3}$$

$$= 6080$$

$$H_g = AT/g = 6080/0.02 = 304000 \text{ Ampere/meter}$$

where, g is air gap distance

$$B_g = \mu_0 H_g = 4\pi \times 10^{-7} \times 304000$$

$$= 0.382 \text{ Tesla}$$

$$\text{Force exerted on plunger } f_c = \frac{1}{2} \frac{B_g}{\mu_0} \times A$$

$$\begin{aligned}\text{where, cross sectional area(A) coil} &= (\pi/4) \times (d)^2 \\ &= (\pi/4) \times (0.08)^2 \\ &= 5.02\text{e}^{-3} \text{ m}^2\end{aligned}$$

$$\text{Hence, force } f_e = 291.467 \text{ N}$$

The difference in numerical and analytical solution is due to fringing.

Calculation of Leakage Inductance

Ketan P Badgujar

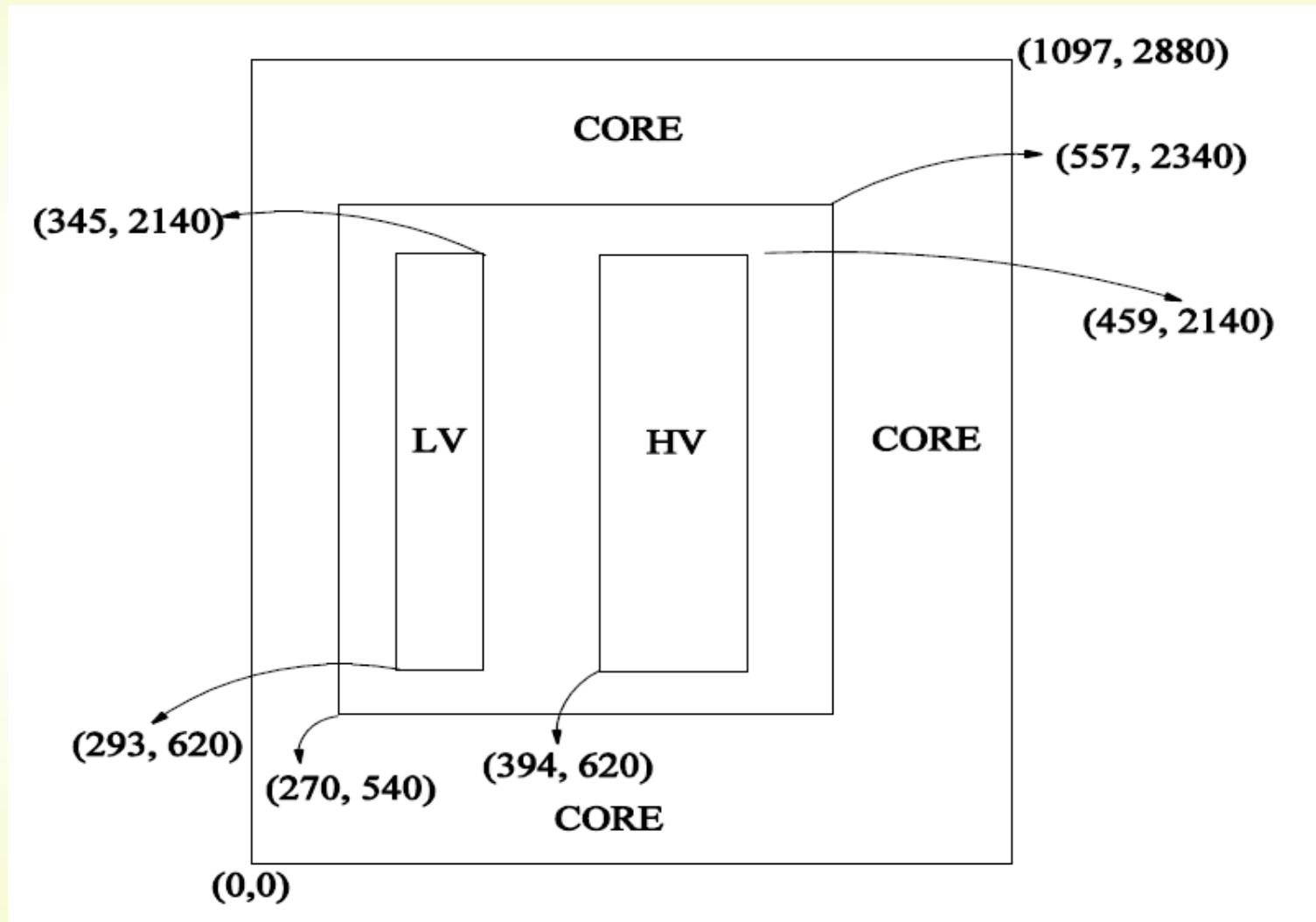


Insulation Diagnostics Lab
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Overview of presentation

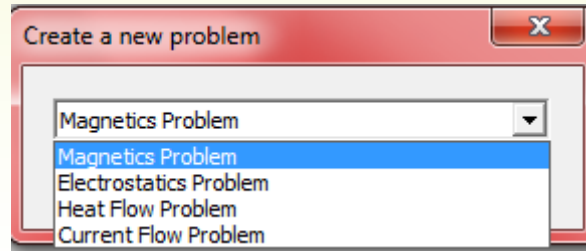
- Problem Definition
- Geometry
- Discretization and Applying Boundary conditions
- Solution


Problem Definition



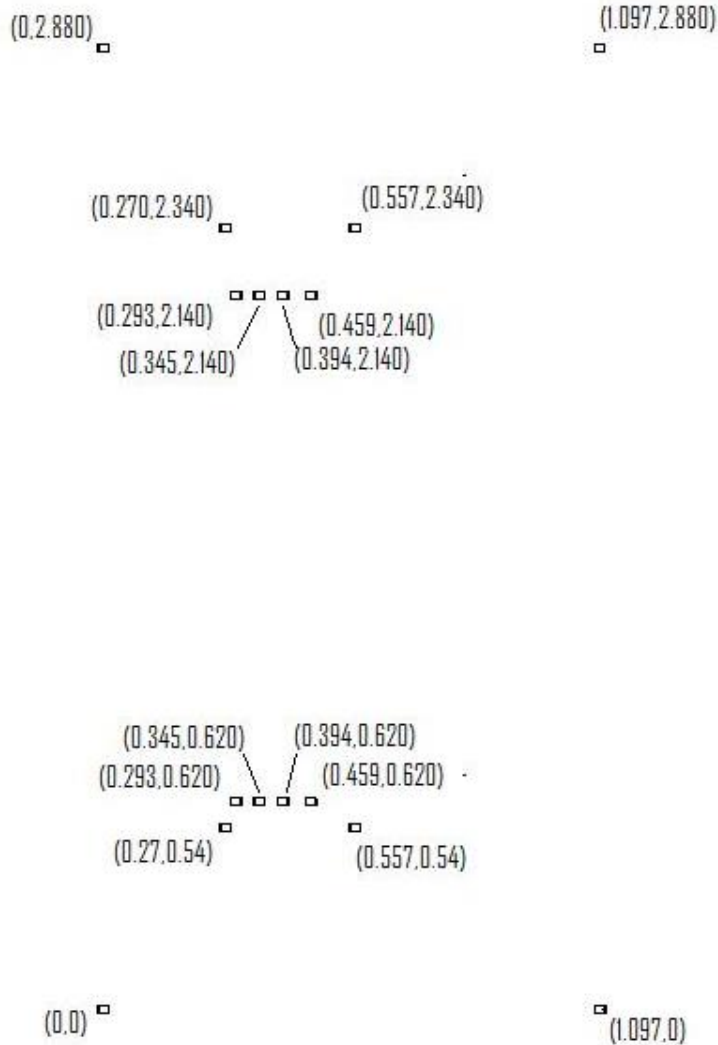
Geometry

- File-> New-> Magnetism problem-> click on 'OK'




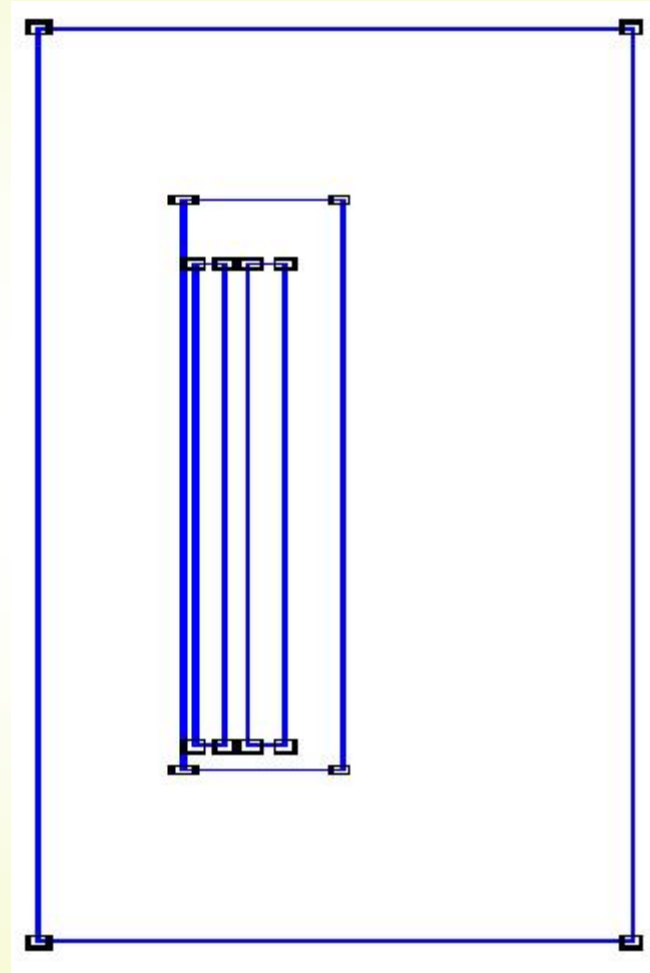
- Draw the given transformer diagram-
 - Click on  (operate on nodes)-> left click on the screen -> right click on the node (node becomes red > press Tab key-> enter the new node location point-> click 'OK' similarly for all the points,


