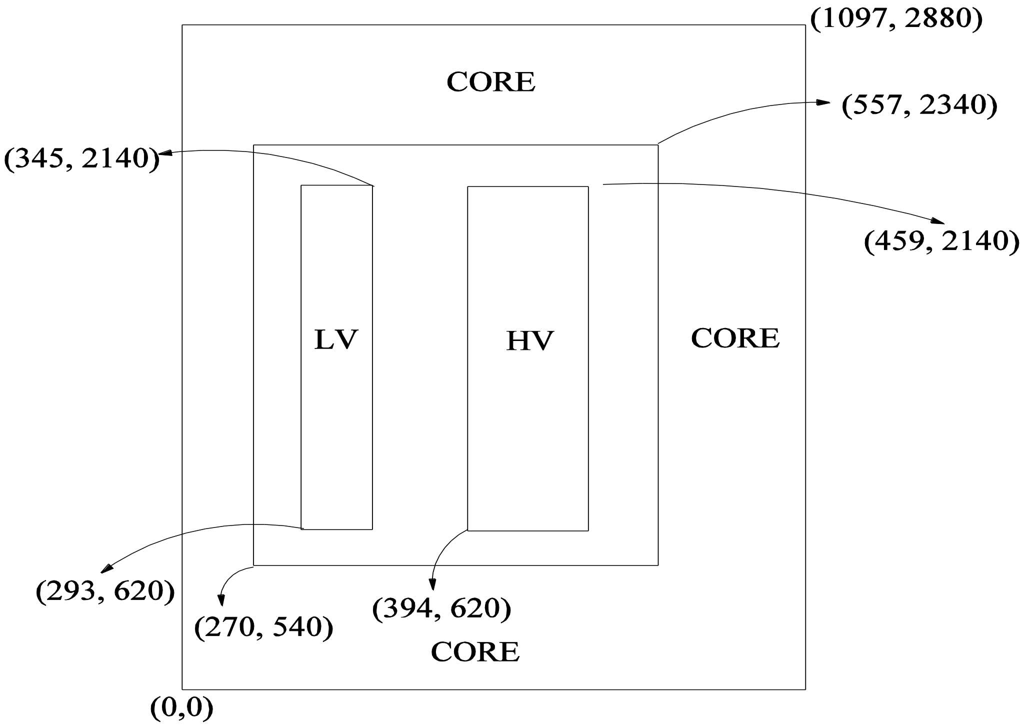
Tutorial One: Calculation of leakage inductance of transformer using FEM

Consider a transformer with the following rating:

31.5 MVA, 132 kV/33kV, Y/Δ, Ampere-turns: 135024, No. of HV turns = 980

Although it is a three-phase transformer, for calculating its leakage impedance on per-phase basis, only a single-phase is modeled as shown in the following figure. LV is inner winding surrounded by outer HV winding. For simplicity, tap winding is not considered. HV to core distance is an equivalent distance calculated by considering 2/3rd window width. This distance has insignificant effect on the leakage inductance.



Dimensions:

Core diameter	:	540 mm	
Core-LV gap	:	23 mm	
LV radial depth	:	52 mm	**LV mean diameter**: 638 mm
LV-HV gap	:	49 mm	
HV radial depth	:	65 mm	**HV mean diameter**: 853 mm

Heights of LV and HV windings : 1520 mm

The steps to simulate the above problem in any commercial FEM software are as follows. Specific commands to be used will vary for different softwares.

1. Draw the given transformer diagram as described above using preprocessor menu. The problem is solved in Cartesian system. The performance figures computed will be for a meter depth in z-direction; this approximation is valid for winding having large diameters. For example, the energy calculated in winding area will then have to be multiplied by its mean turn length to get the total energy stored in it). For smaller diameter windings, axisymmetric model should be used to get more accurate results.
2. Use appropriate finite element type from the menu.
3. Define material properties. It should be remembered that relative permeability of winding zones is 1 (since they consist of copper and insulation). Core should be assigned high permeability (being made of magnetic steel).
4. Choose small enough mesh size to get accurate results.
5. Define sources: enter current and turns information for each winding. Define exactly equal number of ampere-turns for both windings (magnetizing ampere-turns are neglected since we are interested in calculating leakage inductance; shunt branch in the equivalent circuit of the transformer, consisting of parallel combination of Rc and Xm, is neglected).
6. Define flux-parallel boundary condition (magnetic vector potential = 0) on the outermost boundaries.
7. Solve the problem as a magnetostatic one.
8. Plot the equipotential lines (magnetic flux lines in this case). Since the ampere-turns defined for both the windings are equal but with opposite signs, the net ampere-turns enclosed by the magnetic circuit is zero. Thus, there should not be a single line in the core part enclosing both the windings. You will observe a bunch of lines enclosing LV winding and the remaining ones enclosing HV winding, as shown in the figure 1 below.