Co-ordinates

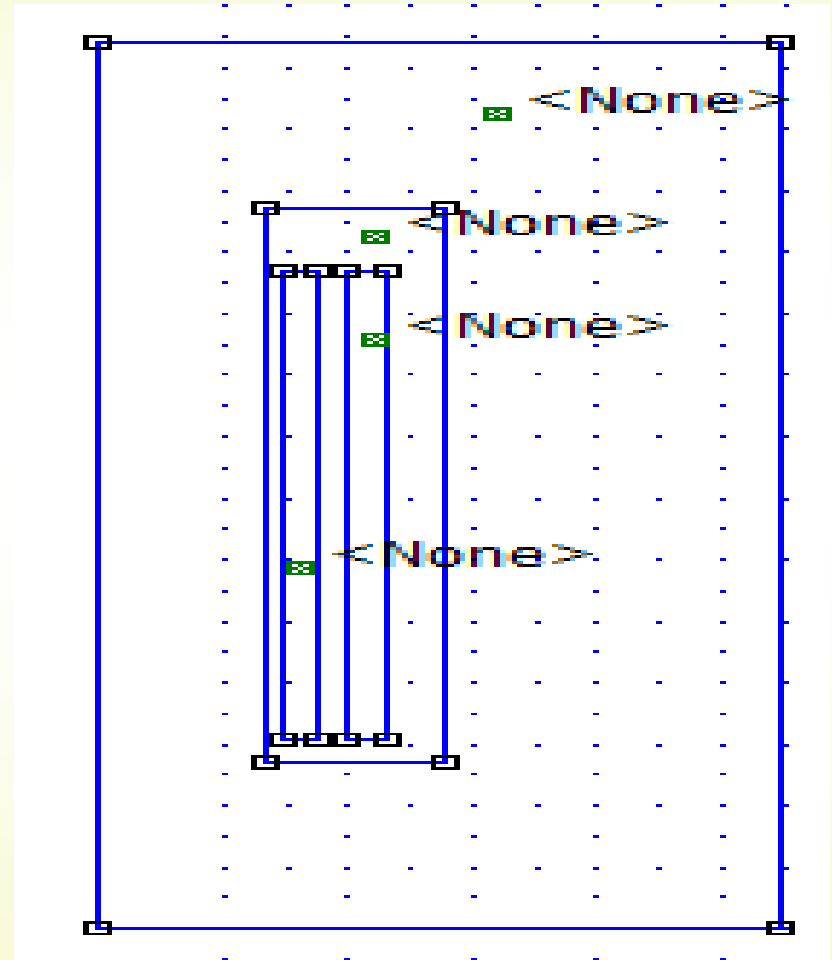


x	y	X	y
0	0	0.293	0.62
0	2.88	0.459	0.62
1.097	2.88	0.345	0.62
1.097	0	0.394	0.62
0.27	0.54	0.293	2.14
0.557	0.54	0.459	2.14
0.27	2.34	0.345	2.14
0.557	2.34	0.394	2.14

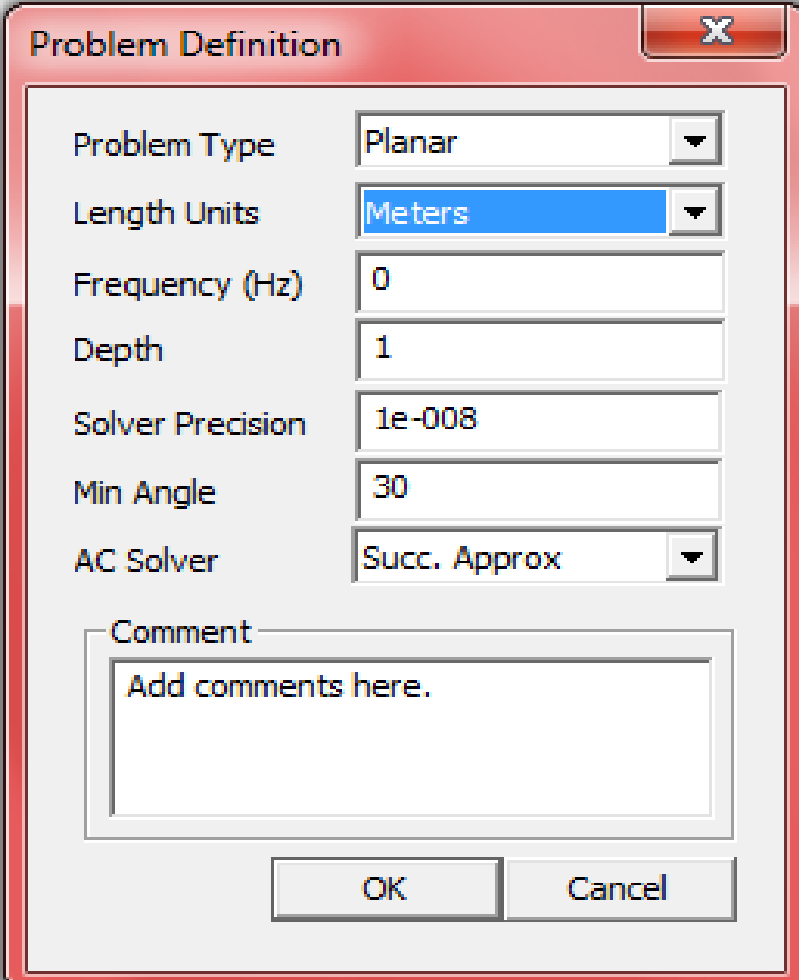
- Click on  (operate on segments) -> left click on first node (node becomes red)-> left click on the second node (now both nodes are connected through a line)



- Click on  (operate on block labels)-> left click on the each area once



- Click on “Problem” from FEMM main menu-
- Click on ‘OK’



The image shows a 'Problem Definition' dialog box with a red title bar and a close button (X) in the top right corner. The dialog contains several input fields and dropdown menus. The 'Problem Type' dropdown is set to 'Planar'. The 'Length Units' dropdown is set to 'Meters'. The 'Frequency (Hz)' text box contains '0'. The 'Depth' text box contains '1'. The 'Solver Precision' text box contains '1e-008'. The 'Min Angle' text box contains '30'. The 'AC Solver' dropdown is set to 'Succ. Approx'. Below these fields is a 'Comment' section with a text area containing the placeholder text 'Add comments here.'. At the bottom of the dialog are 'OK' and 'Cancel' buttons.

Problem Type	Planar
Length Units	Meters
Frequency (Hz)	0
Depth	1
Solver Precision	1e-008
Min Angle	30
AC Solver	Succ. Approx

Comment

Add comments here.

OK Cancel

- Click on “Properties”-
 >”Materials”-> Add
 property
- Click on ‘OK’, similarly
 for all materials