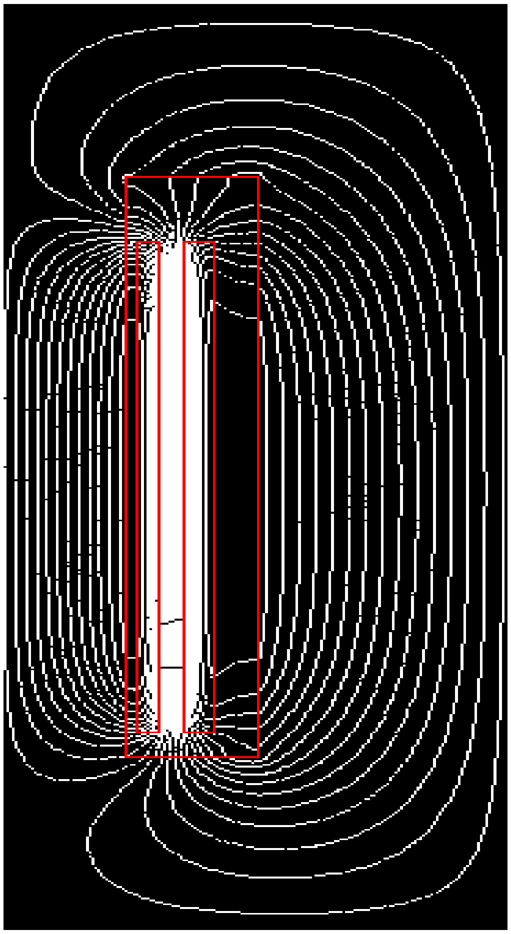


Figure 1. Magnetic vector potential plot

9. Calculate energy in all parts of the problem. The energy values obtained should be close to that given in the following table:

Sr. No.	Part	Energy in Joules per meter depth
1.	CORE	0.0391982
2.	Air	360.805
3.	LV	120.653
4.	HV	147.144

It can be seen from the table that the energy stored in the core is negligible as the flux density in it is insignificant.

It should be noted that if we want to analyze transformer in no-load state, excited winding (LV or HV) should be fed by a rated voltage source with other winding open-circuited along with definition of non-linear B-H curve for the core. In this case, rated flux density (corresponding to mutual flux) would be set up in the core, giving substantial energy in it.

10. Calculate energies in all the parts by multiplying them with corresponding mean turn lengths (pi x mean diameters) as shown in the following table (core energy is neglected).

Sr. No.	Part	Energy in Joules
1.	Air	pi × 0.739 ×360.805 = 837.7 J
2.	LV	pi × 0.638 ×120.653 = 241.8 J
3.	HV	pi × 0.853 × 147.144 = 394.3 J
TOTAL ENERGY = 1473.8 J		

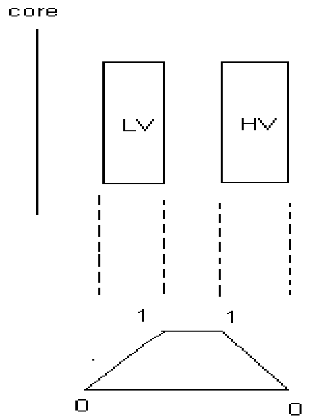
11. Finally, the inductance is calculated by the following formula.

$$\frac{1}{2} L i^2 = 1473.8$$

L (referred to HV side) = $\dfrac{1473.8 \times 2}{i_{HV}^2 = (137.78)^2} = 0.155 \text{ Henry}$

Analytical solution:-

(Ref: S. V. Kulkarni and S. A. Khaparde, Transformer Engineering: Design and Practice, Marcel Dekker, Taylor & Francis Group, New York, 2004, chapter 3).



Ampere-turn diagram

Leakage inductance calculations

$$\text{Effective area} = \left[\frac{1}{3}\ T_1 \times D_1\ +\ T_g \times D_g\ + \frac{1}{3}\ T_2 \times D_2\ \right] \times \pi$$

where, D_1, D_g and D_2 are the mean diameters and T_1, T_g and T_2 are the radial depths of LV, gap and HV respectively

$$\text{Effective area} = [(5.2/3)*63.8+4.9*73.9+(6.5/3)*85.3] \times \Pi \times 10^{-4}\ \text{m}^2$$

$$= 657.5 \times \Pi \times 10^{-4}\ \text{m}^2 = 0.2066\ \text{m}^2$$

Effective height (HT_{eff}) = actual height + {(HVOD-LVID)/2Π} cm

where, HVOD is HV winding outer diameters and LVID is LV winding inner diameter

$$= 152 + (91.8-58.6)/(2\Pi\ ) = 157.3\ \text{cm}$$

$$= 1.573\ \text{m}$$

L (referred to HV side) = $(\mu_0 N^2 A) / HT_{eff} = (4\Pi x10^{-7} \times 980^2 \times 0.2066) / 1.573 = 0.158$ H