Block Property

Name: Air

B-H Curve: Linear B-H Relationship

Linear Material Properties

Relative μ_x : 1 Relative μ_y : 1

ϕ_{hx} , deg: 0 ϕ_{hy} , deg: 0

Nonlinear Material Properties

Edit B-H Curve ϕ_{hmax} , deg: 0

Coercivity

H_c , A/m: 0

Electrical Conductivity

σ , MS/m: 0

Source Current Density

J, MA/m²: 0

Special Attributes: Lamination & Wire Type

Not laminated or stranded

Lam thickness, mm: 0 Lam fill factor: 1

Number of strands: 0 Strand dia, mm: 0

OK Cancel

Block Property

NameCore

B-H CurveLinear B-H Relationship

Linear Material Properties

Relative μ_x 50000

Relative μ_y 50000

ϕ_{hx} , deg0

ϕ_{hy} , deg0

Nonlinear Material Properties

Edit B-H Curve

ϕ_{hmax} , deg0

Coercivity

H_c , A/m0

Electrical Conductivity

σ , MS/m0

Source Current Density

J, MA/m^20

Special Attributes: Lamination & Wire Type

Not laminated or stranded

Lam thickness, mm0

Lam fill factor1

Number of strands0

Strand dia, mm0

OK

Cancel

Block Property

NameLV

B-H CurveLinear B-H Relationship

Linear Material Properties

Relative μ_x 1

Relative μ_y 1

ϕ_{hx} , deg0

ϕ_{hy} , deg0

Nonlinear Material Properties

Edit B-H Curve

ϕ_{hmax} , deg0

Coercivity

H_c , A/m0

Electrical Conductivity

σ , MS/m0

Source Current Density

J, MA/m^21.708299595

Special Attributes: Lamination & Wire Type

Not laminated or stranded

Lam thickness, mm0

Lam fill factor1

Number of strands0

Strand dia, mm0

OK

Cancel

Block Property

NameHV

B-H CurveLinear B-H Relationship

Linear Material Properties

Relative μ_x 1

Relative μ_y 1

ϕ_{hx} , deg0

ϕ_{hy} , deg0

Nonlinear Material Properties

Edit B-H Curve

ϕ_{hmax} , deg0

Coercivity

H_c , A/m0

Electrical Conductivity

σ , MS/m0

Source Current Density

J, MA/m^2-1.366639676

Special Attributes: Lamination & Wire Type

Not laminated or stranded

Lam thickness, mm0

Lam fill factor1

Number of strands0

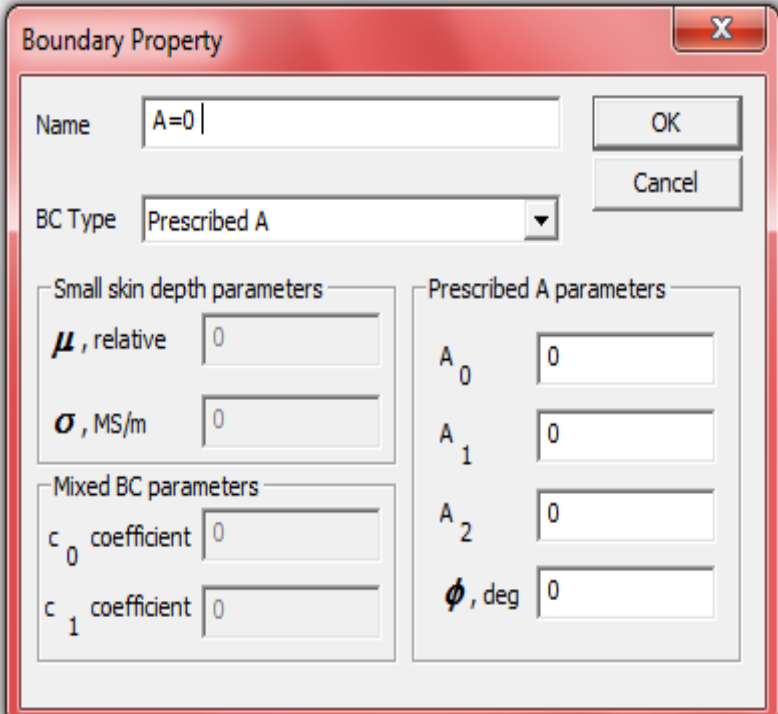
Strand dia, mm0

OK

Cancel




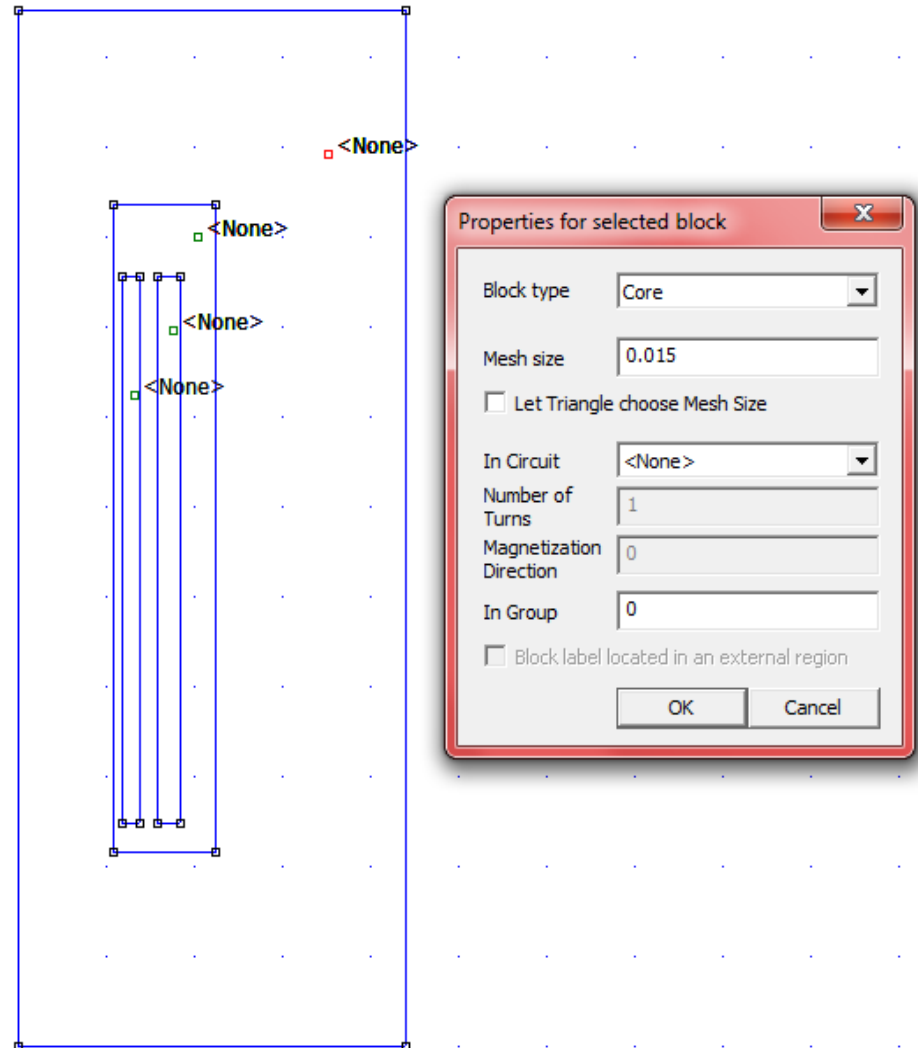
- Click on “Properties”-
->”Boundary”->Add
property -> click on
‘OK’




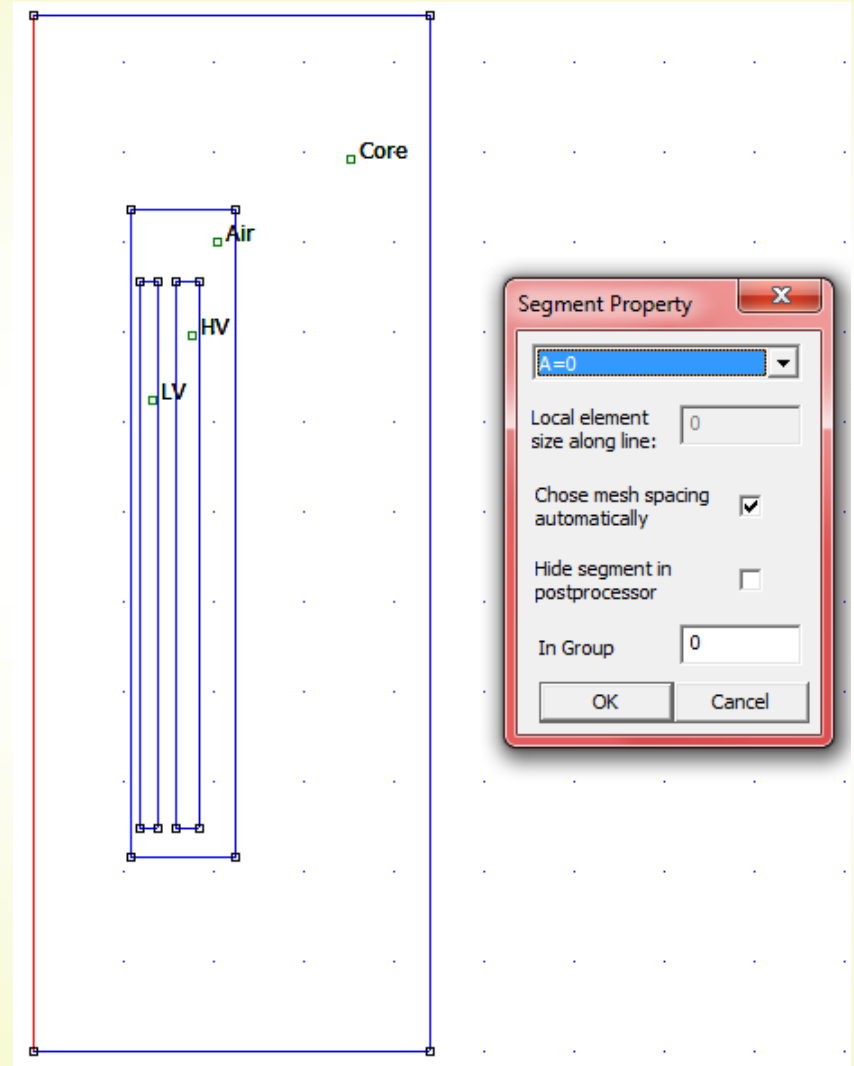
The image shows a software dialog box titled "Boundary Property". It has a red title bar with a close button (X) in the top right corner. The dialog contains several input fields and buttons. At the top, there is a "Name" field with the text "A=0" and two buttons, "OK" and "Cancel", to its right. Below this is a "BC Type" dropdown menu currently set to "Prescribed A". The main area of the dialog is divided into three sections: "Small skin depth parameters" on the left, "Mixed BC parameters" at the bottom left, and "Prescribed A parameters" on the right. The "Small skin depth parameters" section includes fields for μ , relative (0) and σ , MS/m (0). The "Mixed BC parameters" section includes fields for c_0 coefficient (0) and c_1 coefficient (0). The "Prescribed A parameters" section includes fields for A_0 (0), A_1 (0), A_2 (0), and ϕ , deg (0).

Section	Parameter	Value
Small skin depth parameters	μ , relative	0
	σ , MS/m	0
Mixed BC parameters	c_0 coefficient	0
	c_1 coefficient	0
Prescribed A parameters	A_0	0
	A_1	0
	A_2	0
	ϕ , deg	0


- Click on  (operate on block labels)-> Right click on a block (point of block becomes red)-> press the space key
- Click on 'OK' similarly for all the blocks
- Here we are defining meshing size to each material

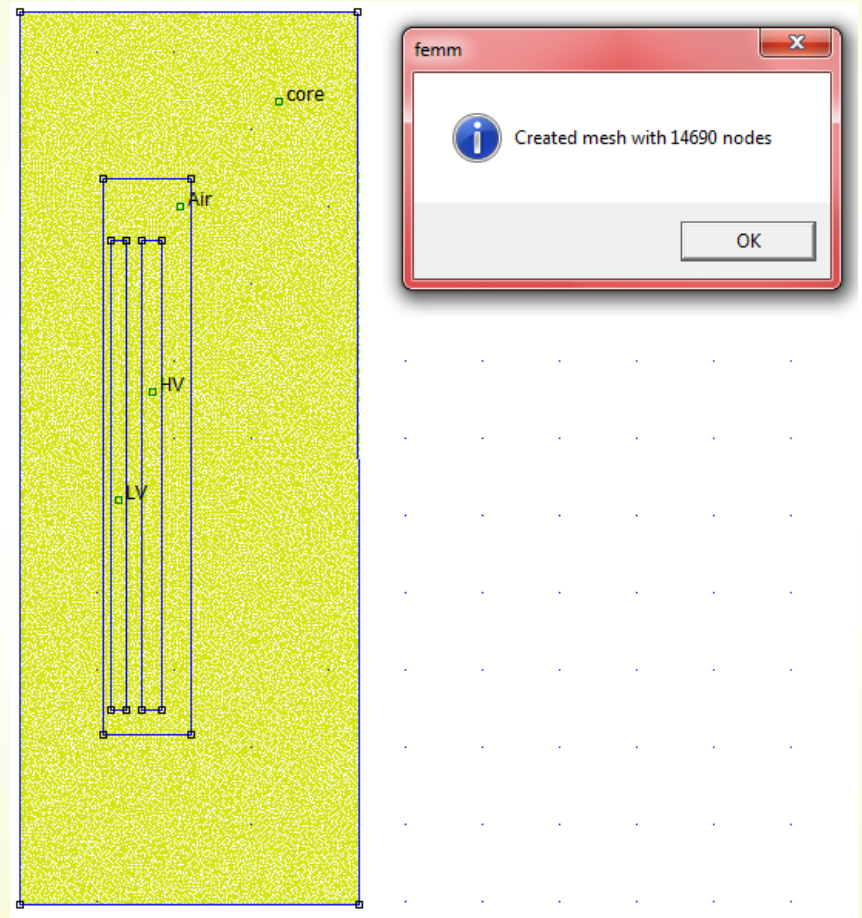


- Click on  (operate on segments)-> Right click on the one of the four outer most lines (Line becomes red)-> press space key
- Click on 'OK' similarly for three remaining lines
- In this step we are assigning boundary conditions





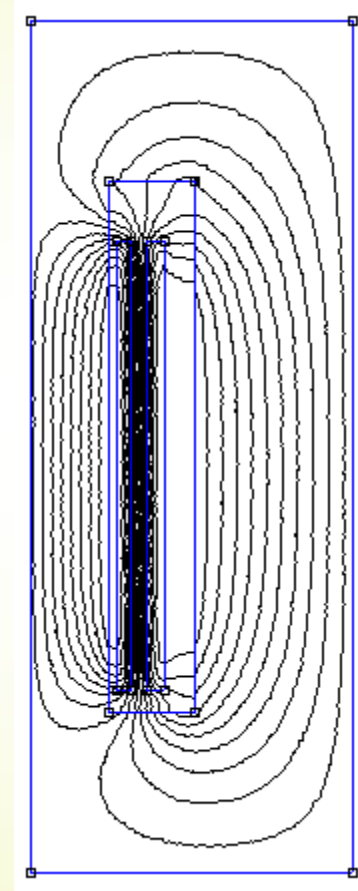
Meshing



- Click on  -> click on 'OK'
- Above button runs mesh generator
- This step creates mesh for the given geometry

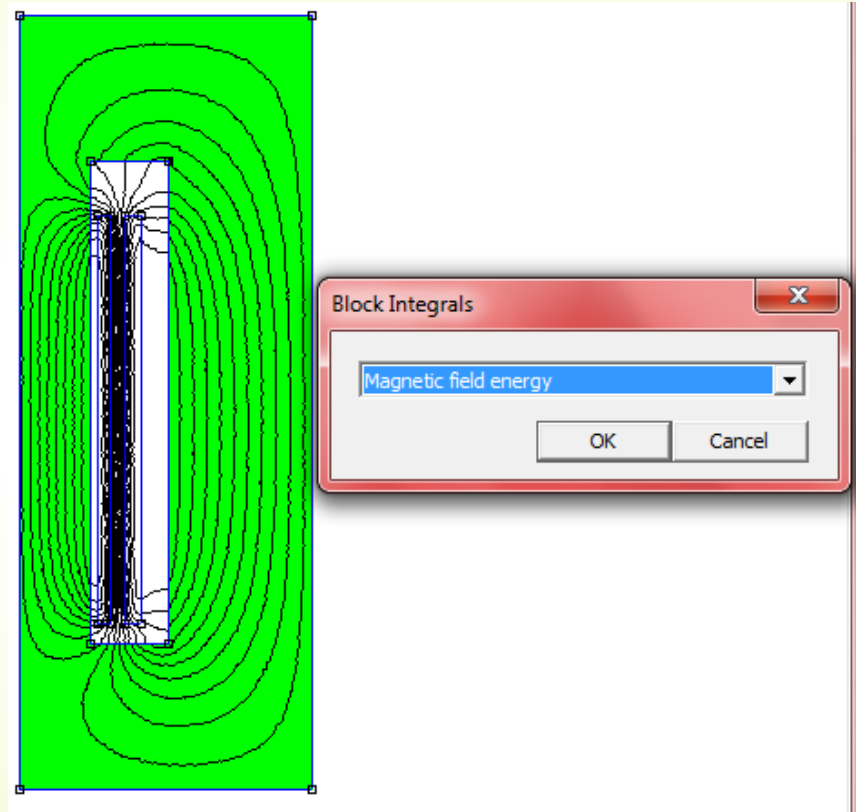


Output

- Click on ‘hand crank’ icon , it executes the solver
- Now click on . This is ‘glass’ button to display the results in post processing window



- Click on  -> Left click on one of the four areas (area becomes colored)
- Click on  ->
- Click on 'OK'
- Similarly find energy for all the areas



Results

Sr. No.	Part	Energy in Joules per meter depth
1	CORE	0.0262184
2	Air	360.794
3	LV	120.653
4	HV	147.144

Sr. No.	Part	Energy in Joules
1	CORE	$\text{Pi} \times 0.540 \times 0.0262184 = 0.0445 \text{ J}$
2	Air	$\text{Pi} \times 0.739 \times 360.794 = 837.6327 \text{ J}$
3	LV	$\text{Pi} \times 0.638 \times 120.653 = 241.8292 \text{ J}$
4	HV	$\text{Pi} \times 0.853 \times 147.144 = 394.3133 \text{ J}$
TOTAL ENERGY = 1473.8672 J		

$$\frac{1}{2} L i^2 = 1473.8672$$

$$L \text{ (referred to HV side)} = \frac{1473.8672 \times 2}{i_{\text{HV}}^2 = (137.7808104)^2} = 0.1553 \text{ Henry}$$

Analytical solution

- **Leakage inductance calculations**

$$\begin{aligned}\text{Effective area} &= [(5.2/3)*63.8+4.9*73.9+ (6.5/3)*85.3] \times \Pi \times 10^{-4} \text{ m}^2 \\ &= 657.5 \times \Pi \times 10^{-4} \text{ m}^2 = 0.2066 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Effective height} &= 152 + \{(\text{HVOD}-\text{LVID})/2\Pi\} \\ &= 152 + (91.8-58.6)/(2\Pi) \\ &= 157.3 \text{ cm} \\ &= 1.573 \text{ m}\end{aligned}$$

– (Ref: *S. V. Kulkarni and S. A. Khaparde, Transformer Engineering: Design and Practice, Marcel Dekker, Taylor & Francis Group, New York, 2004.*)

$$\begin{aligned}L (\text{referred to HV side}) &= (\mu_0 N^2 A) / HT_{\text{eff}} \\ &= (4\Pi \times 10^{-7} \times 980^2 \times 0.2066) / 1.573 \\ &= 0.158 \text{ H}\end{aligned}$$

Force on a plunger due to magnetic field in a nonlinear solenoid

Ketan Badgujar
Research Scholar
EE, IIT Bombay

Presentation Layout

Step:1



Problem Definition

Step:2



Geometry

Step:3



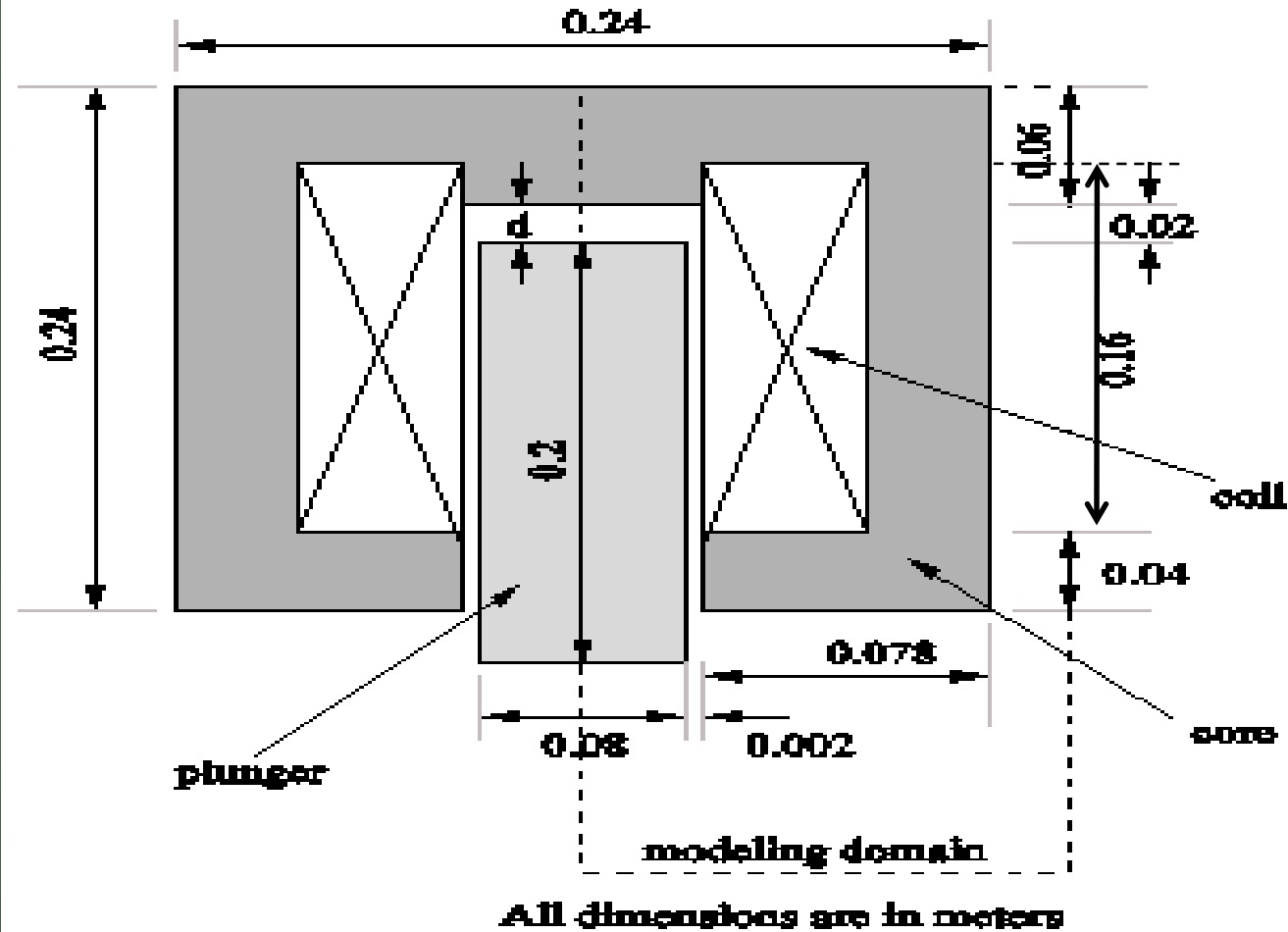
Discretization and applying boundary conditions

Step:4



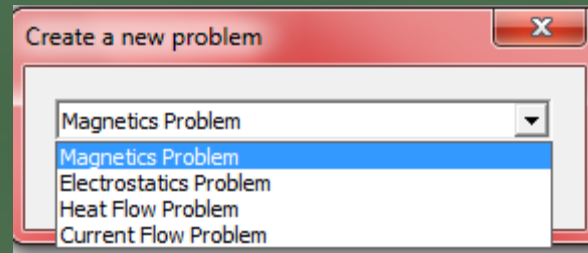
Solution and post processing


Problem Definition



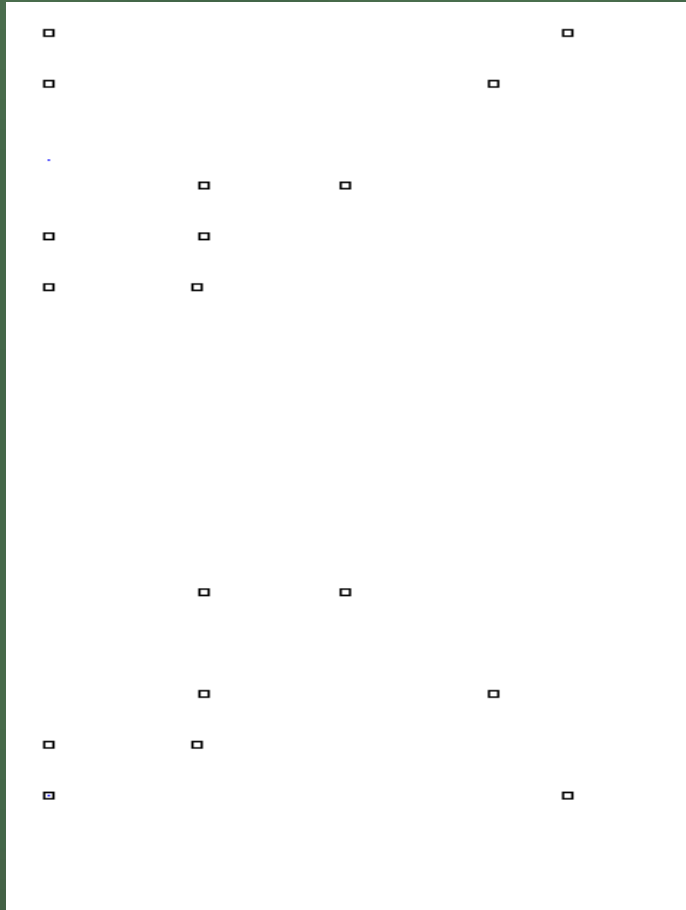
•Geometry

- File-> New-> Magnetis problem-> click on `OK`




- Draw the given transformer diagram-
 - Click on  (operate on nodes)-> left click on the screen -> right click on the node (node becomes red > press Tab key-> enter the new node location point-> click `OK` similarly for all the points

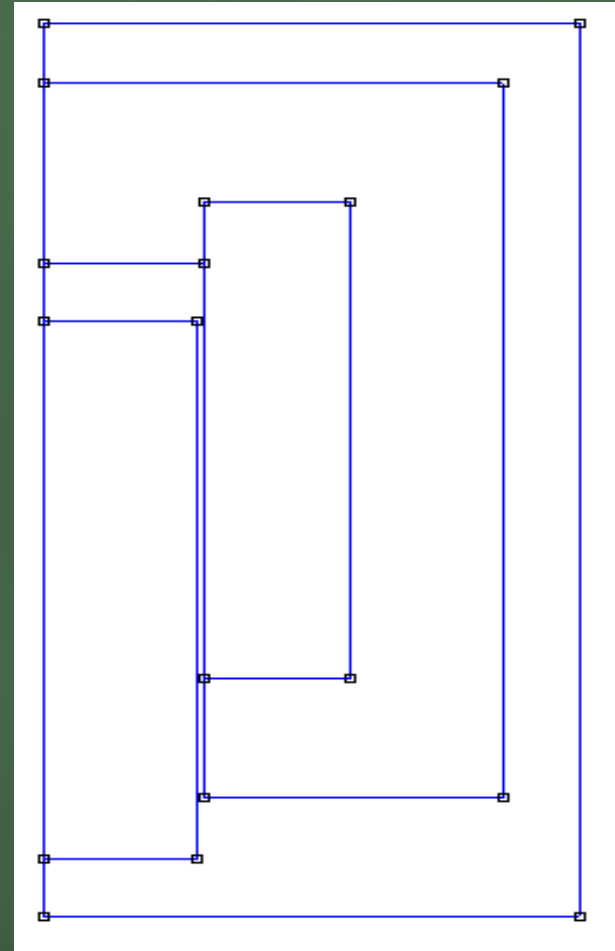
•Co-ordinates




x	y	x	y
0	0	0	0.2
0.14	0	0	0.22
0.14	0.30	0	0.28
0	0.30	0.12	0.28
0	0.02	0.08	0.24
0.042	0.04	0.042	0.24
0.12	0.04	0.042	0.22
0.042	0.08	0.04	0.2
0.08	0.08	0.04	0.02

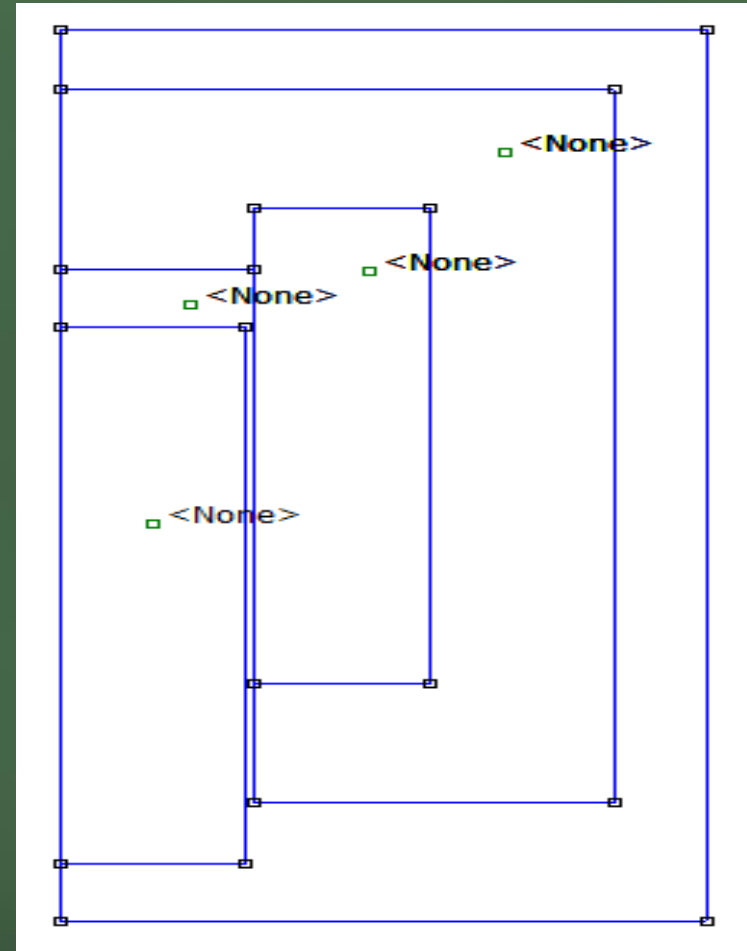
Operate on segments

- Click on  (operate on segments) -> left click on first node (node becomes red)-> left click on the second node (now both nodes are connected through a line)



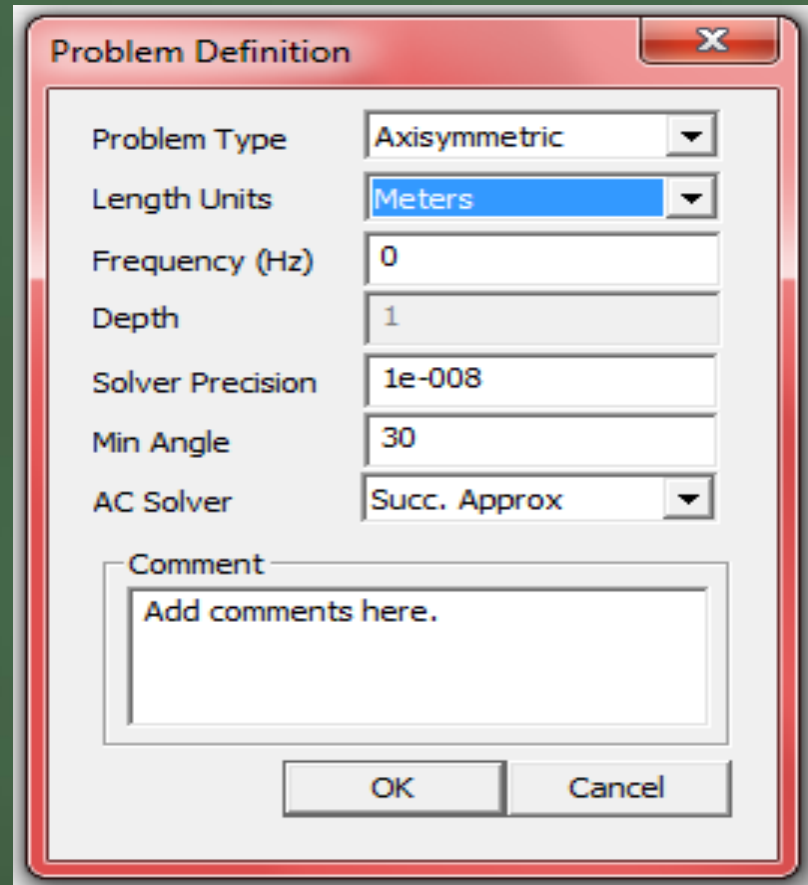
•Operate on blocks

- Click on  (operate on block labels)-> left click on the each area once



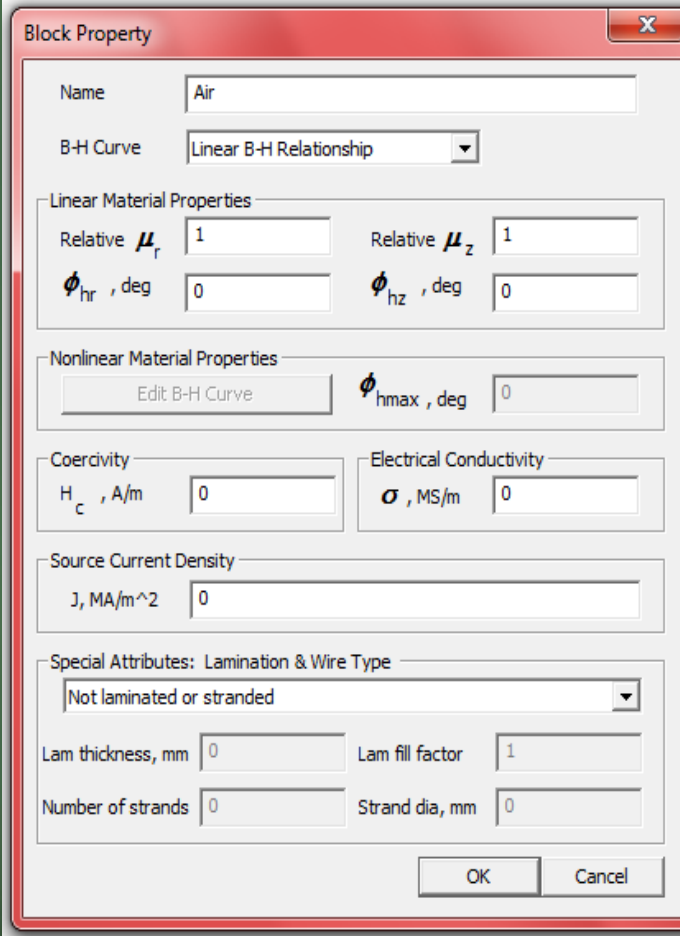
Problem type and length units

- Click on “Problem” from FEMM main menu-
- Click on ‘OK’

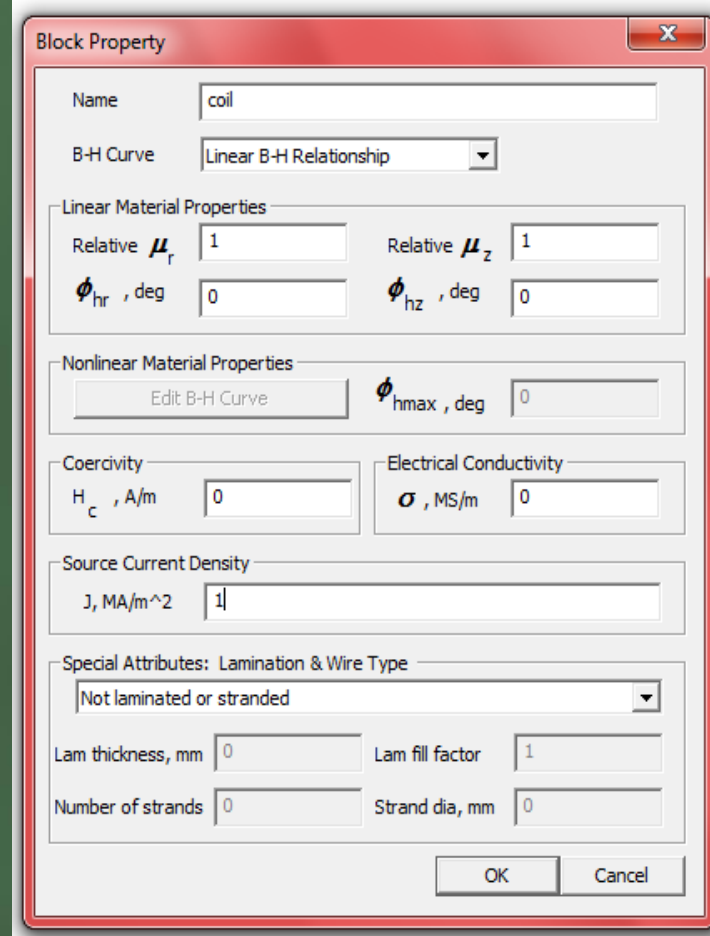


Material properties

- Click on “Properties”->”Materials”->Add property
- Set properties for air and coil as shown in figure



Block Property dialog box for Air. The Name field is set to "Air". The B-H Curve is set to "Linear B-H Relationship". The Linear Material Properties section shows Relative μ_r as 1, Relative μ_z as 1, ϕ_{hr} , deg as 0, and ϕ_{hz} , deg as 0. The Nonlinear Material Properties section has an "Edit B-H Curve" button and ϕ_{hmax} , deg as 0. The Coercivity section shows H_c , A/m as 0. The Electrical Conductivity section shows σ , MS/m as 0. The Source Current Density section shows J , MA/m² as 0. The Special Attributes: Lamination & Wire Type section shows "Not laminated or stranded". The Lam thickness, mm is 0, Lam fill factor is 1, Number of strands is 0, and Strand dia, mm is 0. The OK and Cancel buttons are at the bottom right.



Block Property dialog box for coil. The Name field is set to "coil". The B-H Curve is set to "Linear B-H Relationship". The Linear Material Properties section shows Relative μ_r as 1, Relative μ_z as 1, ϕ_{hr} , deg as 0, and ϕ_{hz} , deg as 0. The Nonlinear Material Properties section has an "Edit B-H Curve" button and ϕ_{hmax} , deg as 0. The Coercivity section shows H_c , A/m as 0. The Electrical Conductivity section shows σ , MS/m as 0. The Source Current Density section shows J , MA/m² as 1. The Special Attributes: Lamination & Wire Type section shows "Not laminated or stranded". The Lam thickness, mm is 0, Lam fill factor is 1, Number of strands is 0, and Strand dia, mm is 0. The OK and Cancel buttons are at the bottom right.

Material properties

- For, core & plunger “Properties”->”Materials”-> Add property ->B-H Curve (Nonlinear BH-Curve)->Click on “Edit B-H Curve”-> click on “OK”

Block Property

Name: core

B-H Curve: Nonlinear B-H Curve

Linear Material Properties

Relative μ_r : 1 Relative μ_z : 1

ϕ_{hr} , deg: 0 ϕ_{hz} , deg: 0

Nonlinear Material Properties

Edit B-H Curve ϕ_{hmax} , deg: 0

Coercivity

H_c , A/m: 0

Electrical Conductivity

σ , MS/m: 0

Source Current Density

J , MA/m²: 0

Special Attributes: Lamination & Wire Type

Not laminated or stranded

Lam thickness, mm: 0 Lam fill factor: 1

Number of strands: 0 Strand dia, mm: 0

OK Cancel

B-H Curve Data

B-H Curve for: core

B, Tesla	H, Amp/m
0.0	0
0.8	460
0.95	640
1.0	720
1.10	890
1.25	1280
1.40	1900
1.55	3400
1.65	6000

Plot B-H Curve

Log Plot B-H Curve

Read B-H points from text file

OK Cancel

Block Property

Name: plunger

B-H Curve: Nonlinear B-H Curve

Linear Material Properties

Relative μ_r : 1 Relative μ_z : 1

ϕ_{hr} , deg: 0 ϕ_{hz} , deg: 0

Nonlinear Material Properties

Edit B-H Curve ϕ_{hmax} , deg: 0

Coercivity

H_c , A/m: 0

Electrical Conductivity

σ , MS/m: 0

Source Current Density

J , MA/m²: 0

Special Attributes: Lamination & Wire Type

Not laminated or stranded

Lam thickness, mm: 0 Lam fill factor: 1

Number of strands: 0 Strand dia, mm: 0

OK Cancel

B-H Curve Data

B-H Curve for: plunger

B, Tesla	H, Amp/m
0.0	0
0.8	460
0.95	640
1.0	720
1.10	890
1.25	1280
1.40	1900
1.55	3400
1.65	6000

Plot B-H Curve

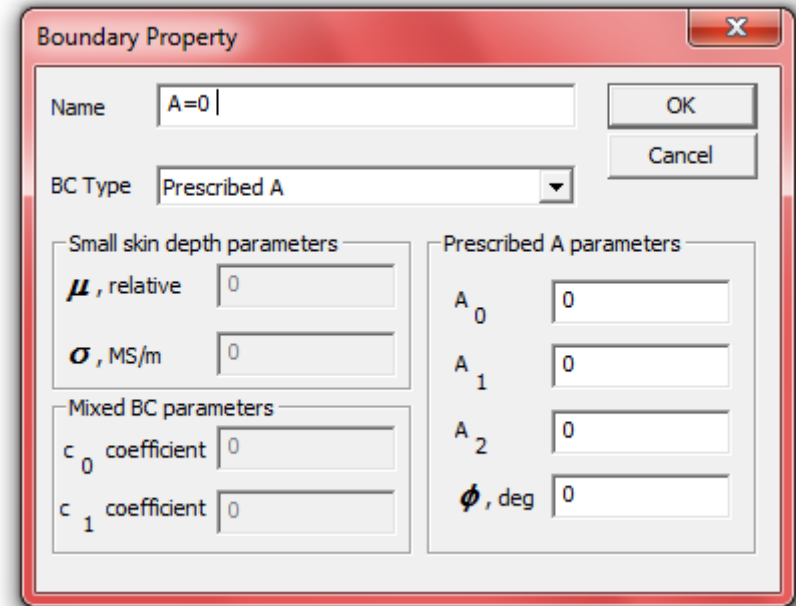
Log Plot B-H Curve

Read B-H points from text file

OK Cancel


Boundary property

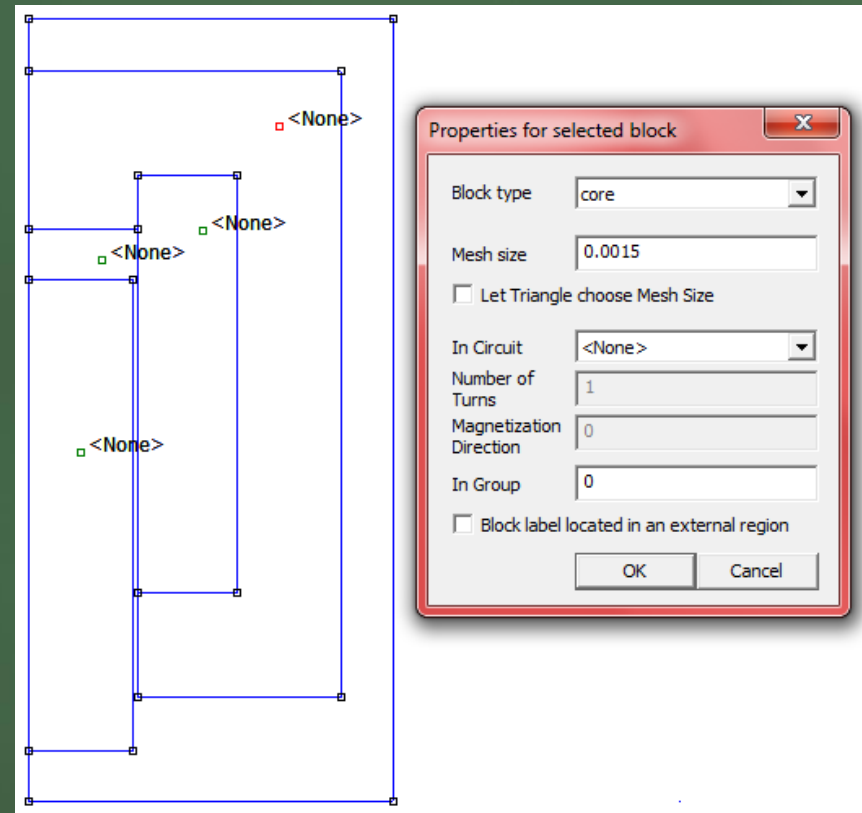
- Click on “Properties” -
>”Boundary”->Add property -
> click on ‘OK




The image shows a software dialog box titled "Boundary Property". It has a red title bar with a close button (X) in the top right corner. The dialog contains several input fields and buttons. At the top, there is a "Name" field with the text "A=0" and two buttons, "OK" and "Cancel", to its right. Below this is a "BC Type" dropdown menu currently set to "Prescribed A". The main area of the dialog is divided into three sections: "Small skin depth parameters" on the left, "Mixed BC parameters" below it, and "Prescribed A parameters" on the right. Each section contains two input fields with numerical values.

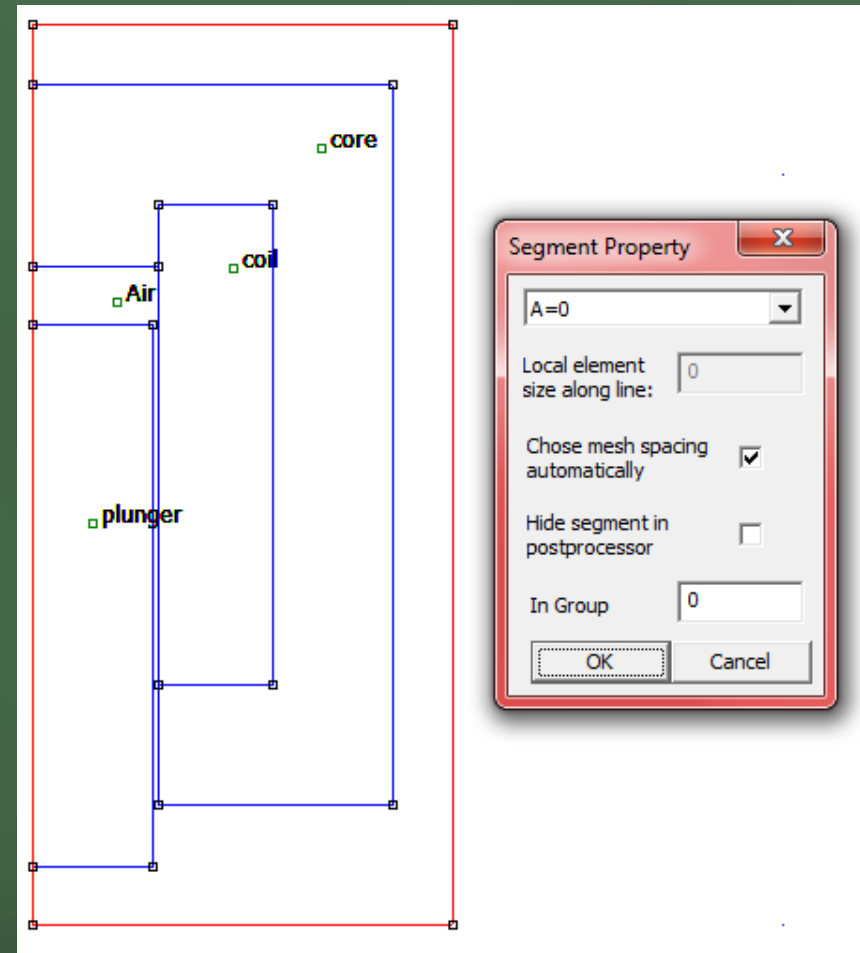
Section	Parameter	Value
Small skin depth parameters	μ , relative	0
	σ , MS/m	0
Mixed BC parameters	c_0 coefficient	0
	c_1 coefficient	0
Prescribed A parameters	A_0	0
	A_1	0
	A_2	0
	ϕ , deg	0

- Click on  (operate on block labels)-> Right click on a block (point of block becomes red)-> press the space key
- Click on 'OK' similarly for all the blocks
- Here we are defining meshing size to each material




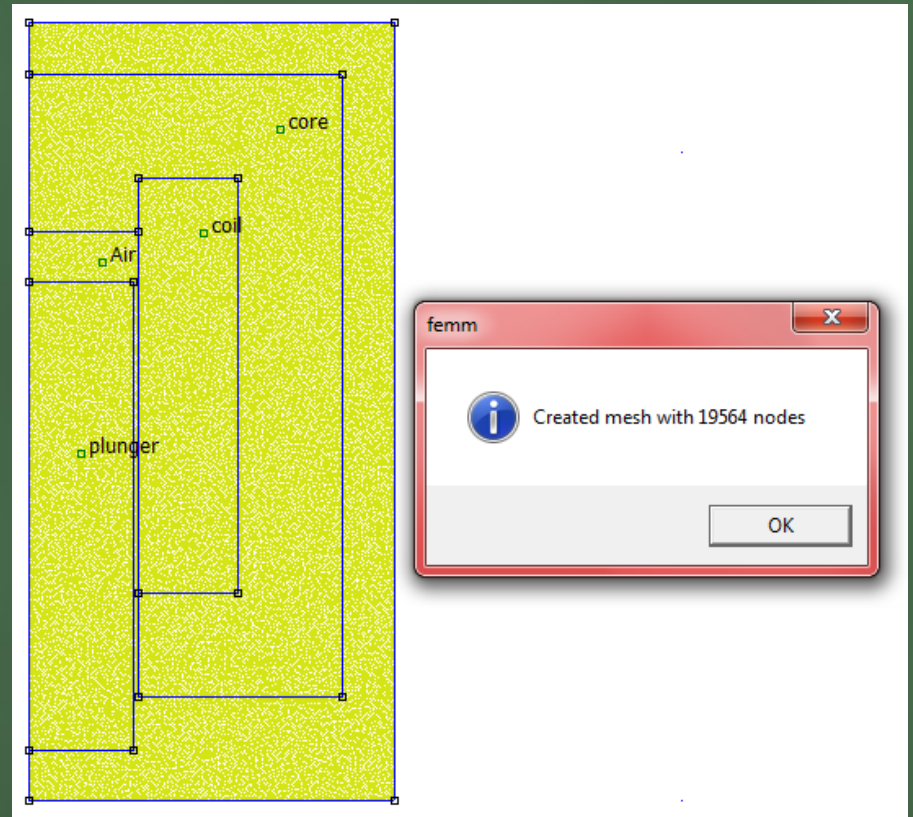
Set boundary conditions

- Click on  (operate on segments)-> Right click on the one of the four outer most lines (Line becomes red)-> press space key
- Click on 'OK' similarly for three remaining lines
- In this step we are assigning boundary conditions





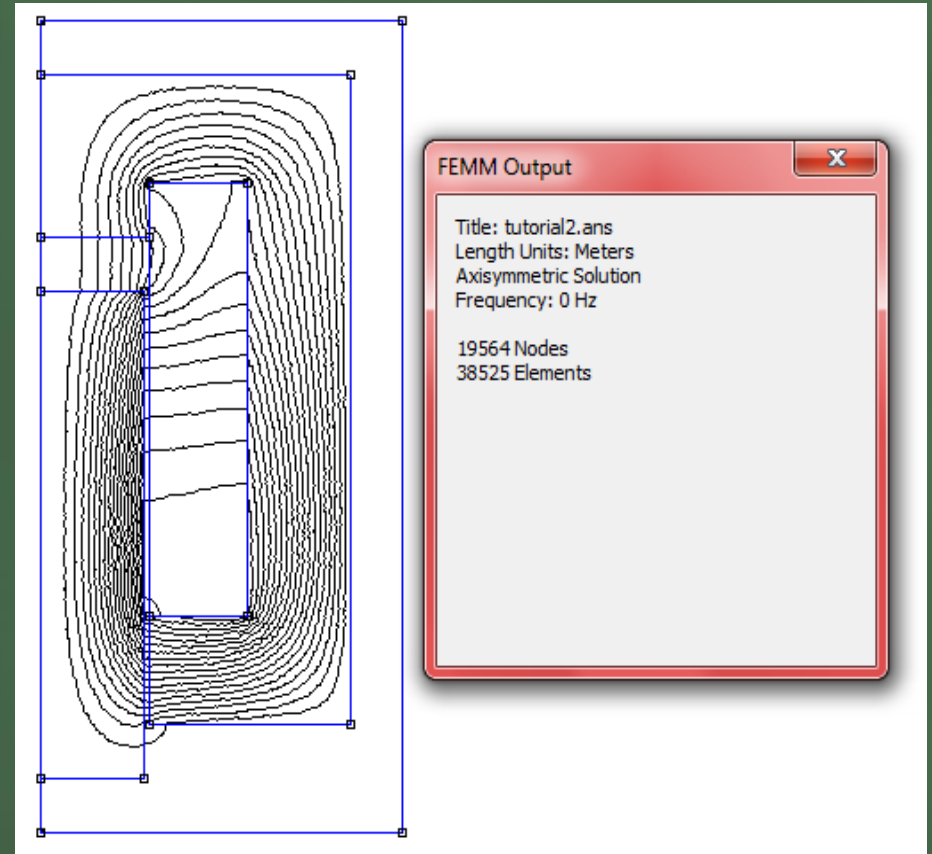
Meshing

- Click on  -> click on 'OK'
- Above button runs mesh generator
- This step creates mesh for the given geometry





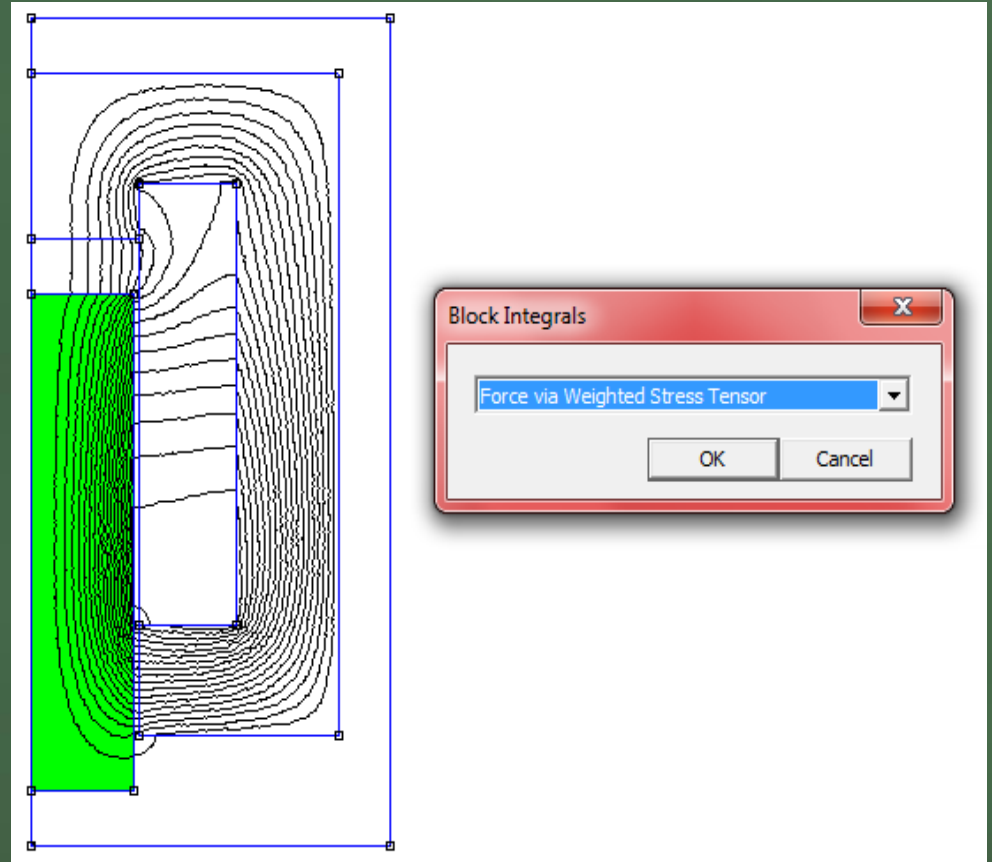
Output

- Click on 'hand crank' icon , it executes the solver
- Now click on . This is 'glass' button to display the results in post processing window

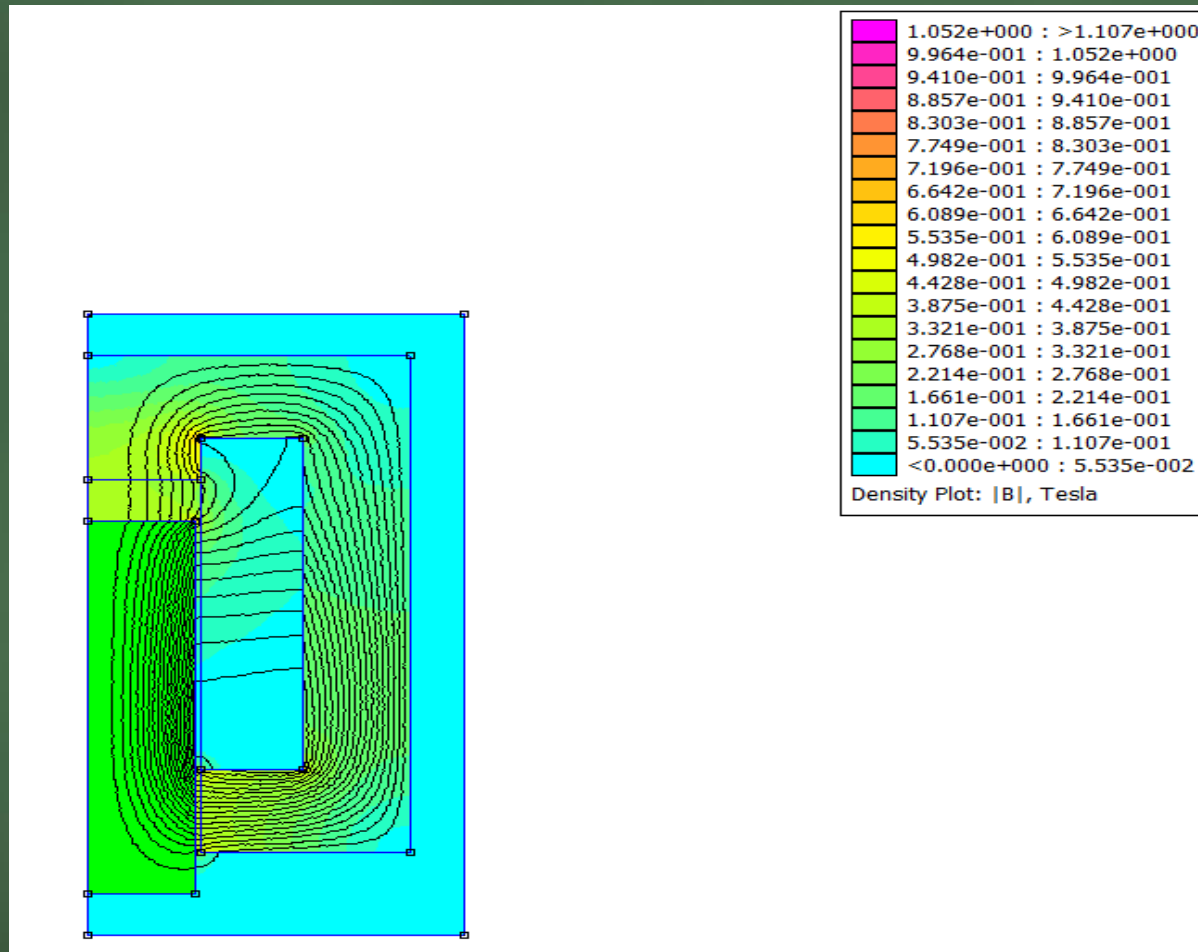


Post processing

- Click on  -> Left click on the plunger (area becomes colored)
- Click on  ->
- Click on 'OK'
- **Force = 363 N**



Density plot



Analytical solution

The core is assumed to have infinite permeability requiring no magnetizing mmf

Let, Ampere current density (ATD) = Current density (J)

$$\text{Ampere current (AT)} = 6080$$

$$H_g = AT/g = 304000$$

where, g air gap distance

$$B_g = \mu_0 H_g = 0.382$$

$$f_e = \frac{1}{2} \frac{B_g^2}{\mu_0} \times A$$

where, A=area of cross section of coil

- $f_e = 261 \text{ N}$
- The difference in numerical and analytical solution is due to fringing

Thank you